Hydraulically Actuated of Half Car Active Suspension System

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Abstract: The studies of the half active suspension have been performed using various suspension models. In the early days, the modeling considered the inputs to the active suspension as the linear forces. Recently, due to the development of new control theory, the forces input to the half car active suspension system has been replaced by an actual input to the hydraulic actuators. Therefore, the dynamic of the active suspension system now consists of the dynamic of half car suspension system plus the dynamic of the hydraulic actuators. This paper proposed a new modeling technique in integrating both dynamic models. The proportional integral sliding mode control technique is utilized to control the hydraulically actuated of the half car active suspension system. The performance of the half car hydraulically actuated active suspension system is simulated with a bump input. The results show that the proposed modeling technique and the proportional integral sliding mode controller are improved the ride comfort and ride handling of the half car active suspension system.

Keywords: Active Suspension, Automotive Control, Sliding Mode Control, Mismatched Uncertainties

1. INTRODUCTION

Modeling of the active suspension system as a linear force input is the most simple but it does not give an accurate model of the system because the actuator's dynamics have been ignored in the design. Thus the controller developed and the result presented may have problem when applying to the active suspension system in the real world. In order to overcome the problem, [1,2,3,4,5,6,7] have considered the hydraulic actuator dynamics in the design of active suspension system for the quarter car model. The hydraulic actuator dynamics has also been included in the controller design for the active suspension system for half car model by [8]. All these researchers have utilized the hydraulic actuator dynamics formulated by [9]. However, the mathematical derivation of the active suspension systems using this approach is very complex and require high computing capabilities of the system.

To circumvent the problem, [10,11] has proposed a hydraulic actuator of the actuating ram type in the development of active suspension systems for half car model. The mathematical derivation of this approach is much more simpler as compared to the previous approach. However, in the mathematical derivation, [10,11] has assumed that the derivative term in the rate of change of the pressure difference in the cylinder which is nonlinear is insignificant as compared to the large value of the effective bulk modulus of the oil and thus can be replaced by a linear term. Therefore, the rate of change of the pressure difference in the cylinder has been assumed to be a linear parameter in their modeling. However, the rate of change of the pressure difference in the cylinder is nonlinear, and this nonlinearity cannot be ignored if a complete mathematical modeling of the hydraulic actuator dynamics is required.

And thus in this paper, the complete formulation of the active suspension system actuated by hydraulic actuator based on the formulation presented by [10,11] is proposed for half car model but with the nonlinear rate of change of the pressure difference in the hydraulic cylinder is included in the mathematical model. Furthermore, the mathematical equation will be presented in state space form for ease of the controller design. Finally, the Proportional Integral Sliding Mode Control (PISMC) approach will be utilized to the control the hydraulically actuated active suspension system for half car model.

2. HALF CAR SUSPENSION MODELING

In this section, a mathematical model of the active suspension for a half car model, as shown in Figure 1 is derived based on the approach as presented in [11]. The passive suspension for the half car model consists of the front and rear wheels and also the axles that are connected to the half portion of the car body through the passive springs-dampers combination, while the tyres are modeled as a simple spring without damper. It is assumed that all springs and dampers are linear. Let f_f and f_r be the forces input for the front and rear actuators, respectively. Therefore, the motion equations of the active suspension for the half car model may be determined as follows:

$$\frac{m_b}{L}(L_f \ddot{x}_{bf} + L_r \ddot{x}_{br}) + c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) + k_{bf}(x_{bf} - x_{wf}) + c_{br}(\dot{x}_{br} - \dot{x}_{wr}) + k_{br}(x_{br} - x_{wr}) - f_f - f_r = 0 \quad (1)$$

$$\frac{I_b}{L}(\ddot{x}_{bf} - \ddot{x}_{wf}) + L_f[c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) + k_{bf}(x_{bf} - x_{wf}) - f_f] - L_r[c_{br}(\dot{x}_{br} - \dot{x}_{wr}) + k_{br}(x_{br} - x_{wr}) - f_r] = 0 \quad (2)$$

$$m_{wf}\ddot{x} - c_{bf}(\dot{x}_{bf} - \dot{x}_{wf}) - k_{bf}(x_{bf} - x_{wf}) + k_{wf}(x_{wf} - w_f) + f_f = 0 \quad (3)$$

$$m_{wr}\ddot{x}_{wr} - c_{br}(\dot{x}_{br} - \dot{x}_{wr}) - k_{br}(x_{br} - x_{wr}) + k_{wr}(x_{wr} - w_r) + f_r = 0 \quad (4)$$

