Spacecraft Attitude Control with a Two-axis Variable Speed Control Momentum Gyro

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Abstract: CMG(Control Momentum Gyro) is a control device being used for spacecraft attitude control constructing relatively large amount of torque compared to conventional body-fixed reaction wheels. The CMG produces gyroscopic control torque by continuously varying the angular momentum vector direction with respect to the spacecraft body. The VSCMG(Variable Speed Control Momentum Gyro) has favorable advantages with variable speed to lead to better control authority as well as singularity avoidance capability. Attitude dynamics with a VSCMG mounted on a two-axis gimbal system are derived in this study. The dynamic equation may be considered as an extension of the single-axis counterpart. Also, a feedback control law design is addressed in conjunction with the dynamic equations of motion.

Keywords: Attitude Control, CMG, VSCMG, Attitude Dynamics, Attitude Maneuver, Feedback Control

1. INTRODUCTION

A device generating torque command continuously is needed to control spacecraft attitude precisely. Reaction wheels(RW) are continuous torque generators onboard most spacecraft lately.[1] RW make the best use of reaction torque provided by the wheel speed control using servo motors. A reaction wheel consists of a spinning rotor whose spin axis is fixed to the spacecraft and its speed is increased or decreased to generate reaction torque about the spin axis.

The main drawback of a reaction wheel is that there is no torque amplification effect that causes more energy consumption than CMG for a given rotation. Also, as the direction of their angular momentum is fixed, control command is restricted. Particularly, if a large torque command is demanded on, it leads to a RW saturation problem. As the moment of inertia can not be desired in most RW arbitrarily, wheel saturation is an important issue for the efficiency of spacecraft attitude control.

For those reasons, CMG(Control Momentum Gyro) is widely studied lately.[2-5] Unlike RW, A CMG contains a spinning rotor with large, constant angular momentum, whose angular momentum vector can be changed with respect to the spacecraft. That is, angular momentum vector of CMG wheel has an angular velocity component generating gyroscopic torque with respect to the body axis. Even for a small wheel angular momentum, a large control torque can be produced by enlarging the magnitude of angular velocity relatively. Such a CMG has been used in many military satellites requiring rapid attitude maneuver traditionally but is being loaded on the LEO earth observation satellites at large recently. That is, CMGs are indispensable equipments to make the body-axis aligned with a stereo camera pointing to the earth quickly and precisely. As the earth observation satellites are minimized and required precious attitude control, CMGs are becoming smaller and general gradually.

In CMGs, there are two methods to generate control torque. At first, making the angular momentum fixed and angular momentum vector variable, only momentum vector is used to generate control torque. In the second, making not only momentum vector but also angular momentum changeable, we use the VSCMG(Variable Speed Control Momentum Gyro) to produce control torque. On the other hand, there are one-axis and two-axis gimbal systems according to the method of wheel alignment. One-axis gimbal CMGs require a minimum of four units in pyramid configuration for full 3-axis attitude control even when one of those CMGs is out of order[5]. The most significant drawback with SGCMGs is the problem of singularities. This is the condition for which no torque can be produced for a certain set of gimbal angles as representing the control torque in three body axis using 4 wheels[6-7] .

Generally, VSCMGs or the CMG in two-axis gimbal system is robust about the problem of singularities. The technique to solve singularity problem form is a very important factor in operating the CMGs system. Various control laws have been presented using CMGs. CMGs system are mainly used for the large angle attitude maneuver laws according to the original characteristics.[4-7] In case of external disturbances, control laws using CMGs is considered.[13] Also, control laws with CMGs have been investigated in mode of storing energy.[14,15] Mostly CMGs model in which angular momentum is fixed in one-axis gimbal is chosen. Control laws are expressed on the quaternion parameters. But recently, Attitude control laws using the MRP(Modified Rodrigues Parameters) are studied.

Attitude dynamics with a VSCMG mounted on a two-axis gimbal system are derived in this study. The dynamic equations may be considered as an extension of the single-axis counterpart. Also, a feedback control law design is addressed in conjunction with the dynamic equations of motion.[16] As the degree of freedom is increased at the two-axis gimbal system in comparison with the single-axis gimbal system, there is a difference in the attitude dynamic equation. As a result, a different control law is derived. This study is based on the reference [16] and presents the generalized dynamic equation and gimbal steering laws.

2. EQUATIONS OF MOTION

2.1 Two-axis Gimbal CMG System

Before deriving equations of motion, let us consider the principal of two-axis gimbal system briefly. It is represented as the configuration of a CMG attached to a two-axis gimbal fixed in the body-axis of satellite as Fig. 1. It is assumed that the gimbal has a constant moment of inertia.

