Problems of Special Causes in Feedback Adjustment

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Abstract

Process adjustment is a complimentary tool to process monitoring in process control. Process adjustment directs on maintaining a process output close to a target value by manipulating another controllable variable, by which significant process improvement can be achieved. Therefore, this approach can be applied to the 'Improve' stage of Six Sigma strategy. Though the optimal control rule minimizes process variability in general, it may not properly function when special causes occur in underlying process, resulting in off-target bias and increased variability in the adjusted output process, possibly for long periods. In this paper, we consider a responsive feedback control system and the minimum mean square error control rule. The bias in the adjusted output process is investigated in a general framework, especially focussing on stationary underlying process and the special cause of level shift type. Illustrative examples are employed to illustrate the issues discussed.

1. Introduction

Process adjustment, also called engineering process control (EPC), focuses on keeping a process output close to a desired target value, i.e., minimizing the process variability around the target value by manipulation of another controllable variable. In this paper, we consider a *responsive feedback control system* in which all the effects of a change in the manipulable variable will be realized on the output within one period. It is well known that the minimum mean square error (MMSE) control rule is optimal. However, when some special causes such as sudden changes in environmental conditions or mistakes by the operator occur, additional variations in underlying

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process may be introduced. In such cases, the effects of those causes may not be properly entertained by the MMSE control rule in process adjustment, resulting in off-target bias and increased variability in output process possibly for long periods.

In this paper, we consider the case that the underlying process is a stationary process and contaminated by special causes and the MMSE control rule is applied to process adjustment. Three types of special causes, AO, IO, and LS are considered. The effects of the causes on adjusted output process are derived under a general framework. Finally, illustrative examples are employed to understand and interpret the issues discussed.

2. Special Cause Problems

We consider a responsive feedback control system represented by

$$U_t = Y_t + Z_t \qquad (1)$$

where Z_t and U_t are the amount of deviation from target in the system output when a control action is and is not applied, respectively. In (1), $Y_t = gX_{t-1}$ is the amount of compensation on the output and g is the steady state gain [Box et al. (1994)]. In this paper Z_t, X_t, and U_t will be called underlying process (or unadjusted output), input variable, and adjusted output process, respectively.

2.1 MMSE Control Rule

We assumed that Z_t follows ARMA(p,q) model with *known* parameters, defined by

$$\phi(B)Z_t = \theta(B)a_t \qquad (2)$$

where a_t is a white noise process.

The one-step-ahead MMSE forecast (MMSEF) of Z_t and its error can be expressed as

$$\begin{aligned} Z_{t-1}(1) &= \psi_1 a_t + \psi_2 a_{t-1} + \cdots \\ e_{t-1}(1) &= a_t \end{aligned} \qquad (3)$$

The MMSEF (3) can be expressed in terms of present and past observations as

$$Z_{t-1}(1) = \pi_1 Z_{t-1} + \pi_2 Z_{t-2} + \cdots \qquad (4)$$

When no special cause occurs in the underlying process,

$$X_{t-1} = -Z_{t-1}(1)/g \qquad (5)$$

is the MMSE control rule. Then, the adjusted output at time t will then be

$$U_t = e_{t-1}(1) = a_t. \qquad (6)$$

Therefore, by applying process adjustment, the variation of the output process is reduced from the variance of unadjusted output, σ_Z^2, to the variance of adjusted output, σ_a^2.

2.2 Effects of Special Causes on Adjusted Output

If special cause occur, Z_t is influenced by the cause and we actually observe

$$N_t = Z_t + \omega\xi(B)I_t(T) \qquad (7)$$

where T is the time of occurrence, ω is the impact parameter of the cause, and $I_t(T)$ is the pulse indicator at time T. Three special causes types are considered: $\xi(B) = 1$ for AO, $\xi(B) = \psi(B)$ for IO, and $\xi(B)=1/(1-B)$ for LS type [see, Tsay (1986), Chen and Liu (1993a,b)].

We now consider such a situation that the MMSE control rule (5) is applied to N_t. Then, MMSEF will be computed with N by $\hat{Z}_{t-1}(1)=\pi^{(1)}(B)N_t$. Then,

$$\hat{Z}_{t-1}(1)=Z_{t-1}(1) + \omega\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i} \quad (8)$$

Therefor, the biases are $\omega\pi_{t-T}$ for AO, $\omega\psi_{t-T}$ for IO, and $\omega\sum_{i=1}^{t-T}\pi_i$ for LS type special cause, respectively.

When (5) is applied to N_t, X will be set as $\hat{X}_{t-1}=-\hat{Z}_{t-1}(1)/g$ and the bias on input variable can be expressed as

$$\hat{X}_{t-1}=X_{t-1}-(\omega/g)\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i}. \quad (9)$$

Then, the adjusted output can be expressed, in terms of U_t, as

$$\hat{U}_t = U_t+\omega(\xi_{t-T}-\sum_{i=1}^{t-T}\pi_i\xi_{t-T-i}) \quad (10)$$

It is to be noted that at t, the bias in the adjusted output ω for all types. However, for time $t>T$, the bias is $-\omega\pi_{t-T}$ for AO, zero for IO, and $\omega(1-\sum_{i=1}^{t-T}\pi_i)$ for LS type cause, which are summarized in Table 1.