The control forces, f_f and f_r in equations (1), (2), (3) and

(4) are generated by the actuating rams in the hydraulic cylinder as presented in [10,11].

3. THE ACTUATOR DYNAMICS

The flow rates from the servo-valves to the hydraulic cylinders are regulated by the control signals u_f and u_r. Then, the control pressures in the hydraulic actuators are given by,

$$P_{uf} = \frac{u_f}{A_{pf}} \tag{5}$$

and

$$P_{ur} = \frac{u_r}{A_{pr}} \tag{6}$$

where A_{pf} and A_{pr} are the sectional areas of the front and rear pistons, respectively. P_{uf} and P_{ur} are the control pressures in the front and rear cylinders, respectively.

The pressure differences in the hydraulic cylinders will move the actuating rams which produce the control forces represented by

$$f_f = A_{pf} P_{lf} \tag{7}$$

and

$$f_r = A_{pr} P_{lr} \tag{8}$$

where P_{lf} and P_{lr} are the pressures difference in the front and rear cylinders, respectively. The rate of change of the pressure differences for the front and rear cylinders are given by

$$\frac{V_{lf}}{2\beta_e}\dot{P}_{lf} = -\frac{P_{lf}}{r_p} - A_{pf}(\dot{x}_{bf} - \dot{x}_{wf}) + \frac{P_{uf}}{r_p} \tag{9}$$

and

$$\frac{V_{lr}}{2\beta_e}\dot{P}_{lr} = -\frac{P_{lr}}{r_p} - A_{pr}(\dot{x}_{br} - \dot{x}_{wr}) + \frac{P_{ur}}{r_p} \tag{10}$$

where V_{lf} and V_{lr} are the total volume in the front and rear cylinders, respectively, β_e is an effective bulk modulus of oil and r_p is the resistance factor.

Let

$$A_{ef} = \frac{V_{lf} r_p}{2\beta_e} \tag{11}$$

and

$$A_{er} = \frac{V_{lr} r_p}{2\beta_e} \tag{12}$$

Therefore equations (9) and (10) can be rewritten as

$$\dot{P}_{lf} = -\frac{1}{A_{ef}} P_{lf} - \frac{A_{pf} r_p}{A_{ef}}(\dot{x}_{bf} - \dot{x}_{wf}) + \frac{1}{A_{ef}} P_{uf} \tag{13}$$

and

$$\dot{P}_{lr} = -\frac{1}{A_{er}} P_{lr} - \frac{A_{pr} r_p}{A_{er}}(\dot{x}_{br} - \dot{x}_{wr}) + \frac{1}{A_{er}} P_{ur} \tag{14}$$

Hence, the rate of change of the control forces for the front and rear hydraulic actuators can be written as:

$$\dot{f}_f = -\frac{A_{pf}}{A_{ef}} P_{lf} - \frac{A_{pf}^2 r_p}{A_{ef}}(\dot{x}_{bf} - \dot{x}_{wf}) + \frac{A_{pf}}{A_{ef}} P_{uf} \tag{15}$$

and

$$\dot{f}_r = -\frac{A_{pr}}{A_{er}} P_{lr} - \frac{A_{pr}^2 r_p}{A_{er}}(\dot{x}_{br} - \dot{x}_{wr}) + \frac{A_{pr}}{A_{er}} P_{ur} \tag{16}$$