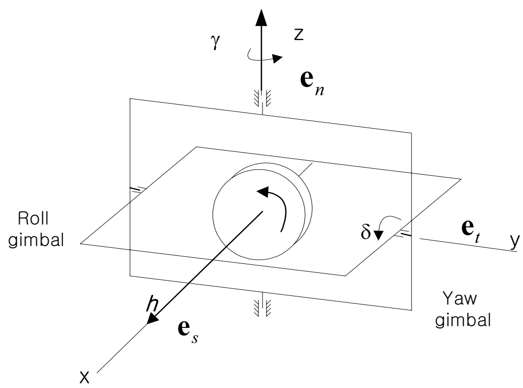

Fig. 1 A CMG attached to a two-axis gimbal

The wheel angular momentum($\mathbf{H}_W$) can be changed by adjusting the wheel speed. Also, gyroscopic torque can be generated by using gimbal rate(δ, γ). Actually, gyroscopic torque is generated from the angular velocity motion($\dot{\delta}, \dot{\gamma}$) about the gimbal-axis with respect to body-axis. { $\mathbf{e}_t, \mathbf{e}_n$ } is the unit vectors attached to gimbal frames and $\mathbf{e}_s$ is the unit vector according with wheel spin-axis. In this way, equations of motion are represented with the unit vector attached to gimbal axis and wheel-axis in which torque is given by the motion of wheel or gimbal. For reference, only γ -axis rotation may be generated for the case of the one-axis gimbal.

2.2 Equations of Motion

To derive equations of motion, the whole system is divided into satellite body, gimbal and wheel. The total angular momentum of a satellite with a CMG cluster can be expressed in the satellite body frame as[16-18]

$$\mathbf{H} = \mathbf{H}_B + \mathbf{H}_W + \mathbf{H}_G \tag{1}$$

where $\mathbf{H}_B$ is the angular momentum vector of a satellite, $\mathbf{H}_G$ is the angular momentum vector of the gimbal, and $\mathbf{H}_W$ denotes the angular momentum vector of the wheel.

First, the angular momentum vector of the gimbal is[16]

$$\mathbf{H}_G = [I_G]\boldsymbol{\omega}_{G/N} \tag{2}$$

where $\boldsymbol{\omega}_{G/N}$ represents the angular velocity vector of the gimbal with respect to the inertia frame and can be expressed as

$$\boldsymbol{\omega}_{G/N} = \boldsymbol{\omega}_{G/B} + \boldsymbol{\omega}_{B/N} \tag{3}$$

Therefore, $\mathbf{H}_G$ is

$$\mathbf{H}_G = [I_G]^B \boldsymbol{\omega}_{G/B} + [I_G]^B \boldsymbol{\omega}_{B/N} \tag{4}$$

where $[I_G]^B$ is the moment of inertia of the gimbal with respect to the inertia frame and the time-variant function with respect to the body frame. If $\{I_{Gs}, I_{Gt}, I_{Gn}\}$ is written as the principal inertia about the gimbal-axis, then $[I_G]^B$ can be defined as.

$$\begin{aligned} [I_G]^B &= [BG]^T [I_G][BG] \\ &= I_{Gs}\mathbf{e}_s\mathbf{e}_s^T + I_{Gt}\mathbf{e}_t\mathbf{e}_t^T + I_{Gn}\mathbf{e}_n\mathbf{e}_n^T \end{aligned} \tag{5}$$

where the matrix $[BG]$ represents the direction cosine matrix between gimbal-axis and body-axis. As a result, the angular momentum vector of the gimbal is

$$\mathbf{H}_G = (I_{Gs}\mathbf{e}_s\mathbf{e}_s^T + I_{Gt}\mathbf{e}_t\mathbf{e}_t^T + I_{Gn}\mathbf{e}_n\mathbf{e}_n^T)\boldsymbol{\omega}_{B/N} + [I_G]\boldsymbol{\omega}_{G/B} \tag{6}$$

The components of projected angular velocity vector about the body-axis into the gimbal-axis is defined as

$$\begin{aligned} \omega_s &= \mathbf{e}_s^T \boldsymbol{\omega}_{B/N} \\ \omega_t &= \mathbf{e}_t^T \boldsymbol{\omega}_{B/N} \\ \omega_n &= \mathbf{e}_n^T \boldsymbol{\omega}_{B/N} \end{aligned} \tag{7}$$

And, if the angular motions(δ, γ) of the gimbal with respect to the body frame is considered, then Eq. (6) implies

$$\mathbf{H}_G = I_{Gs}\omega_s\mathbf{e}_s + I_{Gt}\left(\omega_t + \dot{\delta}\right)\mathbf{e}_t + I_{Gn}\left(\omega_n + \dot{\gamma}\right)\mathbf{e}_n \tag{8}$$

Meanwhile, the angular momentum vector of the reaction wheel is

$$\mathbf{H}_W = [I_W]\boldsymbol{\omega}_{W/N} \tag{9}$$

By applying the similar method of the equation of gimbal motion to Eq. (9), we can derive the following:

$$\mathbf{H}_W = I_{Ws}\left(\omega_s + \Omega\right)\mathbf{e}_s + I_{Wt}\left(\omega_t + \dot{\delta}\right)\mathbf{e}_t + I_{Wt}\left(\omega_n + \dot{\gamma}\right)\mathbf{e}_n \tag{10}$$

In case of the reaction wheel, assume that the components of the inertia perpendicular to the wheel spin axis are the same because of the symmetrical structure geometrically.

