Effect of viscous interfaces on bending of orthotropic rectangular laminate

Geun Woo Kim, Kang Yong Lee, W. Q. Chen

직교 이방성 적층판의 굽힘에 대한 점성 경계면의 영향

김근우† · 이강용* · 첸웨이쵸**

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Abstract

This paper investigates asimply supported orthotropic rectangular laminate with viscous interfaces subjected to bending. Additional mathematical difficulty is involved due to the presence of viscous interfaces because the behavior of the laminate depends on time. A step-by-step state-space approach is suggested, which is directly based on the three-dimensional theory of elasticity. In particular, Taylor's expansion theorem is employed to model the variations of field variables with time. The proposed method is suitable for analyzing laminated plate of arbitrary thickness. Numerical calculations are performed and it is shown that the viscous interfaces have a significant fluence on the response.

1. Introduction

Recently, many researchers have devoted themselves to laminated plates and shells featuring interlaminar bonding imperfections [1-10]. The imperfections of interlaminar interfaces can be modeled by various simplified models [11-13] . Most above-mentioned works employed the linear interfacial model (or the spring-layer model) [14,15] , using which the static deformations and stresses of laminates are solely functions of the spatial coordinates.

Experimental evidence showed that viscous interfacial sliding could be induced in fiber-matrix composites due to high temperature [16] . In the conventional design of sensors and transducers, viscous interfacial couplants are usually adopted to integrate different functional components [17] . It is also noted that viscous interfacial layers are sometimes introduced artificially to tailor the mechanical properties of laminates, such as the damping performance [18] . He and Jiang [18] recently made a first step in this respect. They derived an exact solution of layered isotropic strips in cylindrical bending by extending the famous Pagano's solution [19] .

As pointed by Noor and Burton [20] , the conventional three dimensional analyses, such as the Pagano's method, is computationally expensive when the laminate has a great number of layers. In the case of an anisotropic laminate with viscous interfaces, the analysis presented by He and Jiang [18] will become more expensive from the view point of computation. On the other hand, the state-space approach has been proved to be very effective in analyzing laminated structures because the number of the final solving equations keeps unchanged regardless of the layer number [21-24] .

† Yonsei University , Korea
E-mail : kgw0723@yahoo.co.kr
TEL : (02)2123-2813 FAX : (02) 2123-2813
* Yonsei University , Korea
** Zhejiang University, China

In this paper, the time-dependent response of a simply supported thick orthotropic rectangular laminate featuring interfacial viscosity is considered.

2. State-space formulations

The state-space formulations for an orthotropic body have been derived by Fan and Ye [23] . Here we just give a brief review for completeness. When σ_z, u, v, w, τ_{xz} and τ_{yz} are chosen as the state variables, the state equation can be derived from the constitutive equations and equilibrium equations as [23]

Where

$$\frac{\partial}{\partial z}\begin{Bmatrix}\sigma_z\\ u\\ v\\ w\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix}=\begin{bmatrix} & \mathbf{0} & & 0 & -\frac{\partial}{\partial x} & -\frac{\partial}{\partial y}\\ & & & & \frac{1}{c_{55}} & 0\\ & & & \text{sym.} & & \frac{1}{c_{44}}\\ \frac{1}{c_{33}} & -\frac{c_{13}}{c_{33}}\frac{\partial}{\partial x} & -\frac{c_{23}}{c_{33}}\frac{\partial}{\partial y} & & & \\ & -\beta_1\frac{\partial^2}{\partial x^2}-c_{66}\frac{\partial^2}{\partial y^2} & -\beta_2\frac{\partial^2}{\partial x\partial y} & & \mathbf{0} & \\ \text{sym.} & & -\beta_3\frac{\partial^2}{\partial y^2}-c_{66}\frac{\partial^2}{\partial x^2} & & & \end{bmatrix}\begin{Bmatrix}\sigma_z\\ u\\ v\\ w\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix}, \quad (1)$$

$$\beta_1=c_{11}-\frac{c_{13}^2}{c_{33}},\quad \beta_2=c_{12}+c_{66}-\frac{c_{13}c_{23}}{c_{33}},\quad \beta_3=c_{22}-\frac{c_{23}^2}{c_{33}} \quad (2)$$

It is noted that in deriving Eq. (1), the inertia effect is neglected as the deformation of the plate is considered to be very slow [18] .