Defining

$$A_{yf} = A_{pf}^2 r_p \tag{17}$$

and

$$A_{yr} = A_{pr}^2 r_p \tag{18}$$

Substituting equations (7-8) and (17-18) into equations (15) and (16), respectively, gives

$$\dot{f}_f = -\frac{1}{A_{ef}} f_f - \frac{A_{yf}}{A_{ef}}(\dot{x}_{bf} - \dot{x}_{wf}) + \frac{1}{A_{ef}} u_f \tag{19}$$

and

$$\dot{f}_r = -\frac{1}{A_{er}} f_r - \frac{A_{yr}}{A_{er}}(\dot{x}_{br} - \dot{x}_{wr}) + \frac{1}{A_{er}} u_r \tag{20}$$

By rearranging equations (1 - 4) and (19 - 20) then introduced the new state vector $x_h = [\dot{x}_{bf} \;\; \dot{x}_{wf} \;\; \dot{x}_{br} \;\; \dot{x}_{wr} \;\; x_{bf} \;\; x_{wr} \;\; x_{br} \;\; x_{wr} \;\; f_f \;\; f_r]^T$, the state space representation of the hydraulically actuated half car active suspension may be written as follows:

$$\dot{x}_h = A_h x_h + B_h u_h + G_h w_h(t) \tag{21}$$

where the matrices A_h, B_h and G_h are as given in Appendix A.

It can be observed from equations (2.103) and (2.104) that the non-zero elements of the disturbance matrix G_h are not in phase with the input matrix B_h. Therefore the system suffers from the mismatched condition.

4. SLIDING MODE CONTROL DESIGN

In this study, the proportional integral sliding surface for a half car suspension model is defined as follows:

$$\sigma_h(t) = C_h x_h(t) - \int_0^t [C_h A_h + C_h B_h K_h] x_h(\tau) d(\tau) \tag{22}$$

where $B_h \in \Re^{m \times n}$ is the input matrix for a half car model, $C_h \in \Re^{m \times n}$ and $K_h \in \Re^{m \times n}$ are the constant matrices, respectively, *m* is the number of inputs and *n* is the number of system states. Thus, it can be seen that the active suspension system for the half car model has two sliding surfaces. Let the sliding surface for front and rear suspensions be defined as $\sigma_{hf}(t)$ and $\sigma_{hr}(t)$, respectively.

The matrix C_h is chosen such that $C_h B_h \in \Re^{m \times m}$ is nonsingular, while the matrix K_h is chosen such that $\lambda_{\max}(A_h + B_h K_h) < 0$, therefore its guarantees that all the desired closed loop poles are located in the left half of the s-plane to ensure stability.

The control input of the sliding mode control for the half car active suspension system can be written as

$$u_h(t) = u_{h_{eq}}(t) + u_{h_s}(t) \tag{23}$$

The equivalent control $u_{h_{eq}}(t)$ is presented by

$$u_{h_{eq}}(t) = K_h x_h(t) - [C_h B_h]^{-1} C_h f_h(x,t) \tag{24}$$

The switching control $u_{h_s}(t)$ is selected as follows:

$$u_{h_s}(t) = (C_h B_h)^{-1} \rho_h \operatorname{sgn}(\sigma_h(t)) \quad (25)$$

It can be seen from equation (25) that the switching control $u_{h_s}(t)$ is nonlinear and discontinuous. The chattering effect caused by the $\operatorname{sgn}(\sigma_h(t))$ function may be replaced by the continuous function. Hence the switching control becomes:

$$u_{h_s}(t) = (C_h B_h)^{-1} \rho_h \frac{\sigma_h(t)}{\|\sigma_h(t)\| + \delta_h} \quad (26)$$

where δ_h is the boundary layer thickness which is selected to reduce the chattering problem and ρ_h is the design parameter which is specified by the designer. Therefore, the proposed proportional sliding mode controller for the half car active suspension model is given as follows:

$$u_h(t) = K_h x_h(t) - (C_h B_h)^{-1} C_h f_h(t) - (C_h B_h)^{-1} \rho_h \frac{\sigma_h(t)}{\|\sigma_h(t)\| + \delta_h} \quad (27)$$

where $\rho_h > 0$.