From now on, let the body angular velocity vector with respect to the inertia frame, $\boldsymbol{\omega}_{B/N}$ as $\boldsymbol{\omega}$ for the sake of convenience. By the way, the angular momentum vector of a satellite body is

$$\mathbf{H}_B = [I_B]\boldsymbol{\omega}_{B/N} = [I_B]\boldsymbol{\omega} \tag{11}$$

where $[I_B]$ is the moment of inertia matrix of a satellite itself except for the gimbal and the wheel.

Using the angular momentum vectors derived up to now, the total equation of rotational motion by Euler's formula is

$$\dot{\mathbf{H}} = \mathbf{L} \tag{12}$$

where $\mathbf{L}$ is an external torque.[17] To take the time derivative of the total angular momentum of the system, the time derivative of the unit vector with respect to the gimbal axis has to be derived. Because the angular momentum vector is defined about the gimbal axis and gimbal axis rotates about the body axis continually. They are then calculated as

$$\begin{aligned} \dot{\mathbf{e}}_s &= -\left(\dot{\delta} + \omega_t\right)\mathbf{e}_n + \left(\dot{\gamma} + \omega_n\right)\mathbf{e}_t \\ \dot{\mathbf{e}}_t &= -\left(\dot{\gamma} - \omega_n\right)\mathbf{e}_s + \omega_s \mathbf{e}_n \\ \dot{\mathbf{e}}_n &= \left(\dot{\delta} + \omega_n\right)\mathbf{e}_s - \omega_s \mathbf{e}_t \end{aligned} \tag{13}$$

Also, the time derivative of the gimbal axis components of the angular velocity about the body axis can be derived as

$$\begin{aligned} \dot{\omega}_s &= \dot{\mathbf{e}}_s^T \boldsymbol{\omega} + \mathbf{e}_s^T \dot{\boldsymbol{\omega}} \\ &= \left(\dot{\gamma}\omega_t - \dot{\delta}\omega_n\right) + \mathbf{e}_s^T \dot{\boldsymbol{\omega}} \\ \dot{\omega}_t &= \dot{\mathbf{e}}_t^T \boldsymbol{\omega} + \mathbf{e}_t^T \dot{\boldsymbol{\omega}} = -\dot{\gamma}\omega_s + \mathbf{e}_t^T \dot{\boldsymbol{\omega}} \\ \dot{\omega}_n &= \dot{\mathbf{e}}_n^T \boldsymbol{\omega} + \mathbf{e}_n^T \dot{\boldsymbol{\omega}} = \dot{\delta}\omega_s + \mathbf{e}_n^T \dot{\boldsymbol{\omega}} \end{aligned} \tag{14}$$

Using the relation equation derived till now, the angular momentum vector of the wheel can be shown to be

$$\begin{aligned} \dot{\mathbf{H}}_W &= \left[I_{Ws}\left(\dot{\Omega} + \mathbf{e}_s^T \dot{\boldsymbol{\omega}} + \dot{\gamma}\omega_t - \dot{\delta}\omega_n\right)\right]\mathbf{e}_s \\ &+ [I_{Ws}\left(\dot{\gamma}\left(\omega_s + \Omega\right) + \Omega\omega_n\right) + \left(I_{Ws} - I_{Wt}\right)\omega_s\omega_n \\ &- 2I_{Wt}\dot{\gamma}\omega_s + I_{Wt}\left(\mathbf{e}_t^T \dot{\boldsymbol{\omega}} + \ddot{\delta}\right)]\mathbf{e}_t \\ &+ [I_{Wt}\left(\ddot{\gamma} + \mathbf{e}_n^T \dot{\boldsymbol{\omega}}\right) + 2I_{Wt}\dot{\delta}\omega_s - I_{Ws}\left(\omega_s + \Omega\right)\dot{\delta} \\ &+ \left(I_{Wt} - I_{Ws}\right)\omega_t\omega_s - I_{Ws}\Omega\omega_t]\mathbf{e}_n \end{aligned} \tag{15}$$

To operate the wheel, a torque input should be provided. Because the wheel spin axis is the same with the axis $\mathbf{e}_s$, torque input is imposed into this direction by using a servo motor. As a result, the variance of the angular momentum in the same axis comes into existence. Therefore, using the components of the axis e_s in Eq. (15), a torque equilibrium equation can be expressed as

$$u_s = I_{Ws}\left(\dot{\Omega} + \mathbf{e}_s^T \dot{\boldsymbol{\omega}} + \dot{\gamma}\omega_t - \dot{\delta}\omega_n\right) \tag{16}$$

where u_s is the torque input into the wheel spin axis. To acquire the time derivative of the angular momentum in a gimbal system, the angular momentum vector defined in Eq. (8) must be used. In consequence of that, one obtains

$$\begin{aligned} \dot{\mathbf{H}}_G &= I_{Gs}\dot{\omega}_s \mathbf{e}_s + I_{Gt}\left(\dot{\omega}_t + \ddot{\delta}\right)\mathbf{e}_t + I_{Gn}\left(\dot{\omega}_n + \ddot{\gamma}\right)\mathbf{e}_n \\ &+ I_{Gs}\omega_s \dot{\mathbf{e}}_s + I_{Gt}\left(\omega_t + \dot{\delta}\right)\dot{\mathbf{e}}_t + I_{Gn}\left(\omega_n + \dot{\gamma}\right)\dot{\mathbf{e}}_n \end{aligned} \tag{17}$$