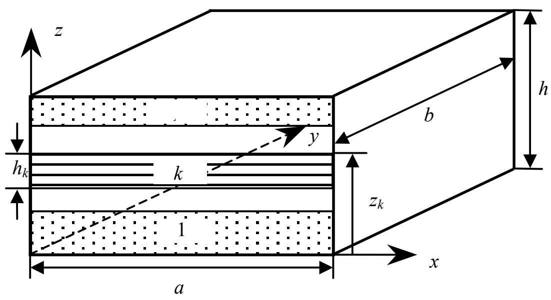


Fig. 1 Geometry of a rectangular laminated plate and the coordinate.

An N -layered orthotropic rectangular plate is depicted in Fig. 1. If the plate is simply supported at all four edges, we can assume

$$\begin{Bmatrix}\sigma_z\\ u\\ v\\ w\\ \tau_{xz}\\ \tau_{yz}\end{Bmatrix}=\begin{Bmatrix}-c_{44}^{(1)}\overline{\sigma}_z(\zeta,t)\sin(m\pi\xi)\sin(n\pi\eta)\\ h\overline{u}(\zeta,t)\cos(m\pi\xi)\sin(n\pi\eta)\\ h\overline{v}(\zeta,t)\sin(m\pi\xi)\cos(n\pi\eta)\\ h\overline{w}(\zeta,t)\sin(m\pi\xi)\sin(n\pi\eta)\\ c_{44}^{(1)}\overline{\tau}_{xz}(\zeta,t)\cos(m\pi\xi)\sin(n\pi\eta)\\ c_{44}^{(1)}\overline{\tau}_{yz}(\zeta,t)\sin(m\pi\xi)\cos(n\pi\eta)\end{Bmatrix}, \quad (3)$$

where $\zeta=z/h$, $\xi=x/a$ and $\eta=y/b$ are dimensionless coordinates, m and n are half-wave numbers, and $c_{44}^{(1)}$ represents the elastic constant of the first layer (the bottom layer). The substitution of Eq. (3) into Eq. (1) yields

$$\frac{\partial}{\partial\zeta}\mathbf{V}(\zeta,t)=\mathbf{A}\mathbf{V}(\zeta,t), \quad (4)$$

where $\mathbf{V}(\zeta,t)=[\overline{\sigma}_z,\overline{u},\overline{v},\overline{w},\overline{\tau}_{xz},\overline{\tau}_{yz}]^{\mathrm{T}}$, and

$$\mathbf{A}=\begin{bmatrix} & \mathbf{0} & & 0 & -t_1 & -t_2\\ & & & & \frac{c_{44}^{(1)}}{c_{55}} & 0\\ & & & \text{sym.} & & \frac{c_{44}^{(1)}}{c_{44}}\\ -\frac{c_{44}^{(1)}}{c_{33}} & \frac{c_{13}}{c_{33}}t_1 & \frac{c_{23}}{c_{33}}t_2 & & & \\ & \frac{\beta_1}{c_{44}^{(1)}}t_1^2+\frac{c_{66}}{c_{44}^{(1)}}t_2^2 & \frac{\beta_2}{c_{44}^{(1)}}t_1t_2 & & \mathbf{0} & \\ \text{sym.} & & \frac{\beta_3}{c_{44}^{(1)}}t_2^2+\frac{c_{66}}{c_{44}^{(1)}}t_1^2 & & x & \end{bmatrix}, \quad (5)$$

with $t_1=m\pi h/a$ and $t_2=n\pi h/b$. The solution to Eq. (4) can be obtained as [25]

$$\mathbf{V}(\zeta,t)=\exp[\mathbf{A}(\zeta-\zeta_{k-1})]\mathbf{V}(\zeta_{k-1},t),$$

$$(\zeta_{k-1}\le\zeta\le\zeta_k,\ k=1,2,\cdots,N), \quad (6)$$

where $\zeta_0=0$, $\zeta_k=z_k/h=\sum_{j=1}^{k}h_j/h$, and h_k is the thickness of the k th layer.