[12] has proved using the Lyapunov stability analysis that the proposed controller is boundedly stable.

5. SIMULATIONS AND DISCUSSION

The mathematical model for the active suspension system with hydraulic actuator dynamics for half car model is given by equations (1-4) and equations (19– 20). The parameter values used for the active suspension system are as follows:

Mass of the car body, m_b	:	1794.4 Kg
Moment of inertia for the car body, I_b	:	3443.05 Kgm^2
Mass of the front wheel, m_{wf}	:	87.15 Kg
Mass of the rear wheel, m_{wr}	:	140.04 Kg
Stiffness of the front car body spring, k_{bf}	:	66824.2 N/m
Stiffness of the front car body spring, k_{br}	:	18615.0 N/m
Stiffness of the front car tire, k_{wf}	:	101115 N/m
Stiffness of the rear car tire, k_{wr}	:	101115 N/m
Damping of the front car damper, c_{bf}	:	1190 Ns/m
Damping of the rear car damper, c_{br}	:	1000 Ns/m

The parameter values for the front and rear hydraulic actuators are $A_{ef} = A_{er} = 1\,s$ and $A_{yf} = A_{yf} = 10\ kNs/m$ as presented in Yoshimura *et al.* (1999) [11].

Substituting these parameter values into equations (1-4) and equations (19– 20) yields the dynamic equation for the hydraulically actuated active suspension system for the half car model in state space form as in equation (21). The performance of the hydraulically actuated active suspension systems will be investigated and compared to the passive suspension system through computer simulations. The simulations are carried out using MATLAB and SIMULINK software.

The passive and active suspension systems are simulated using such software. To prove the effectiveness of the hydraulically actuated active suspension as given by equation (21) in improving the road handling and the ride comfort for the active suspension system of the half car model, a road profiles as in figure 2 will be utilized.

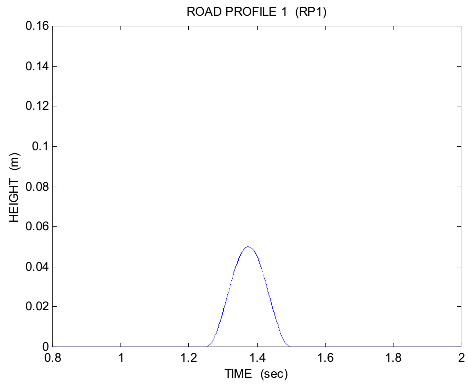

Figure 2 A single 5 cm bump

For comparison purposes, the performance of the PISMC is compared to the active suspension system for the half car model using linear state feedback (LSF) control approach. The following linear state feedback law is used:

$$u_{LSF}(t) = K_h x_h(t) \quad (28)$$

The suspension travel for the front and rear suspensions for the active suspension system using PISMC and LSF controllers and also the passive suspension system are shown in Figures 3 and 4. The results show that the PISMC technique perform better as compared to the other especially for the rear suspension performance. Figures 5 and 6 describe the response of the wheel deflection for the passive suspension system and also for the active suspension system using the PISMC and LSF controllers. The simulation results show that the active suspension system with the PISMC approach has a better tyre to road surface contact, hence directly improved the car handling as compared to the SLF method and the passive suspension system. Figures 7 and 8 show that the body acceleration of the proposed controller is slightly reduced as compared to the SLF method and the passive suspension system. The sliding surfaces of the front and rear active suspension utilizing the PISMC approach are illustrated in Figures 9 and 10. The simulation results show that the state trajectories slide onto the sliding manifold and remains on them thereafter.