If the time derivative of the gimbal axis unit vector and of body angular velocity components with respect to the gimbal axis are handled like the case of the wheel, the time derivative of the angular momentum vector in a gimbal system becomes

$$\begin{aligned} \dot{\mathbf{H}}_G &= [\left(I_{Gs} - I_{Gt} + I_{Gn}\right)\dot{\gamma}\omega_t + \left(I_{Gn} - I_{Gt}\right)\omega_t\omega_n \\ &+ \left(I_{Gn} - I_{Gt}\right)\left(\dot{\delta}\omega_n + \dot{\delta}\dot{\gamma}\right) - I_{Gs}\dot{\delta}\omega_n + I_{Gs}\mathbf{e}_s^T \dot{\boldsymbol{\omega}}]\mathbf{e}_s \\ &+ [\left(I_{Gs} - I_{Gt} - I_{Gn}\right)\dot{\gamma}\omega_s + I_{Gt}\mathbf{e}_t^T \dot{\boldsymbol{\omega}} \\ &+ \left(I_{Gs} - I_{Gn}\right)\omega_n\omega_s + I_{Gt}\ddot{\delta}]\mathbf{e}_t \\ &+ [I_{Gn}\left(\dot{\delta}\omega_s + \ddot{\gamma} + \mathbf{e}_n^T \dot{\boldsymbol{\omega}}\right) \\ &+ \left(I_{Gt} - I_{Gs}\right)\omega_s\omega_t + \left(I_{Gt} - I_{Gs}\right)\dot{\delta}\omega_s]\mathbf{e}_n \end{aligned} \tag{18}$$

The governing equations for the gimbal system and wheel angular momentum vector have been derived by this time. In the one-axis gimbal system of reference [16], the equation is defined in term of γ. But in the two-axis gimbal system, the gimbal motion variable, δ is added to the equation. To simplify the equation, the sum of the moment of inertia about the wheel and that about the gimbal is defined as

$$\begin{aligned} [I] &= [I_G] + [I_W] \\ &= \begin{bmatrix} J_s & 0 & 0 \\ 0 & J_t & 0 \\ 0 & 0 & J_n \end{bmatrix} \end{aligned} \tag{19}$$

In the two-axis gimbal system, the variance of the angular momentum about the wheel and the gimbal arises from the torque produced. That is, a gimbal torque maintains equilibrium with the variance of the angular momentum in a wheel and gimbal system. This equation takes the form

$$\dot{\mathbf{H}}_G + \dot{\mathbf{H}}_W = \mathbf{L}_G \tag{20}$$

where L_G represents the torque applied to the gimbal. An gimbal torque is given to the axis e_t, e_n. Therefore, using the time derivative of the angular momentum vector induced from Eqs. (15) and (16), the torque equilibrium equations can be rewritten

$$\begin{aligned} u_g^t &= J_t\left(\ddot{\delta} + \mathbf{e}_t^T \dot{\boldsymbol{\omega}}\right) + \left(J_s - J_n\right)\omega_s\omega_n \\ &+ \left(J_s - J_t - J_n\right)\dot{\gamma}\omega_s + I_{Ws}\left(\dot{\gamma} + \omega_n\right)\Omega \end{aligned} \tag{21}$$

$$\begin{aligned} u_g^n &= J_n\left(\ddot{\gamma} + \mathbf{e}_n^T \dot{\boldsymbol{\omega}}\right) + \left(J_t - J_s\right)\omega_s\omega_t \\ &+ \left(J_n + J_t - J_s\right)\dot{\delta}\omega_s - I_{Ws}\left(\dot{\delta} + \omega_t\right)\Omega + I_{Wt}\dot{\delta}\omega_s \end{aligned} \tag{22}$$

where u_g^t, u_g^n are the torque inputs into the gimbal axis, e_t, e_n, respectively.

From the time derivative of the angular momentum vectors derived from Eqs. (11), (15), and (17) and the Euler equation derived from Eq. (12), the total equation of the rotational motion is expressed in a compact form as

$$\begin{aligned}&[I]\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}\times[I]\boldsymbol{\omega}\\&=-\mathbf{e}_s[J_s\dot{\gamma}\omega_t+I_{Ws}\left(\dot{\Omega}-\dot{\delta}\omega_n\right)-\left(J_t-J_g\right)\omega_t\dot{\gamma}\\&\quad+\left(I_{Gn}-I_{Gt}-I_{Gs}\right)\dot{\delta}\omega_n+\left(I_{Gn}-I_{Gt}\right)\dot{\delta}\dot{\gamma}]\\&\quad-\mathbf{e}_t[J_s\dot{\gamma}\omega_s+I_{Ws}\left(\dot{\gamma}+\omega_n\right)\Omega-\left(J_t+J_n\right)\dot{\gamma}\omega_s+J_t\ddot{\delta}]\\&\quad-\mathbf{e}_n\left[J_n\ddot{\gamma}-I_{Ws}\Omega\omega_t-J_s\dot{\delta}\omega_s+\left(J_t+J_n\right)\dot{\delta}\omega_s\right]+\mathbf{L}\end{aligned}\tag{23}$$