Setting $\zeta=\zeta_k$ in Eq. (6), yields

$$\mathbf{V}_1^{(k)}=\mathbf{M}_k\mathbf{V}_0^{(k)}, \quad (7)$$

where $\mathbf{V}_1^{(k)}$ and $\mathbf{V}_0^{(k)}$ are the state vectors at the upper and lower surfaces of the k th layer, respectively, and $\mathbf{M}_k=\exp[\mathbf{A}(\zeta_k-\zeta_{k-1})]$ is the transfer matrix. Similarly, we get for the $(k+1)$ th layer

$$\mathbf{V}_1^{(k+1)}=\mathbf{M}_{k+1}\mathbf{V}_0^{(k+1)}. \quad (8)$$

3. Viscous interface conditions

For the viscous interface between the k th and $(k+1)$ th layer, we have [18]

$$\dot{\delta}_x^{(k)}=\frac{\tau_{xz}^{(k)}}{\eta_x^{(k)}},\quad \dot{\delta}_y^{(k)}=\frac{\tau_{yz}^{(k)}}{\eta_y^{(k)}},\quad \text{at } z=z_k, \quad (9)$$

where a dot indicates differentiation with respect to time, $\eta_x^{(k)}$ and $\eta_y^{(k)}$ are the viscous coefficients in x and y directions, respectively, and $\delta_x^{(k)}$ and $\delta_y^{(k)}$ are the relative sliding displacements. The initial condition is $\delta_x^{(k)} = \delta_y^{(k)} = 0$ at $t = 0$. The conditions at the interfaces then read as

$$\sigma_z^{(k+1)} = \sigma_z^{(k)}, \quad \tau_{xz}^{(k+1)} = \tau_{xz}^{(k)}, \quad \tau_{yz}^{(k+1)} = \tau_{yz}^{(k)},$$
$$u^{(k+1)} = u^{(k)} + \delta_x^{(k)}, \quad v^{(k+1)} = v^{(k)} + \delta_y^{(k)}, \tag{10}$$
$$w^{(k+1)} = w^{(k)}, \text{ at } z = z_k .$$

In view of Eq. (3), we assume

$$\delta_x^{(k)} = h\bar{\delta}_x^{(k)}(t)\cos(m\pi\xi)\sin(n\pi\eta),$$
$$\delta_y^{(k)} = h\bar{\delta}_y^{(k)}(t)\sin(m\pi\xi)\cos(n\pi\eta). \tag{11}$$

Thus, Eq. (10) can be rewritten as

$$\mathbf{V}_0^{(k+1)} = \mathbf{V}_1^{(k)} + \mathbf{Q}^{(k)}, \tag{12}$$

where $\mathbf{Q}^{(k)} = [0, \bar{\delta}_x^{(k)}, \bar{\delta}_y^{(k)}, 0, 0, 0]^{\mathrm{T}}$. It can be shown that $\bar{\delta}_x^{(k)}$ and $\bar{\delta}_y^{(k)}$ satisfy

$$\frac{\mathrm{d}\bar{\delta}_x^{(k)}}{d\tau} = \bar{\tau}_{xz}^{(k)}\bar{\eta}_x^{(k)}, \quad \frac{\mathrm{d}\bar{\delta}_y^{(k)}}{d\tau} = \bar{\tau}_{yz}^{(k)}\bar{\eta}_y^{(k)}, \text{ at } z = z_k, \tag{13}$$

where $\tau = c_{44}^{(1)} t / [\eta_x^{(1)} h]$ is the dimensionless time, and $\bar{\eta}_i^{(k)} = \eta_x^{(1)} / \eta_i^{(k)}$ ($i = x, y$) are the viscosity ratios. Note that for a perfect interface, we have $\mathbf{Q}^{(k)} = 0$.