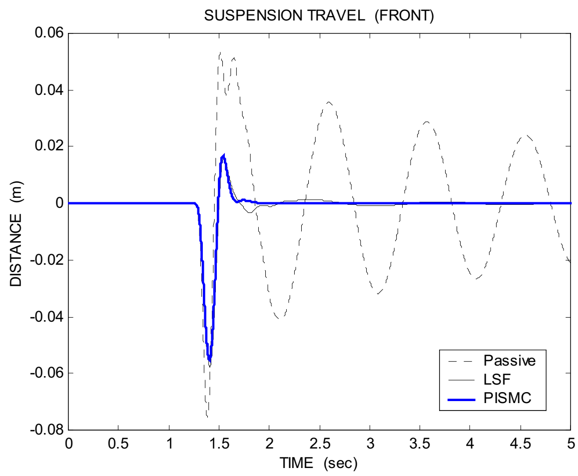

Figure 3 Suspension travel for front suspension

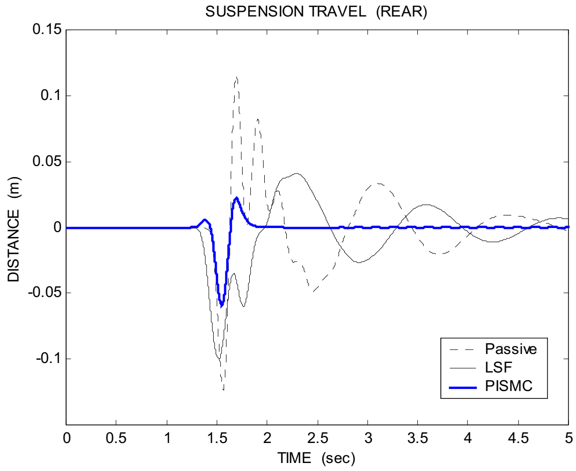

Figure 4 Suspension travel for rear suspension

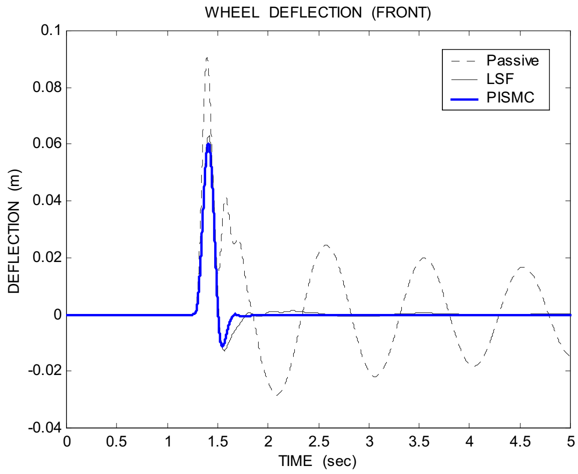

Figure 5 Wheel deflection for front suspension

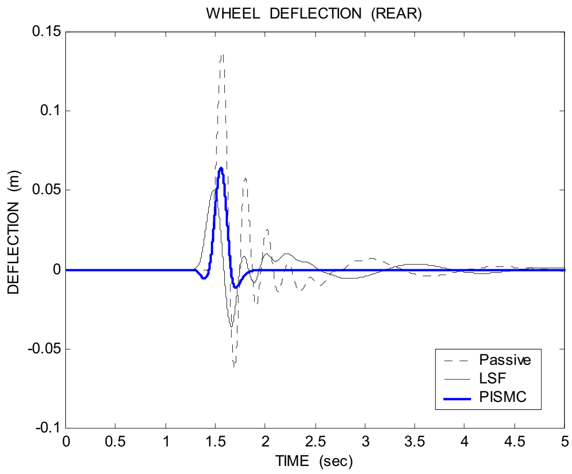

Figure 6 Wheel deflection for rear suspension

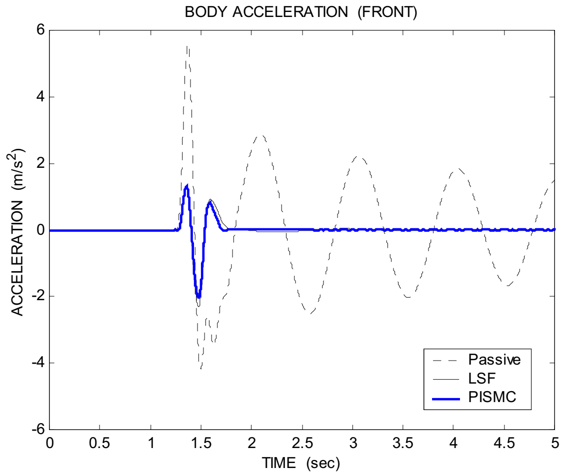

Figure 7 Body acceleration for front suspension

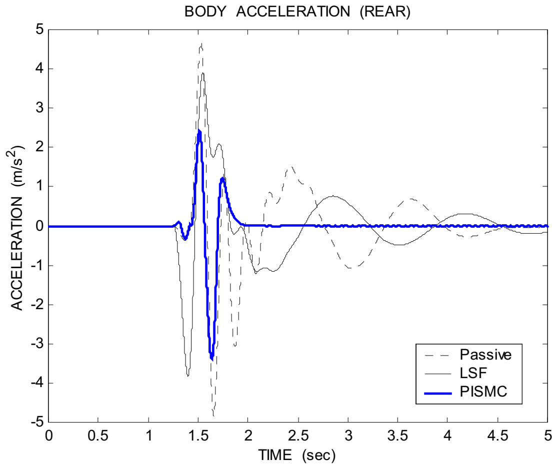

Figure 8 Body acceleration for rear suspension

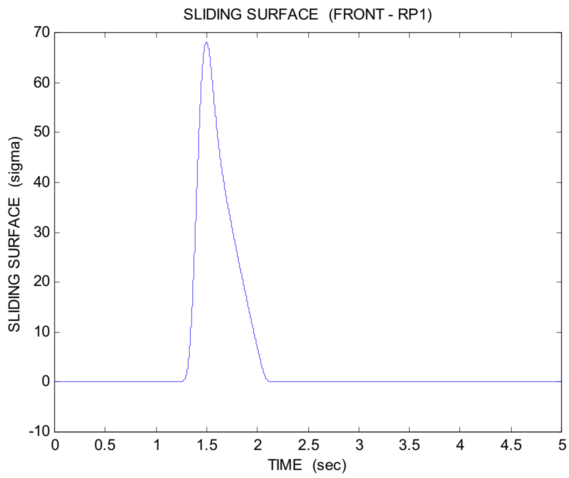

Figure 9 Sliding surface of the front suspension