A new parameter, $[I]\left(=[I_B]+[J]\right)$ is the inertia matrix of the total system containing the satellite body, gimbal, and wheel. Meanwhile, the inertia of the wheel and gimbal $[J]$ is changed with respect to the body frame continuatively. Therefore, using Eq. (5), we can rewrite such that[16]

$$\begin{aligned}[I]&=[I_B]+[J]\\&=[I_B]+J_s\mathbf{e}_s\mathbf{e}_s^T+J_t\mathbf{e}_t\mathbf{e}_t^T+J_n\mathbf{e}_n\mathbf{e}_n^T\end{aligned}\tag{24}$$

As shown in Eq. (23), the three-axis's attitude maneuver is possible by varying the two-axis motion of the gimbal($\dot{\delta},\dot{\gamma}$) and the angular momentum change of the wheel($I_{Ws}\dot{\Omega}$). In the case of an one-axis gimbal system studied conventionally, the degree of freedom of the gimbal is restricted within the term, γ . So, Eq. (23) can be expressed as

$$\begin{aligned}&[I]\dot{\boldsymbol{\omega}}+\boldsymbol{\omega}\times[I]\boldsymbol{\omega}\\&=-\mathbf{e}_s\left[J_s\dot{\gamma}\omega_t+I_{Ws}\dot{\Omega}-\left(J_t-J_g\right)\omega_t\dot{\gamma}\right]\\&\quad-\mathbf{e}_t\left[(J_s\omega_s+I_{Ws}\Omega)\dot{\gamma}-\left(J_t+J_n\right)\dot{\gamma}\omega_s+I_{Ws}\Omega\omega_n\right]\\&\quad-\mathbf{e}_n\left(J_n\ddot{\gamma}-I_{Ws}\Omega\omega_t\right)+\mathbf{L}\end{aligned}\tag{25}$$

This equation is equal to the equation of the reference [16] completely. Therefore, one can say that the equation about a two-axis gimbal system derived in this study is the general equation. Also, if we consider that the wheel angular velocity is invariant in Eq. (23), torques are generated about two-axis by the motion of the gimbal.

3. ATTITUDE CONTROL LAW

In this section, we perform the design of a controller using the equations of motion derived in the previous section. In the design of the control law about the satellite attitude, there are two methods in general- linear and non-linear. In this study a non-linear control law is used like it is. To design a control law, we describe the attitude dynamics shown for the momentum equilibrium during the attitude maneuver, as well as the attitude kinematics. First, as the variable to describe the attitude kinematics, we use the so-called MRPs(Modified Rodrigues Parameter) which has been used in the study of satellite attitude control recently. MRPs' vectors, $\boldsymbol{\sigma}$ are defined as

$$\sigma_i=\frac{\beta_i}{1+\beta_0}\quad i=1,2,3\tag{26}$$

where β_i are the Euler parameters. Also, the MRPs are defined in terms of the Euler principal unit vector $\mathbf{l}$ and angle ϕ by

$$\boldsymbol{\sigma}=\tan\frac{\phi}{4}\mathbf{l}\tag{27}$$

And MRPs and the angular velocity of a satellite body satisfy the differential equation given by

$$\dot{\boldsymbol{\sigma}}=\frac{1}{4}\Gamma(\boldsymbol{\sigma})\begin{pmatrix}\omega_1\\\omega_2\\\omega_3\end{pmatrix}\tag{28}$$

where matrix $\Gamma(\boldsymbol{\sigma})$ is

$$\Gamma(\boldsymbol{\sigma})=\begin{bmatrix}1-\sigma^2+2\sigma_1^2 & 2(\sigma_1\sigma_2-\sigma_3) & 2(\sigma_1\sigma_3+\sigma_2)\\2(\sigma_2\sigma_1+\sigma_3) & 1-\sigma^2+2\sigma_2^2 & 2(\sigma_2\sigma_3-\sigma_1)\\2(\sigma_3\sigma_1-\sigma_2) & 2(\sigma_3\sigma_2+\sigma_1) & \left(1-\sigma^2+2\sigma_3^2\right)\end{bmatrix}$$

Eq. (27) can be transformed in type of vector to

$$\begin{aligned}\dot{\boldsymbol{\sigma}}&=\frac{1}{4}\left[\left(1-\boldsymbol{\sigma}^2\right)[I_{3\times3}]+2[\tilde{\boldsymbol{\sigma}}]+2\boldsymbol{\sigma}\boldsymbol{\sigma}^T\right]\boldsymbol{\omega}\\&=\frac{1}{4}\left[\Gamma(\boldsymbol{\sigma})\right]\boldsymbol{\omega}\end{aligned}\tag{29}$$

