An Electromechanical ΣΔ Modulator for MEMS Gyroscope

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Abstract: This paper presents a design and analysis of electromechanical sigma-delta modulator for MEMS gyroscope, which enables us to control the proof mass and to obtain an exact digital output without additional A/D conversion. The system structure and the circuit realization of the sigma-delta modulation are simpler than those of the analog sensing and feedback circuit. Based on the electrical sigma-delta modulator theory, a compensator is designed to improve the closed loop resolution of the sensor. With the designed compensator, we could obtain enhanced closed-loop performances of the gyroscope such as larger bandwidth, lower noise, and digital output comparing with the results of analog open-loop system.

Keywords: Sigma-delta modulator, Gyroscope, MEMS

1. INTRODUCTION

The inertial sensors are basic elements that compose Inertial Navigation System (INS). The gyroscope is basic inertial sensor, which can measure external angular rate. The MEMS gyroscope is an inertial angular rate sensor fabricated using MEMS technology. It has many good properties, which are small, light, and low power consuming. Moreover, it can be mass-produced inexpensively. Therefore, it has many applications in various fields.

The gyroscope is a high-Q system, which results in small bandwidth and has theoretically nonlinear characteristics. When external angular rate is applied to a micro gyroscope, excited proof mass vibrating at resonant frequency is forced to vibrate in different direction due to the Coriolis force. The angular rate can be estimated by measuring the amplitude of the vibration. In this case, the dynamic range of sensor is limited and the sensor becomes more sensitive to measurement noise and less robust. Furthermore, nonlinearity becomes larger as the amplitude of the vibration becomes larger and scale factor changes with the vibration of sensor parameters.[1] To overcome these disadvantages, a closed loop controller named rebalance loop can be used. A rebalance loop is a kind of feedback controller that makes position error small. Therefore rebalance loop improve the performances of gyroscope such as dynamic range, bandwidth, linearity, and robustness. When a sensor is used as a closed loop one, the input rate can be measured by detecting and demodulating the controller output, since the controller output is proportional to the input angular rate.

Most of feedback control technique is analog control. Analog feedback has good performance and high sensitivity. However, it is difficult to implement the feedback loop. Because we consider a system dynamics, modeling error and disturbance when we design feedback loop. In addition, design of high performance analog circuit is very difficult.

Sigma-delta modulation, which has been widely used in high-resolution A/D and D/A converters [2], is has been studied to realize high-performance MEMS sensor systems. It combines sampling at rates well above the Nyquist rate with negative feedback and filtering in order to exchange resolution in time for that in amplitude. It is especially attractive in sensor applications where the signals are relatively narrow-band and a large oversampling ratio can be easily achieved.

Using a 2^{nd} order transducer dynamics as the loop filter , a electromechanical sigma-delta modulator can be achieved. In MEMS gyroscope, using electromechanical sigma-delta feedback, we can get directly a digital output. We don't need to add an A/D converter. To implement the sigma-delta circuit is easier than analog feedback and we need not consider the characteristics of the system (system dynamics, modeling error and disturbance). [4]

Electromechanical sigma-delta feedback, however, is different from electrical sigma-delta modulator. Electrical sigma-delta modulator consists of pure electric signals, but electromechanical sigma-delta feedback is a very complicated mixed signal feedback system. It has building blocks in multiple domains, such as mechanical and electrical, continuous-time and discrete-time, analog and digital, as well as linear and nonlinear. Mapping such an electromechanical sigma-delta feedback into pure electrical sigma-delta modulator will be helpful in understanding and designing the circuit. After system mapping, we can design closed loop system using the theory of conventional sigma-delta modulator.

Rigorous analysis of a sigma-delta modulator in the frequency domain is a difficult task due to the nonlinearity of quantizer. To simplify this analysis, one can use the input-independent additive white-noise approximation for the quantization error. The linearized model of the sigma-delta modulator consists of signal transfer function (STF) and noise transfer function (NTF). STF represents the relation between the modulator input and output signal, whereas NTF represents the relation between the quantization error and output. Then well-designed sigma-delta modulator has the loop filter, which has unity gain STF and almost zero gain NTF in demanded signal band.