Now we divide the time domain into a series of equal intervals $[p\Delta\tau, (p+1)\Delta\tau]$ $(p = 0, 1, 2, \cdots)$, each with a small length of $\Delta\tau$. For a typical interval, according to the Taylor's expansion theorem, we have

$$\bar{\delta}_x^{(k)} = \bar{\delta}_{xp,0}^{(k)} + (\tau - p\Delta\tau)\bar{\delta}_{xp,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{\delta}_{xp,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{\delta}_{xp,3}^{(k)} + \cdots,$$
$$\bar{\delta}_y^{(k)} = \bar{\delta}_{yp,0}^{(k)} + (\tau - p\Delta\tau)\bar{\delta}_{yp,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{\delta}_{yp,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{\delta}_{yp,3}^{(k)} + \cdots,$$
$$\bar{\sigma}_z^{(k)} = \bar{\sigma}_{p,0}^{(k)} + (\tau - p\Delta\tau)\bar{\sigma}_{p,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{\sigma}_{p,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{\sigma}_{p,3}^{(k)} + \cdots,$$
$$\bar{u}^{(k)} = \bar{u}_{p,0}^{(k)} + (\tau - p\Delta\tau)\bar{u}_{p,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{u}_{p,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{u}_{p,3}^{(k)} + \cdots,$$
$$\bar{v}^{(k)} = \bar{v}_{p,0}^{(k)} + (\tau - p\Delta\tau)\bar{v}_{p,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{v}_{p,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{v}_{p,3}^{(k)} + \cdots,$$
$$\bar{w}^{(k)} = \bar{w}_{p,0}^{(k)} + (\tau - p\Delta\tau)\bar{w}_{p,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{w}_{p,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{w}_{p,3}^{(k)} + \cdots,$$
$$\bar{\tau}_{xz}^{(k)} = \bar{\tau}_{xp,0}^{(k)} + (\tau - p\Delta\tau)\bar{\tau}_{xp,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{\tau}_{xp,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{\tau}_{xp,3}^{(k)} + \cdots,$$
$$\bar{\tau}_{yz}^{(k)} = \bar{\tau}_{yp,0}^{(k)} + (\tau - p\Delta\tau)\bar{\tau}_{yp,1}^{(k)} + (\tau - p\Delta\tau)^2\bar{\tau}_{yp,2}^{(k)} + (\tau - p\Delta\tau)^3\bar{\tau}_{yp,3}^{(k)} + \cdots. \tag{14}$$

Obviously, we have $\bar{\delta}_{x0,0}^{(k)} = \bar{\delta}_{y0,0}^{(k)} = 0$ because of the zero initial condition at $t = \tau = 0$. Thus Eq. (12) can be written as

$$\mathbf{V}_{0,p,i}^{(k+1)} = \mathbf{V}_{1,p,i}^{(k)} + \mathbf{Q}_{p,i}^{(k)}, \quad (p, i = 0, 1, 2, \cdots), \tag{15}$$

where $\mathbf{Q}_{p,i}^{(k)} = [0, \bar{\delta}_{xp,i}^{(k)}, \bar{\delta}_{yp,i}^{(k)}, 0, 0, 0]^{\mathrm{T}}$. We also obtain from Eq. (13) by equating coefficients of the same order of τ at the two sides

$$\bar{\delta}_{xp,i}^{(k)} = \bar{\tau}_{xp,i-1}^{(k)}\bar{\eta}_x^{(k)} / i, \quad \bar{\delta}_{yp,i}^{(k)} = \bar{\tau}_{yp,i-1}^{(k)}\bar{\eta}_y^{(k)} / i,$$
$$(i = 1, 2, 3, \cdots). \tag{16}$$