Matrix $\left[\Gamma(\boldsymbol{\sigma})\right]$ satisfies the relation equation as

$$[\Gamma]^{-1}=\frac{1}{\left(1+\boldsymbol{\sigma}^2\right)^2}[\Gamma]^T\tag{30}$$

Therefore, let inverse transform Eq. (28) into

$$\boldsymbol{\omega}=\frac{4}{\left(1+\boldsymbol{\sigma}^2\right)^2}[\Gamma]^T\dot{\boldsymbol{\sigma}}\tag{31}$$

So far, attitude dynamics and kinematics have been derived in terms of MRPs. Next, error parameters about the body angular velocity vector are introduced to design a controller[15,16].

$$\Delta\boldsymbol{\omega}=\boldsymbol{\omega}-\boldsymbol{\omega}_r\tag{32}$$

where $\boldsymbol{\omega}_r$ is the body angular velocity with respect to the reference frame. Then, the attitude variables $\boldsymbol{\sigma}$ imply the attitude errors between the body frame and the reference frame and satisfy the relation equation of the kinematics as

$$\dot{\boldsymbol{\sigma}}=\frac{1}{4}\left[\Gamma(\boldsymbol{\sigma})\right]\Delta\boldsymbol{\omega}\tag{33}$$

Then, note that the attitude error variables $\boldsymbol{\sigma}$ are defined as the attitude angle between the body frame and the reference frame. That is, $\boldsymbol{\sigma}$ are not related with the inertia frame. To design a controller, Lyapunov stability theory is derived. To do this, candidate Lyapunov function are introduced as

$$U=\frac{1}{2}\Delta\boldsymbol{\omega}^T[I]\Delta\boldsymbol{\omega}+2K\log\left(1+\boldsymbol{\sigma}^T\boldsymbol{\sigma}\right)\tag{34}$$

where $K>0$. This function is positive definite and radially unbounded in terms of the attitude errors $\Delta\omega$ and σ. The time derivative of U is

$$\frac{dU}{dt} = \Delta\boldsymbol{\omega}^T\left([I]\Delta\dot{\boldsymbol{\omega}} + \frac{1}{2}\frac{d[I]}{dt}\Delta\boldsymbol{\omega}\right) + 4K\frac{1}{1+\boldsymbol{\sigma}^T\boldsymbol{\sigma}}\dot{\boldsymbol{\sigma}}^T\boldsymbol{\sigma} \tag{35}$$

As the inertia matrix($[I]$) is the time variant function like Eq. (24), the time derivative of the inertia caused by the gimbal axis's spin with respect the body frame is calculated as

$$\frac{d[I]}{dt} = \dot{\gamma}(J_s - J_t)\left(\mathbf{e}_s\mathbf{e}_t^T + \mathbf{e}_t\mathbf{e}_s^T\right) + \dot{\delta}(J_n - J_s)\left(\mathbf{e}_s\mathbf{e}_n^T + \mathbf{e}_n\mathbf{e}_s^T\right) \tag{36}$$

Meanwhile, let the governing equation of the angular velocity error $\Delta\dot{\boldsymbol{\omega}}$ and the kinematics of Eq. (29) put into Eq. (35). Then, we can rewrite as

$$\frac{dU}{dt} = \Delta\boldsymbol{\omega}^T\left([I](\dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}_r) + \frac{1}{2}\frac{d[I]}{dt}\Delta\boldsymbol{\omega} + K\boldsymbol{\sigma}\right) \tag{37}$$

To stabilize the system, the preceding equation suggests that, for Lyapunov stability theorem, the relation equation about control input

$$[I](\dot{\boldsymbol{\omega}} - \dot{\boldsymbol{\omega}}_r) + \frac{d[I]}{2dt}\Delta\boldsymbol{\omega} + K\boldsymbol{\sigma} = -D\Delta\boldsymbol{\omega} \tag{38}$$

where D is the payoff matrix of positive definition. And if the time derivative of Lyapunov function satisfies the condition as

$$\frac{dU}{dt} = -\Delta\boldsymbol{\omega}^T D\Delta\boldsymbol{\omega} \le 0 \tag{39}$$

Then, the system is stabilized. Using the equation of motion in Eq. (23), the control law given by Eq. (38) can be rewritten as

$$\begin{aligned}
&-\mathbf{e}_s[J_s\dot{\gamma}\omega_t + I_{Ws}\left(\dot{\Omega} - \dot{\delta}\omega_n\right) - \left(J_t - J_g\right)\omega_t\dot{\gamma} \\
&+\left(I_{Gn} - I_{Gt} - I_{Gs}\right)\dot{\delta}\omega_n + \left(I_{Gn} - I_{Gt}\right)\dot{\delta}\dot{\gamma}] \\
&-\mathbf{e}_t\left[J_{Ws}\dot{\gamma}\omega_s + I_{Ws}\left(\dot{\gamma} + \omega_n\right)\Omega - \left(J_t + J_n\right)\dot{\gamma}\omega_s + J_t\dot{\delta}\right] \\
&-\mathbf{e}_n\left[J_n\ddot{\gamma} - I_{Ws}\Omega\omega_t + \left(J_t + J_n - J_{Ws}\right)\dot{\delta}\omega_s\right] + \frac{1}{2}\frac{d[I]}{dt}\Delta\boldsymbol{\omega} \\
&= -\boldsymbol{\omega}\times[I]\boldsymbol{\omega} + [I]\dot{\boldsymbol{\omega}}_r - K\boldsymbol{\sigma} - D\Delta\boldsymbol{\omega}
\end{aligned} \tag{40}$$