In this paper, using Simulink, simulate the designed electromechanical electromechanical sigma-delta feedback.

2. MODEL EQUATION OF MEMS GYROSCOPE

MEMS gyroscope is generally composed of a mass and beams that is designed to oscillate. The mass readily oscillates in two orthogonal directions. A simple model of a vibrating gyroscope is shown in Fig. 1.

The governing equations of the vibratory gyroscope can be expressed using equations (1)-(6). When the proof mass is driven by the electrostatic force or $F_{driving}$(t) along the x-axis, the equation of the dynamic motion of the mass can be simply expressed as a second order mass-damper-spring system.

$$M_x\ddot{x}(t)+C_x\dot{x}(t)+K_x x(t)=F_{driving}(t) \quad (1)$$

Where M_x is the mass of the moving plate, C_x, the damping coefficient of the air-damper, and K_x is the spring constant along the driving axis. Then the transfer function from the driving force to the displacement of the plate is given by

$$G_{drive}(s)=\frac{X(s)}{F(s)}=\frac{1/M_x}{s^2+\frac{C_x}{M_x}s+\frac{K_x}{M_x}} \quad (2)$$

If the angular rate input Ω_z(t) is applied along z-axis, Coriolis force is induced, which is given by (3).

$$F_{coriolis}(t)=2M_x\dot{x}(t)\Omega_z(t) \quad (3)$$

The dynamic equation and the transfer function of the sensing mode can be expressed as (4) and (5), respectively.

$$M_y\ddot{y}(t)+C_y\dot{y}(t)+K_y y(t)=F_{coriolis}(t) \quad (4)$$

$$G_{sense}(s)=\frac{Y(s)}{F_{coriolis}(s)}=\frac{1/M_y}{s^2+\frac{C_y}{M_y}s+\frac{K_y}{M_y}} \quad (5)$$

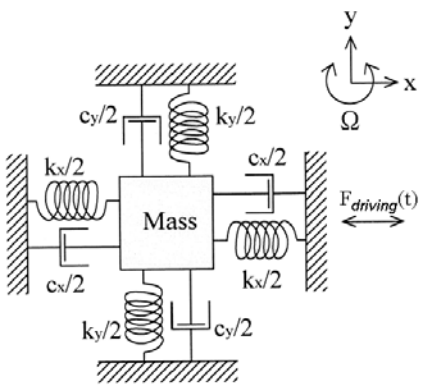

Fig. 1 Simple model of MEMS vibrating gyroscope.

3. SIGMA-DELTA MODULATOR

3.1 Electrical sigma-delta modulator theory

Sigma-delta modulation technique is used in high-resolution A/D and D/A converters widely. Since this system contains a delta modulator and an integrator, they are named sigma-delta modulator, where 'sigma' denoted the summation performed by integrator. The sigma-delta modulator consists of a coarse (low resolution) quantizer preceded by a filter, both embedded into a negative feedback loop. It uses oversampling and quantization error shaping to achieve high accuracy. In other words, it trades speed for resolution, and analog circuit accuracy for digital circuit complexity.

To rigorously analyze a sigma-delta modulator in the frequency domain is a difficult task due to the presence of the nonlinear quantizer. If quantization error is uncorrelated with the input signal and is uniformly distributed in quantizer output band, and is an independent identically distributed (iid.) sequence, then this analysis can be simplified as the input independent additive white noise approximation for the quantization error. [2,3] Therefore, the deterministic nonlinear system is replaced by a linear stochastic system. The linearized model of the sigma-delta modulator is presented in Fig. 2. Then, the calculations became trivial

$$V(z)=(U(z)-V(z))H(z)+E(z) \quad (6)$$

$$\Rightarrow V(z)=\frac{H(z)}{1+H(z)}U(z)+\frac{1}{1+H(z)}E(z) \quad (7)$$

From (7), it results that the sigma-delta modulator process is independently separated into the signal and the noise components. Therefore, its signal transfer function, STF(z) and noise transfer function, NTF(z) can be defined as follows.