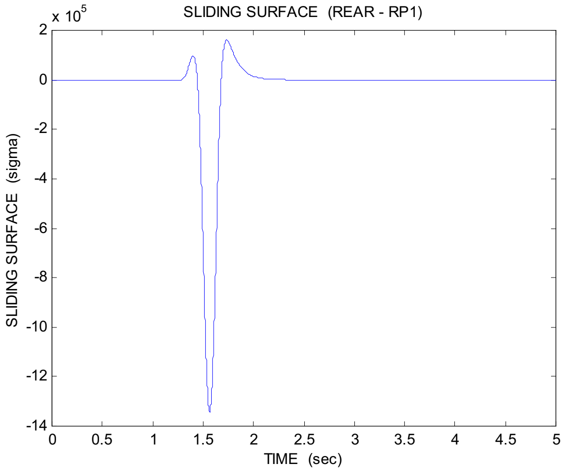

Figure 10 Sliding surface of the rear suspension

6. CONCLUSION

In conclusion, the proposed PISMC approach is capable in improving the ride comfort and road handling of the active suspension for the half car model as compared to the LSF approach and the passive suspension system. Increasing the ride comfort will make the driver and passengers of that particular car more comfortable and relax through a journey. Whereas, increasing the road handling will make the car more stable and can avoid skidding especially when the car is cornering and breaking.