Meanwhile, if an inertial torque is generated in the wheel and gimbal system to control an extraneous torque($\mathbf{L}$), Eq. (40) can be expressed as

$$[F]\begin{Bmatrix}\dot{\Omega}\\ \dot{\gamma}\\ \dot{\delta}\end{Bmatrix} + [G]\begin{Bmatrix}0\\ \ddot{\gamma}\\ \ddot{\delta}\end{Bmatrix} = \mathbf{L}_r \tag{41}$$

If matrix $[F]$ is stated as

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13}\\ f_{21} & f_{22} & f_{23}\\ f_{31} & f_{32} & f_{33}\end{bmatrix} \tag{42}$$

Then, the components of this matrix is defined as

$$\begin{aligned}
&f_{11} = -I_{Ws}, \quad f_{21} = 0, \quad f_{23} = 0, \quad f_{31} = 0, \quad f_{32} = 0\\
&f_{12} = -\left(J_s - J_t + J_n\right)\omega_s + \frac{1}{2}\left(J_s - J_t\right)\omega_t + \frac{1}{2}\left(J_n - J_t\right)\omega_n\\
&f_{13} = -\left(I_{Gn} - I_{Gt} - I_{Gs}\right)\omega_n\\
&f_{22} = -\left(I_{Ws} - J_t - J_n\right)\omega_s - I_{Ws}\Omega + \frac{1}{2}\left(J_s - J_t\right)\omega_s\\
&f_{33} = -\left(J_t + J_n - J_s\right)\omega_s + \frac{1}{2}\left(J_n - J_t\right)\omega_s
\end{aligned}$$

Also, matrix $[G]$ is

$$[G] = \begin{bmatrix} 0 & 0 & 0\\ 0 & -J_n & 0\\ 0 & 0 & -J_t\end{bmatrix} \tag{43}$$

To attain the goal of control, the desired torque input $\mathbf{L}_r$ is given by

$$\begin{aligned}
\mathbf{L}_r = &\frac{1}{2}\frac{d[I]}{dt}\omega_r - \boldsymbol{\omega}\times[I]\boldsymbol{\omega} + [I]\dot{\omega}_r\\
&+ I_{Ws}\omega_n\Omega\mathbf{e}_t - I_{Ws}\omega_t\Omega\mathbf{e}_n - K\boldsymbol{\sigma} - D\Delta\boldsymbol{\omega}
\end{aligned} \tag{44}$$

Conventionally, CMG doesn't have depended on the gimbal acceleration($\ddot{\delta}, \ddot{\gamma}$) command but has generated the gyroscopic torque by the gimbal angular velocity($\dot{\delta}, \dot{\gamma}$). Or, VSCMG can generate the torque about three axis by varying the wheel's spin angular velocity(Ω). Therefore, by eliminating the gimbal acceleration input in Eq. (41) and arranging, we can express as

$$\begin{Bmatrix}\dot{\Omega}\\ \dot{\gamma}\\ \dot{\delta}\end{Bmatrix} = [F]^{-1}\mathbf{L}_r \tag{45}$$

From the previous equation, we can calculate the wheel acceleration($\dot{\Omega}$) input and gimbal axis angular velocity ($\dot{\delta}, \dot{\gamma}$) command to organize the torque input $\mathbf{L}_r$ required for the attitude maneuver. But if the inverse of the matrix $[F]$ is out of existence, singular points happen. To come over the problem, various techniques have been studied by far. Because this is out of this study range, this will be not analyzed any longer. If singularity condition happens while applying the simulation in this study, the numerical method is substituted properly. That is, the inverse matrix is always existent. So, we can obtain the unique solution in Eq. (44). Therefore, using CMG in the two axis gimbal system contained the wheel with the variable angular velocity, we know that the attitude control is possible in three axis. Using the linear equation, the similar conclusion has been proved already in reference [18, 19]. Also, it is radically different from the CMG in one axis gimbal system having the pyramid configuration.

If the wheel angular velocity is constant in CMG system, then, $\dot{\Omega} = 0$ and Eq. (41) can be rewritten as

$$\begin{Bmatrix} \dot{\gamma} \\ \dot{\delta} \end{Bmatrix} = \begin{bmatrix} f_{22} & f_{23} \\ f_{32} & f_{33} \end{bmatrix}^{-1} \mathbf{L}_r \tag{46}$$

Thus, we cannot obtain the unique $\dot{\gamma}, \dot{\delta}$ that satisfy Eq. (46) about the given $\mathbf{L}_r$. That is, the attitude control about the three axes is impossible in the two axes gimbal system using an invariant speed wheel of general. Therefore, to control about the three axes, we must consider the control function about terms of the wheel speed.