From Eqs. (7), (8), (14) and (15), we can establish

$$\mathbf{V}_{1,p,i}^{(k+1)} = \mathbf{M}_{k+1}\mathbf{V}_{1,p,i}^{(k)} + \mathbf{M}_{k+1}\mathbf{Q}_{p,i}^{(k)}, (p, i = 0, 1, 2, \cdots). \tag{17}$$

Continuing the above procedure layer by layer, we finally obtain

$$\mathbf{V}_{1,p,i}^{(N)} = \mathbf{T}\mathbf{V}_{0,p,i}^{(1)} + \mathbf{S}_{p,i}, \quad (p, i = 0, 1, 2, \cdots), \tag{18}$$

where $\mathbf{T} = \prod_{j=N}^{1}\mathbf{M}_j$ is the global transfer matrix and

$$\mathbf{S}_{p,i} = \mathbf{M}_N\mathbf{Q}_{p,i}^{(N-1)} + \mathbf{M}_N\mathbf{M}_{N-1}\mathbf{Q}_{p,i}^{(N-2)} + \cdots + \prod_{j=N}^{2}\mathbf{M}_j\mathbf{Q}_{p,i}^{(1)}, \tag{19}$$

are the inhomogeneous terms associated with viscous interfaces, which vanish in the case of a perfectly bonded laminate.

If the state variables at the bottom surface are determined from Eq. (18) after applying the boundary conditions (see the next section), the ones at any interior point can be calculated as

$$\mathbf{V}_{p,i}^{(k)}(\zeta) = \exp[\mathbf{A}(\zeta - \zeta_{k-1})]\left(\prod_{j=k-1}^{1}\mathbf{M}_j\mathbf{V}_{0,p,i}^{(1)} + \mathbf{M}_{k-1}\mathbf{Q}_{p,i}^{(k-2)} + \mathbf{M}_{k-1}\mathbf{M}_{k-2}\mathbf{Q}_{p,i}^{(k-3)} + \cdots + \prod_{j=k-1}^{2}\mathbf{M}_j\mathbf{Q}_{p,i}^{(1)}\right), \tag{20}$$
$$(\zeta_{k-1} \le \zeta \le \zeta_k; \; p, i = 0, 1, 2, \cdots).$$

Then, the other three variables can be determined as

$$\sigma_x = \frac{c_{13}}{c_{33}}\sigma_z + \beta_1\frac{\partial u}{\partial x} + (\beta_2 - c_{66})\frac{\partial v}{\partial y}, \quad \sigma_y = \frac{c_{23}}{c_{33}}\sigma_z + (\beta_2 - c_{66})\frac{\partial u}{\partial x} + \beta_3\frac{\partial v}{\partial y},$$
$$\tau_{xy} = c_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \tag{21}$$

4. Boundary conditions and solution

In this paper, it is assumed that generally distributed normal pressures $q_b(x)$ and $q_t(x)$ are applied on the bottom and top surfaces, respectively. These loads can be expanded in terms of double sine functions as follows

$$q_b(x) = c_{44}^{(1)}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin(m\pi\xi)\sin(n\pi\eta),$$

$$q_t(x) = c_{44}^{(1)} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} b_{mn} \sin(m\pi\xi)\sin(n\pi\eta), \qquad (22)$$

where

$[a_n, b_n] = [4/c_{44}^{(1)}]\int_0^1\int_0^1 [q_b(\xi), q_t(\xi)]\sin(m\pi\xi)\sin(n\pi\eta)\,\mathrm{d}\xi d\eta$. In view of Eqs. (3) and (14), the boundary conditions for arbitrary m and n are

$$\bar{\sigma}_{p,0}^{(1)}\Big|_{\zeta=0} = a_{mn},\ \bar{\sigma}_{p,0}^{(N)}\Big|_{\zeta=1} = b_{mn},\ \bar{\sigma}_{p,i}^{(1)}\Big|_{\zeta=0} = \bar{\sigma}_{p,i}^{(N)}\Big|_{\zeta=1} = 0,\ \ (i = 1, 2, 3, \cdots),$$

$$\bar{\tau}_{xp,i}^{(1)}\Big|_{\zeta=0} = \bar{\tau}_{xp,i}^{(N)}\Big|_{\zeta=1} = \bar{\tau}_{yp,i}^{(1)}\Big|_{\zeta=0} = \bar{\tau}_{yp,i}^{(N)}\Big|_{\zeta=1} = 0,\ \ (i = 0, 1, 2, \cdots). \qquad (23)$$

