Quaternary LCZ Sequences Constructed From m-Sequences [1]

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Abstract

In this paper, given a composite integer n, we propose a method of constructing quaternary low correlation zone(LCZ) sequences of period 2^n-1 from binary m-sequences of the same length. The correlation distribution of these new quaternary LCZ sequences is derived.

I. Introduction

In a microcellular communication environment such as wireless LAN where the cell size is very small, transmission delay is relatively small and thus it is possible to maintain the time delay in reverse link within a few chip. In such a system as the quasi-synchronous code-division multiple-access(QS-CDMA) system proposed by Gaudenzi, Elia, and Vilola[1], multiple chip time delay among different users are allowed, which gives more flexibility in designing the wireless communication system.

In the design of sequences for QS-CDMA system, what matters most is to have low correlation zone around orgin rather than to minimize maximum nontrivial correlation value[5]. In fact, low correlation zone(LCZ) sequences show better performance than other well-known sequence sets with optimal correlation property. Let $\mathcal{S}$ be a set of M sequences of period N. If the magnitude of correlation function between any two sequences in $\mathcal{S}$ takes the values less than or equal to ϵ within the range $-L<\tau<L$, of the offset τ, then $\mathcal{S}$ is called an (N, M, L, ϵ) LCZ sequence set.

In this paper, given a composite integer n, we propose a method of constructing quaternary low correlation zone(LCZ) sequences of period 2^n-1 from binary m-sequences of the same length. The correlation distribution of these new quaternary LCZ sequences is derived.

II. Preliminaries

In this section, we introduce some definitions and notations.

Let F_{2^n} be the finite field with 2^n elements. The trace function $\mathrm{tr}_m^n(\cdot)$ from F_{2^n} to F_{2^m} is defined by $\mathrm{tr}_m^n(x)=\sum_{i=0}^{\frac{n}{m}-1} x^{2^{mi}}$, where $x \in F_{2^n}$ and $m|n$. It is well known that $\mathrm{tr}_1^n(\alpha^t)$ is a binary m-sequence of period 2^n-1, where α is a primitive element in F_{2^n}.

In this paper, we only deal with binary and quaternary sequences of period 2^n-1, which can be regarded as mappings from F_{2^n} to F_2 and to an integer ring $Z_4=\{0,1,2,3\}$, respectively. We use the notations $\boxplus$ for the addition in Z_4, only if we think it is necessary.

Let $F_{2^n}^* = F_{2^n} \setminus \{0\}$ and $s(x)$ be a mapping from F_{2^n} to F_2 or Z_4. When we restrict the mapping $s(x)$ to $F_{2^n}^*$ and replace x by α^t, then we can obtain a sequence $s(\alpha^t)$, $0 \le t \le 2^n-2$, of period 2^n-1. Hence, for convenience, we will use the expression 'a binary or quaternary sequence $s(\alpha^t)$ of period 2^n-1' interchangeably with 'a mapping $s(x)$ from F_{2^n} to F_2 or Z_4'.

For $\delta \in F_{2^n}^*$, the crosscorrelation function between two quaternary sequences $s_i(x)$ and $s_j(x)$ is defined as

$$R_{i,j}(\delta)=\sum_{x\in F_{2^n}^*} \omega_4^{s_i(x\delta)-s_j(x)}$$

where w_4 is a complex fourth root of unity.

It is not difficult to see that a quaternary sequences can be decomposed into two constituent binary sequences. Let v_1 and v_2 be variables over Z_2, i.e., Boolean variables. Then a variable v over Z_4 can be expressed as

$$v = v_1 \boxplus 2v_2. \qquad (1)$$

Let us use the notation $v=(v_2,v_1)$ to alternatively represent (1). Let $\phi(\cdot)$ and $\psi(\cdot)$ be the maps defined by

$$\phi(v)=v_1, \quad \psi(v)=v_2.$$

Using the expression (v_2, v_1), we can obtain the truth tables for $\phi(v-w)$ and $\psi(v-w)$ given in Table 1.

Let $v(x)$, $w(x)$, and $d(x)$ be quaternary sequences given as

$$v(x)=v_1(x)\boxplus 2v_2(x), \quad w(x)=w_1(x)\boxplus 2w_2(x)$$

[1]This work was supported in part by BK21 and ITRC program.