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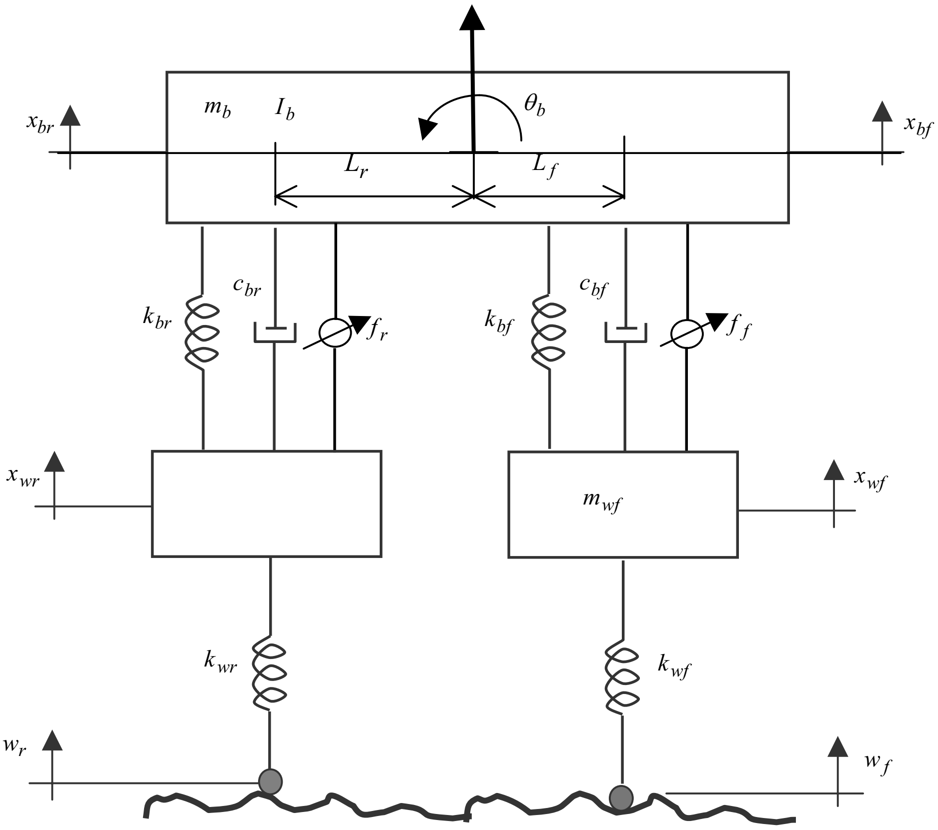

Figure 1 The half car active suspension

Appendix A

$$\begin{bmatrix} \ddot{x}_{bf} \\ \ddot{x}_{wf} \\ \ddot{x}_{br} \\ \ddot{x}_{wr} \\ \dot{x}_{bf} \\ \dot{x}_{wf} \\ \dot{x}_{br} \\ \dot{x}_{wr} \\ f_f \\ f_r \end{bmatrix} = \begin{bmatrix} -1.22 & 1.22 & 0.07 & -0.07 & -68.6 & 68.6 & 1.4 & -1.4 & 0.001 & -7.5x10^{-5} \\ 13.7 & -13.7 & 0 & 0 & 767 & -1930 & 0 & 0 & -0.015 & 0 \\ 0.09 & -0.09 & -1.41 & 1.41 & 5.02 & -5.02 & -26.2 & 26.24 & -7.5x10^{-5} & 0.0014 \\ 0 & 0 & 7.14 & -7.14 & 0 & 0 & 132.98 & -854.97 & 0 & -0.0071 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 13.27 & 13.7 & 0 & 0 & 0 & 0 & 0 & 0 & -0.885 & 0 \\ 0 & 0 & 13.27 & 13.27 & 0 & 0 & 0 & 0 & 0 & -0.885 \end{bmatrix} \begin{bmatrix} \dot{x}_{bf} \\ \dot{x}_{wf} \\ \dot{x}_{br} \\ \dot{x}_{wr} \\ x_{bf} \\ x_{wf} \\ x_{br} \\ x_{wr} \\ f_f \\ f_r \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0.885 & 0 \\ 0 & 0.885 \end{bmatrix} \begin{bmatrix} u_f \\ u_r \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1160 & 0 \\ 0 & 0 \\ 0 & 0.72 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} w_f \\ w_r \end{bmatrix}$$