Applying the above boundary conditions in Eq. (18), leads to

$$\begin{Bmatrix} b_{mn} \\ \bar{u}(1) \\ \bar{v}(1) \\ \bar{w}(1) \\ 0 \\ 0 \end{Bmatrix}_{p,0}^{(N)} = \mathbf{T}\begin{Bmatrix} a_{mn} \\ \bar{u}(0) \\ \bar{v}(0) \\ \bar{w}(0) \\ 0 \\ 0 \end{Bmatrix}_{p,0}^{(1)} + \mathbf{S}_{p,0},\ \ (p = 0, 1, 2, \cdots), \qquad (24)$$

$$\begin{Bmatrix} 0 \\ \bar{u}(1) \\ \bar{v}(1) \\ \bar{w}(1) \\ 0 \\ 0 \end{Bmatrix}_{p,i}^{(N)} = \mathbf{T}\begin{Bmatrix} 0 \\ \bar{u}(0) \\ \bar{v}(0) \\ \bar{w}(0) \\ 0 \\ 0 \end{Bmatrix}_{p,i}^{(1)} + \mathbf{S}_{p,i},\ \ \begin{matrix}(p = 0, 1, 2; \\ i = 1, 2, 3, \cdots)\end{matrix}. \qquad (25)$$

5. Numerical examples

The three-ply laminate subjected to a uniformly distributed normal pressure on the top surface ($q_t = p_0$), which was investigated by Srinivas and Rao [26], is re-considered. Comparison is made in Table 1, where a good agreement can be observed.

Table 1 Bending of a three-layered orthotropic plate under uniform pressure p_0 applied at the top surface.

Quantity	Present study	Srinivas [26]
$c_{11}^{(2)} w(a/2, b/2, z)/(hp_0)$		
At mid surface	-159.38	-159.38
$\sigma_x(a/2, b/2, z)/p_0$		
Top ply at top surface	-65.3743	-65.332
Top ply at interface	-48.8259	-48.857
Mid ply at upper interface	-4.90003	-4.9030
Mid ply at lower interface	4.85997	4.8600
Bottom ply at interface	48.6092	48.609
Bottomplyatbottom surface	65.0828	65.083
$\tau_{xz}(0, b/2, z)/p_0$		
At upper interface	-3.90825	-3.9285
Ad mid surface	-4.09568	-4.0959
At lower interface	-3.51543	-3.5154

Note: The material constants and plate geometry are the same as the three-layered plate considered by Srinivas and Rao [26] with $\gamma = c_{11}^{(1)}/c_{11}^{(2)} = 10$.

In the following, we assume only a normal sinusoidal pressure, $q_t = p_0\sin(\pi\xi)\sin(\pi\eta)$ is applied at the top surface of the plate. In addition, the viscosity coefficients in x and y directions are assumed identical ($\bar{\eta}^{(k)} = \bar{\eta}_x^{(k)} = \bar{\eta}_y^{(k)}$) but may be different for different interfaces. The following non-dimensional quantities are introduced:

$$\sigma = -\frac{\sigma_z(a/2, b/2, z, t)}{p_0},\ \ \omega_1 = \frac{\tau_{xz}(0, b/2, z, t)}{p_0},$$
$$\omega_2 = \frac{\tau_{yz}(a/2, 0, z, t)}{p_0},\ \ w_0 = \frac{c_{44}^{(1)}}{p_0}\frac{w(a/2, b/2, z, t)}{h},$$
$$u_0 = \frac{c_{44}^{(1)}}{p_0}\frac{u(0, b/2, z, t)}{h},\ \ v_0 = \frac{c_{44}^{(1)}}{p_0}\frac{v(a/2, 0, z, t)}{h}, \qquad (26)$$
$$\delta_1 = \frac{c_{44}^{(1)}}{p_0}\frac{\delta_x(0, b/2, t)}{h},\ \ \delta_2 = \frac{c_{44}^{(1)}}{p_0}\frac{\delta_y(a/2, 0, t)}{h}.$$

Table 2 Convergence study of the present method for a two-layered isotropic strip.