$$STF(z)=\left.\frac{V(z)}{U(z)}\right|_{E(z)=0}=\frac{H(z)}{1+H(z)} \quad (8)$$

$$NTF(z)=\left.\frac{V(z)}{E(z)}\right|_{U(z)=0}=\frac{1}{1+H(z)} \quad (9)$$

Also, one can write the output signal *V(z)* as the combination of the input signal, *U(z)* and the quantization noise signal, *Q(z)*. Each of them is filtered by the corresponding transfer function,

$$V(z)=STF(z)U(z)+NTF(z)Q(z). \quad (10)$$

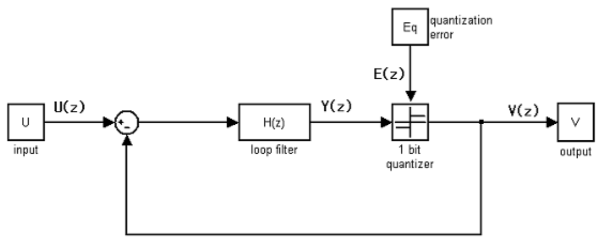

Fig. 2 Linearized model of sigma-delta modulator

If one chooses a low-pass loop filter H(z), which has large magnitude over the low frequencies of interest signal band (also called "signal band" or "in band" or "baseband"), and small magnitude (large attenuation) over high frequencies (also called "out of band"), then the magnitude of the signal transfer function |STF(z)| will be approximated into unity over the signal band, hence it will not distort the signal. However, the magnitude of the noise transfer function, |NTF(z)| will be

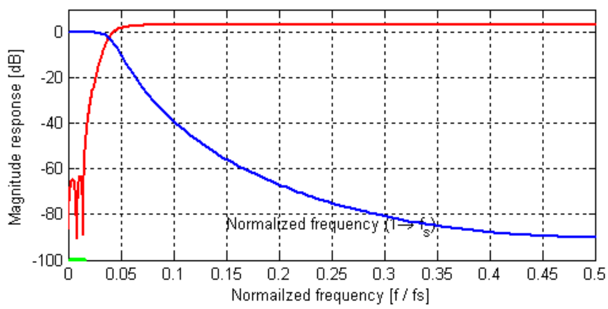

Fig. 3 STF and NTF of conventional sigma-delta modulator

approximated into zero over the same band, hence the quantization noise will be reduced significantly. By doing so, the signal-band spectral composition of the analog input and digital output signals will be linearly related, but outside the signal band, the spectral compositions of them will differ substantially. An example of frequency response of sigma-delta modulator is shown Fig. 3.

3.2 Electromechanical force feedback by sigma-delta modulator

The block diagram of the sigma-delta feedback loop is shown in Fig. 4. The mechanical transducer is embedded in the loop. This modulator is represented in multiple domains, different from the electrical case. Position sensing part converts the displacement to the voltage signal, and actuator part in feedback loop converts the voltage signal to the feedback force.

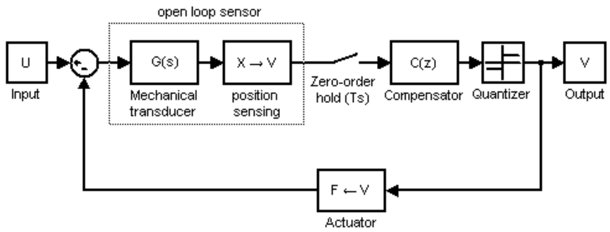

Fig. 4 Block diagram of electromechanical sigma-delta modulator

For easy analysis, mapping of electromechanical sigma-delta modulator into a pure electrical system will be helpful in understanding and designing closed loop sensor. The mechanical transducer, which is embedded in the loop, has 2^{nd} order transfer function. That is represented as (5). Since sigma-delta modulator is discrete system, it is necessary to convert continuous transfer function into discrete one. Assume that sampling period is *Ts* and sampling model is zero-order-hold model [5], then the discrete transfer function of mechanical transducer is represented as (11), (12).

$$G(z) = \frac{z-1}{z} \cdot Z\left\{\frac{1}{s} \cdot G(s)\right\} = \frac{1}{K_y} \cdot \frac{Az+B}{z^2 - 2e^{-aT_s}(\cos bT_s) + e^{-2aT_s}} \quad (11)$$

$$\begin{cases} a = \dfrac{C_y}{2M_y}, \quad b = \sqrt{\dfrac{K_y}{M_y} - a^2} \\ A = 1 - e^{-aT_s}\cos bT_s - \dfrac{a}{b} e^{-aT_s}\sin bT_s \\ B = e^{-2aT_s} + \dfrac{a}{b} e^{-aT_s}\sin bT_s - e^{-aT_s}\cos bT_s \end{cases} \quad (12)$$