4. NUMERICAL SIMULATION

To prove the control law derived in this paper, we perform an simple simulation. At first, let me introduce the model data for the simulation. Assume that the inertia momentum of the satellite body is $[I] = diag[86.2, 85.1, 113.6](kg \cdot m^2)$, the inertia momentum of the wheel is $[I_{Ws}, I_{Wt}] = [0.5, 0.2](kg \cdot m^2)$, and the inertia momentum of the gimbal is $[I_{Gs}, I_{Gt}, I_{Gn}] = [0.6, 0.4, 0.4]$ $(kg \cdot m^2)$. Also, the inertia condition of the attitude error is supposed $[\sigma_1, \sigma_2, \sigma_3] = [0.1, -0.2, 0.3]$ and the locus of reference angular velocity is given by

$$\boldsymbol{\omega}_r(t) = \begin{Bmatrix} 0.03\sin(0.2t) \\ -0.05\cos(0.2t) \\ 0.02\cos(0.2t) \end{Bmatrix} \tag{47}$$

Also, the feedback gain is $K = diag[50, \ 50, \ 50]$ and $D = [40, \ 40, \ 40]$. First, by the result of the simulation, the response of angular velocity is shown in Fig 2.

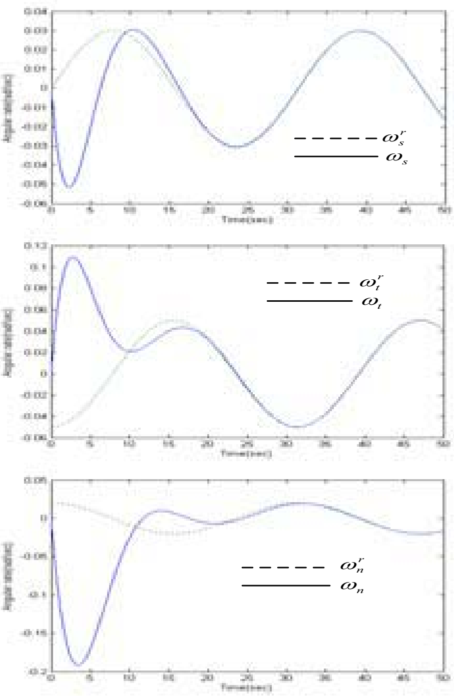

Fig. 2 Time responses of angular velocity

All of the results are shown that the components of angular velocity follow up the locus of reference angular velocity expertly. That is, the Lyapunov controller gives satisfactory performance. After all, final angular velocity errors converge to zero.

And attitude error response in MRP is shown in Fig. 3. We can confirm that final errors converge to zero to our expectations because of the stability of controller design condition.

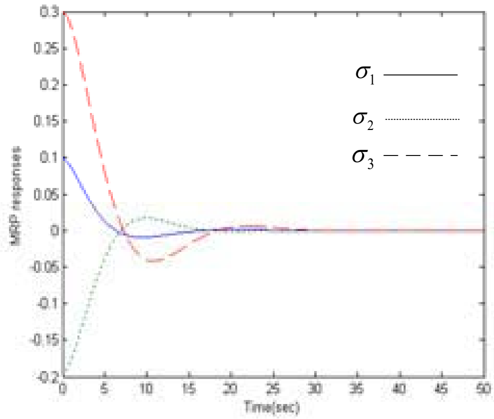

Fig. 3 Attitude error responses in MRP

Also, the response of required torque command($\mathbf{L}_r$) is given by Fig. 4.

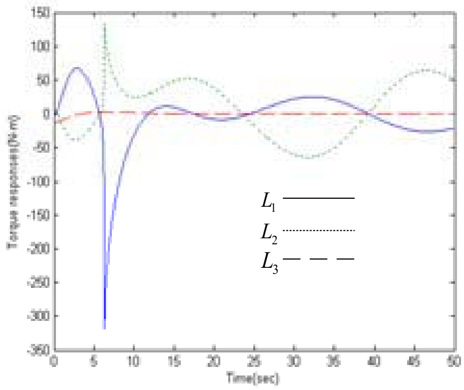

Fig. 4 Required torque command trends

Required torque command is large relatively but can be reduced by regulating the control gain and many other conditions properly. Also, singularity problem happened for this simulation and was treated by numerical method. There are many studies about the singularity problem. In case of the two axis gimbal system, we can predict the result by taking advantage of the existing studies. Through preceding simulations, we can know that designed controller satisfy the desired tracking ability without difficulty.

5. CONCLUSIONS

Attitude dynamics with a VSCMG mounted on a two-axis gimbal system were derived in this study. Also, by using this, the design of a controller was introduced. The dynamic equation was considered as an extension of the single-axis counterpart. We proved that a torque could be generated by the gimbal motion added through the equation obtained from the result. We showed the three-axis control was possible with a CMG. Also, the preceding feedback controller was introduced. Then, we suggested a CMG motion law to operate the control command. And the ability of the designed control law was proved through a simple simulation.

ACKNOWLEDGMENTS

The present work was supported by National Research Lab.(NRL) Program(2002, M1-0203-00-0006) by the Ministry of Science and Technology, Korea. Authors fully appreciate the financial support.

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