τ	$(M, \Delta\tau)$	σ	ω_1	$u_0^{(1)}$	$u_0^{(2)}$	w_0
0	(3, 0.1)	0.4215	-4.3926	-10.745	-10.745	-272.144
	(3,0.05)	0.4215	-4.3926	-10.745	-10.745	-272.144
	(4, 0.1)	0.4215	-4.3926	-10.745	-10.745	-272.144
	(4,0.05)	0.4215	-4.3926	-10.745	-10.745	-272.144
	Ref(18)	0.4215	-4.3926	-10.746	-10.746	-272.169
20	(3, 0.1)	0.3397	-2.2968	26.666	-37.9787	-555.770
	(3,0.05)	0.3397	-2.2968	26.666	-37.9788	-555.771
	(4, 0.1)	0.3397	-2.2968	26.666	-37.9788	-555.771
	(4,0.05)	0.3397	-2.2968	26.666	-37.9788	-555.771
	Ref(18)	0.3397	-2.2965	26.674	-37.986	-555.863

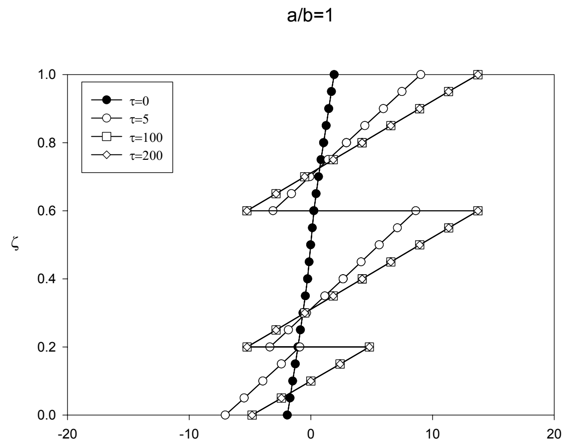
Note: The results are at the interface.

Next, we consider the bending problem of a simply supported five-layered cross-ply rectangular laminate, of which the aspect ratio h/a is fixed at 0.1. The thickness of each layer involved in the laminate is considered to be identical. The second and fourth interfaces are assumed to be perfect, while the first and third ones are viscous with $\eta^{(1)} = 2\eta^{(3)}$. The following typical material properties are adopted:

$$E_L / E_T = 25, G_{LT} / E_T = 0.5,$$

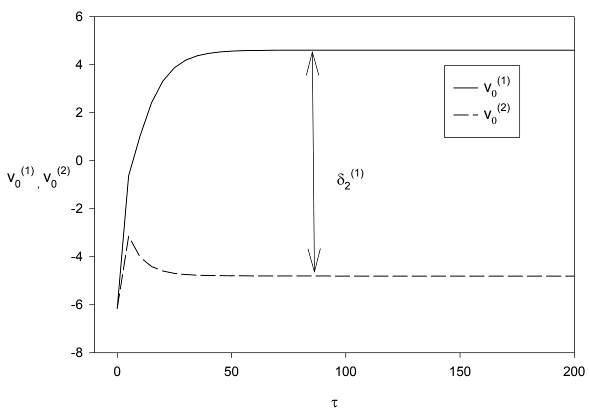
$$G_{TT} / E_T = 0.2, \quad \mu_{LT} = \mu_{TT} = 0.25 \qquad (27)$$

where E is the Young's modulus, G the shear modulus, μ the Poisson's ratio and subscripts L and T indicate, respectively, directions parallel and perpendicular to the fibers. The stacking sequence is (0/90/0/90/0°), from the bottom ($\zeta = 0$) to top ($\zeta = 1$). In the following, $M = 4$ and $\Delta\tau = 0.05$ will always be assumed, for which the results are of high accuracy as shown in the last example.