In addition, the position sensing part and the actuator are modeled as a gain block. Then, a loop filter of electromechanical sigma-delta modulator is expressed as (13).

$$G(z) \cdot K_s \cdot C(z) \quad (13)$$

The linearized model of the electromechanical sigma-delta modulator is presented in Fig. 5. Therefore, the NTF and STF are calculated as (14).

$$NTF(z) = \frac{1}{1 + K_s K_f G(z) C(z)}$$
$$STF(z) = \frac{K_s G(z) C(z)}{1 + K_s K_f G(z) C(z)} \quad (14)$$

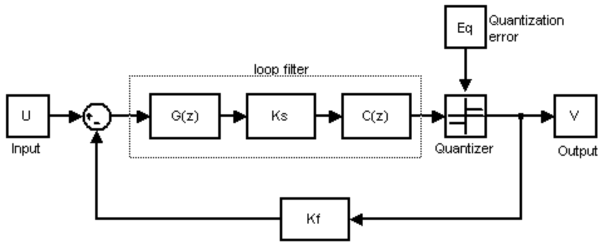

Fig. 5 Linearized model of electromechanical sigma-delta modulator

4. DESIGN COMPENSATOR

In above chapter, we construct an electromechanical sigma-delta feedback loop and map that into a pure electrical sigma-delta modulator. Now we design a compensator block to improve the resolution of the closed loop system.

First, an interested signal band should be determined. In gyroscope system, driving signal is sinusoidal wave whose frequency is a resonance frequency of the mass-string-damper system. Then the external angular rate signal is modulated by the driving signal. Since the bandwidth of conventional gyroscope is about 100Hz, the interested signal band is around the resonance frequency with additional ±100Hz.

In the case of ordinary sigma-delta modulator, the interested signal band is a low frequency region. Therefore, the loop filter of that has a frequency response characteristic like a low-pass filter. In this case, the interested signal band is around the resonance frequency. Therefore, the loop filter of this case has a bandpass property.

Bandpass sigma-delta modulators operate in much the same manner as conventional (low pass) modulator. The design of bandpass converters has much in common with low pass modulator design (The magnitude of the signal transfer function |STF(z)| will be approximated into unity over the signal band, the magnitude of the noise transfer function |NTF(z)| will be approximated into zero over the same band). An example of frequency response of bandpass sigma-delta modulator is shown in Fig. 6.

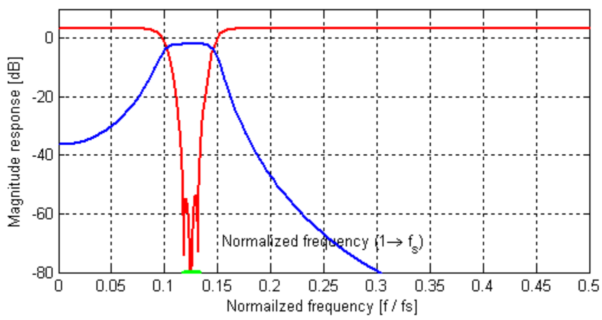

Fig. 6 STF and NTF of bandpass sigma-delta modulator

A much more flexible approach to the transfer function design is via a generalized filter approximator/optimizer [6].

The optimizer adjusts the poles and zeros of NTF such that its amplitude response closely matches the measures of ideality. Table 1 shows the parameters of the MEMS gyroscope. Fig.7 represents NTF of the feedback loop and Fig. 8 is a bode plot of the compensator.