Note that bias can be interpreted as the mean level shift in output process. For an AO type special cause, the bias in U_t at the occurrence time is ω but it decreases with opposite sign because $-\omega\pi_{t-T}$ converges to zero as $t\rightarrow\infty$ because stationarity of Z_t guarantees $\pi_j\rightarrow 0$ as $j\rightarrow\infty$. Meanwhile, an IO type special cause produces effects that follow the pattern of dependent structure of the underlying process Z_t, i.e., the ψ-weights of the model, resulting in no bias in the adjusted output process.

For LS type special cause, the mean level shift $\omega(1-\sum_{i=1}^{t-T}\pi_i)$ is ω at time $t=T$, and converges to some non-zero finite value as time goes on. It is to be noted that when underlying process is stationary, the effects of a special cause of LS type would not vanish eventually, resulting in permanent mean shift in the adjusted output process.

3. Illustrative Examples

We investigate the effects on adjusted outputs for three illustrative models: AR(1), MA(1), and ARMA(1,1), in details.

AR(1) Process

AR(1) process is considered appropriate to many real situations, especially for positive parameter values [Atienza et al. (1998), Montgomery (2001)]. Since $\pi_j=\phi$ for $j=1$ and 0 for $j\geq 2$ and $\psi_j=\phi^j$, the bias is $\omega(\xi_{t-T}-\phi\xi_{t-T-i})$.

For AO type special cause, because of the Markov property of AR(1) model, the biases are ω and $-\omega\phi$ at times T and $T+1$, respectively, but zero for all time $t(\geq T+2)$. For IO type, the bias is ω at time T, but zero at times $t(\geq T+1)$.

Meanwhile, Markov property of AR(1) implies that a special cause of LS type produces bias of ω at time T, and non-zero constant $\omega(1-\phi)$ for all time after T. Therefore, MMSE controller applied to the contaminated series does not compensate all the effects of LS type cause, resulting in permanent mean shift.

MA(1) Process

Using the fact that $\pi_j=\phi$ and $\psi_j=\theta^j$ for $j=1$ and 0 for $j\geq 2$, the bias is

$$\omega(\xi_{t-T}+\sum_{i=1}^{t-T}\theta^i\xi_{t-T-i}).$$

For AO type, the biases are ω at T and $\omega\theta^{t-T}$ decreases geometrically as time t increases, vanishing eventually. For IO type, the bias is ω at T, but zero at time $t(\geq T+1)$.

The bias incurred by a LS type cause is ω at time T and $\omega(1-\theta^{t-T+1})/\lambda$ for $t(\geq T+1)$, where $\lambda=1-\theta$. Thus, the bias decreases as time goes on and converges to ω/λ eventually. That is, an LS type special cause can not be completely compensated by MMSE control.

ARMA(1,1) Process

For this model, $\psi_j=\delta\phi^{j-1}$ and $\pi_j=\delta\theta^{j-1}$ for $j\geq 1$, where $\delta=\phi-\theta$. The bias is simplified as

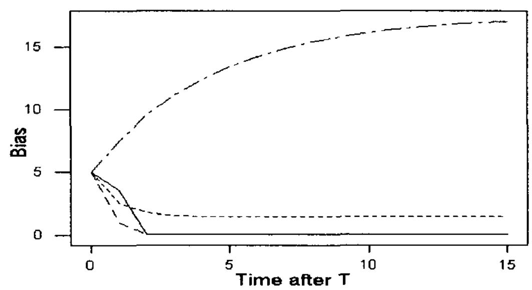
$$\omega(\xi_{t-T}-\delta\sum_{i=1}^{t-T}\theta^{i-1}\xi_{t-T-i}).$$

Note that biases at T are ω for all types of special causes. For $t(\geq T)$, the bias led by AO type is $-\omega\delta\theta^{t-T-1}$ which decreases to zero. For IO type, there will be no bias at time $t(\geq T+1)$.

For the LS type, MMSE control rule produces bias as $\omega\{1-\delta(1-\theta^{t-T})/\lambda\}$, which converges to a non-zero permanent mean shift $\omega(1-\phi)/(1-\theta)$, eventually.

A Numerical Example

As an numerical example, we consider four models such as two AR(1) model with $\phi=$ 0.3 and 0.8, and two ARMA(1,1) models with parameters of $(\phi,\theta)=$ (0.8, 0.3) and (0.3, 0.8). the biases at times $t\geq T$ led by a LS type special cause with $\omega=5.0$ are computed for each model and given in Figure 2.



<Figure 1> Bias of adjusted output at times after T: { $\omega=5.0$ and $(\phi,\theta)=$ (0.3, 0.0), solid; (0.8, 0.0), dash; (0.8, 0.3), dot; (0.3, 0.8), dash 1-dot line}

For AR(1) model, because of the Markov property, the effects of the LS are present at time T and $T+1$ only. It is noted that a model with greater parameter value has smaller effects in process adjustment. For ARMA(1,1) model, the biases depend on the parameters ϕ and θ. The biases given in Table1 depend mainly on $\phi-\theta$ and $\lambda=1-\theta$, and thus relative size of ϕ and θ is crucial for the pattern of biases. When $(\phi,\theta)=$

$(0.8, 0.3)$, $\phi - \theta = 0.5$, biases decreases to the limit vale $\omega(1-\phi)/(1-\theta)$ which is smaller than $\omega = 5.0$. Meanwhile, for the model with $(\phi, \theta) = (0.3, 0.8)$, biases increases to the limit that is larger than the impact parameter, $\omega = 5.0$. In summary, the performance of process adjustment may depend on the degree of dependency in the underlying process.

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