(f)

Fig. 2 Distributions of state variables through thickness: (a) σ; (b) ω_1; (c) ω_2; (d) w_0; (e) u_0; and (f) v_0.



(지면관계상 Fig. 2의 (a),(b),(c),(d),(e)는 싣지 않음)

Fig. 3 Variations of state variables versus time ($\zeta = 0.2$): (a) σ; (b) w_0; (c) ω_1, ω_2; (d) $u_0^{(1)}$, $u_0^{(2)}$; and (e) $v_0^{(1)}, v_0^{(2)}$

(지면관계상 Fig. 3의 (a),(b),(c),(d)는 싣지 않음)

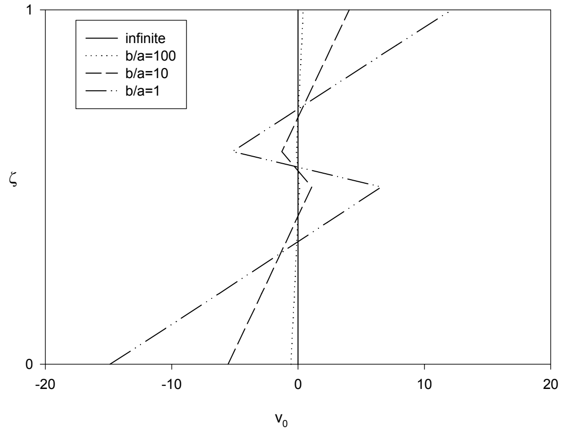


Fig. 4 Distributions of state variables through thickness at $\tau = 10$ for different values of a/b: (a) σ; (b) ω_1; (c) ω_2; (d) w_0; (e) u_0; and (f) v_0.

(지면관계상 Fig. 4의 (a),(b),(c),(d),(e)는 싣지 않음)

The distributions along the thickness direction of the six non-dimensional state variables defined in Eq. (26) are given in Figs. 2(a)-(f), respectively for the laminate of $a/b = 1$. The tangential displacements u_0 and v_0 become discontinuous across the two viscous interfaces when $\tau \neq 0$ due to the relative sliding, as shown in Figs. 2(e) and 2(f). The two transverse shear stresses are nearly zero at the two viscous interfaces when $\tau = 100$ and $\tau = 200$, as shown in Fig. 2(b) and Fig. 2(c), because when $\tau \to \infty$ the interfaces will lose the capability of transferring shear stress gradually. This has also been observed by He and Jiang [18] for an isotropic strip. In this respect, the correctness of the present method is again verified.

The variations of the six state variables versus time at $\zeta = 0.2$ are shown in Fig. 3, indicating a significant mutate at the earlier stage for all variables. Different sliding phenomena are observed in the x and y

directions, as shown in Figs. 3(d) and 3(e).

Figure 4 displays the through-thickness distributions of the state variable for various aspect ratios ($a/b = 1$, 10, 100, ∞) at $\tau = 10$. It is seen that the results for $a/b \geq 10$ are almost the same except for ω_2 and v_0. In the cylindrical bending problem, ω_2 and v_0 are usually ignored, which however, deviate significantly from that of an actual rectangular plate even for $b/a = 10$, as shown in Figs. 4(c) and 4(f).

6. Conclusions

A step-by-step state-space approach is proposed in this paper to study the time-dependent behavior of orthotropic laminated rectangular plates with viscous interfaces. The analysis is directly based on the three-dimensional equations of an orthotropic elastic body without introducing any assumption regarding the through-thickness distributions of deformations and stresses. Comparison with existent results is made and good agreement is obtained. Just like that predicated by the two-dimensional exact solution for an isotropic strip (18), we observe that the behaviors of laminate will gradually approach a final state for which the viscous interfaces lose the function of transferring shear stresses.

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