Table 1 Parameters of MEMS gyroscope

Mass	9.599μg
Quality factor	150
Resonant frequency	3.1kHz
Ky	0.0923N/m
Cy	1.9839e-007N·s/m

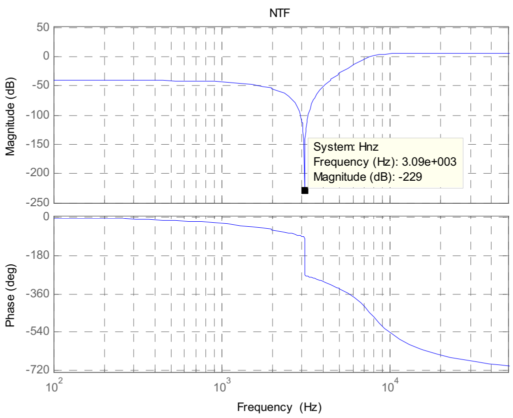

Fig. 7 Designed NTF of sigma-delta modulator for gyroscope

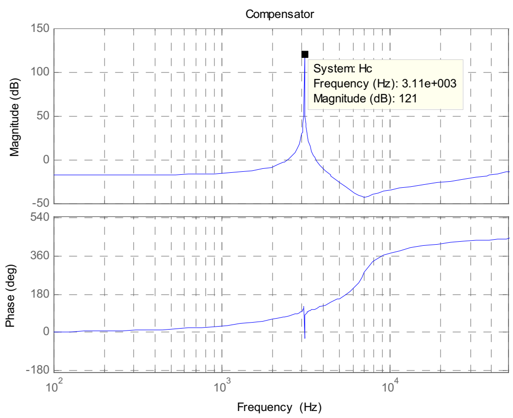

Fig. 8 Designed compensator

5. SIMULATION RESULT

Using the compensator and parameters of the gyroscope, execute simulation by Simulink. Fig. 9 shows displacement output of mass when gyroscope operates open loop case. That is the form, which Input angular rates are modulated sinusoidal driving signal.

Fig. 10 represents displacement output when gyroscope operates with electromechanical sigma-delta feedback. First plot shows output without compensator and second plot shows output with compensator. By the result, Using compensator enhances performance of sigma-delta feedback loop.

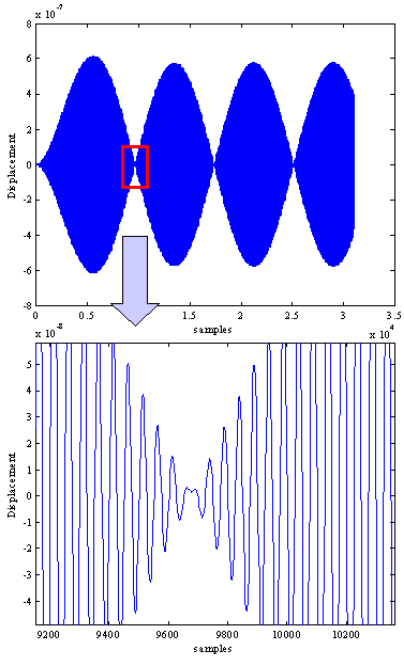

Fig. 9. Displacement of the mass in open loop case

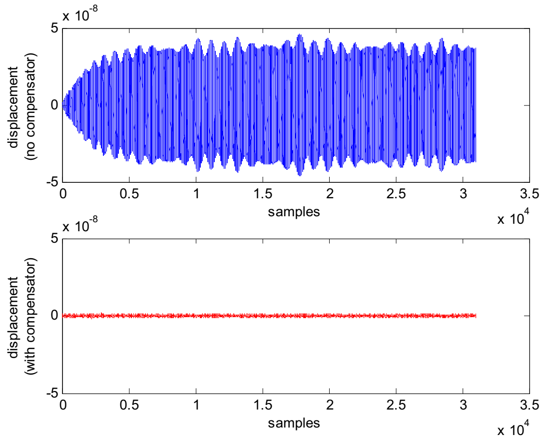

Fig. 10 Displacement of the mass when the gyroscope operates with sigma-delta feedback loop

6. CONCLUSION

In this study, we present the electromechanical sigma-delta modulator for MEMS gyroscope, which enables us to control the proof mass and to obtain a digital output directly. Based on electrical sigma-delta modulator theory, we can design compensator to improve closed loop sensor resolution.

Therefore, we obtain closed loop gyroscope that enhanced performance of the gyroscope such as lower noise, and digital output comparing with the results of analog open-loop system.

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