

# Parameter Recovery for LIDAR Data Calibration Using Natural Surfaces

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**Abstract:** This paper focuses on recovering systematic biases during LIDAR calibration, particularly using natural surfaces as control features. Many previous approaches have utilized all the points overlapping with the control features and often experienced with an inaccurate value converged with a poor rate due to wrong correspondence in pairing a point and the corresponding control features. To overcome these shortcomings, we establish a preventive scheme to select the pairs of high confidence, where the confidence value is based on the error budget associated with the point measurement and the linearity and roughness of the control feature. This approach was then applied to calibrating the LIDAR data simulated with the given systematic biases. The parameters were successfully recovered using the proposed approach with the accuracy and convergence rate superior to those using the previous approaches.

**Keywords:** LIDAR, calibration, natural surfaces, parameter recovery

## 1. Introduction

The LIDAR system effectively produces the precise coordinates of points sampled from the object surfaces. It includes three main subsystems, GPS, INS, and a laser ranging module that provide the position of the platform, the attitude of the platform attitude, and the distance from the platform to a target, respectively. These three kinds of information are integrated to compute the coordinates of a target point. These coordinates may include systematic errors originated from various sources, for examples, the systematic biases inherited from each subsystem, and the integration biases produced from their imperfect alignment [1-3]. The process to remove these systematic errors, called calibration, is essential to produce more accurate coordinates.

A typical calibration process mainly involves modeling systematic errors with bias parameters introduced by analyzing the error sources, and estimating the parameters by comparing the points with given reference data (or called control features). Estimating the parameters, so

called parameter recovery, often use surface patches analytically described as control features, for example, [1-2]. However, such a process may fail to estimate a number of parameters with reasonable accuracy since the parameters are usually highly correlated and it is difficult to find sufficient number of independent analytical surface patches. This brings the use of natural surfaces as control features since they include planar patch of various slopes and directions.

We proposed a robust method for parameter recovery using natural surface as control features. The core of this method is to employ a sophisticated selection scheme that selects a portion of points that corresponds to given control features with high confidence. This selection is based on the error budget associated with the point measurement and the linearity and roughness of the control feature. This paper describes the proposed method with theoretical background and mathematical derivation, introduces experimental results from its application to simulated data, conclude with summary and comment.

## 2. Theory and Method

### 1) Laser Equations

Laser equations are used to compute the 3D coordinates of the point on a surface, which a laser pulse reflected from. A simplified one can be expressed as

$$p' = P' + R' \cdot u \cdot l', \quad (1)$$

where  $p'$  and  $P'$  denotes the locations of the point and the platform, respectively. These are expressed as 3D vectors in a ground coordinate system.  $l'$  denotes the distance between the point and the platform.  $u$  denotes the direction from the platform to the point, expressed a 3D unit vector in the platform coordinate system, which is related with the ground coordinate system based on a rotational matrix,  $R'$ . The prime super-

scripts indicate that the values are not true but observed values, which may include some unknown systematic and random errors. Each vector can be expressed as

$$p' = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}, \quad P' = \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}, \quad u = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}. \quad (2)$$

If we assume that the observed values are affected from systematic errors, the location of the point can be calibrated using

$$p = (P' + \Delta P) + (I_3 + \Delta R)R' \cdot u \cdot (l' + \Delta l), \quad (3)$$

where  $\Delta P$ ,  $\Delta l$ , and  $\Delta R$  denote the systematic biases associated with  $P'$ ,  $l'$ , and  $R'$ , respectively. Each bias can be expressed with its elements as

$$\Delta P = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}, \quad \Delta R = \begin{bmatrix} 0 & -\Delta \kappa & \Delta \phi \\ \Delta \kappa & 0 & -\Delta \omega \\ -\Delta \phi & \Delta \omega & 0 \end{bmatrix} \quad (4)$$

As Eq.(5) rearranged from Eq.(3),  $p$  includes two parts, the observed value  $p$  and the update value  $\Delta p$  based on the introduced biases.  $p$  is expressed as Eq.(6).

$$p = p' + \Delta p \quad (5)$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} \quad (6)$$

$$\Delta p \equiv \Delta P + \Delta R R' \cdot u \cdot l' + R' \cdot u \cdot \Delta l \quad (7)$$

## 2) Observation Equations for Parameter Recovery

A set of control features is given as a 2D surface function expressed as

$$z = f(x, y), \quad (8)$$

where this function can be constructed from a DEM, TIN, or a set of surface points.

A perfectly calibrated point should be on the surface, which presents a condition that its coordinates should satisfy the given surface function. If we substitute  $x$ ,  $y$ ,  $z$  in Eq.(8) with the calibrated coordinates in Eq.(6), we acquire

$$z' + \Delta z = f(x' + \Delta x, y' + \Delta y). \quad (9)$$

If we assume that the updates originated from the systematic biases  $\Delta x$ ,  $\Delta y$ ,  $\Delta z$  are small, Eq.(9) can be linearized and rearranged as

$$z' - f(x', y') = \left[ \frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \quad -1 \right] \cdot \Delta p. \quad (10)$$

Here  $\Delta p$  is a function of the systematic biases as shown in Eq.(7) and further arranged as

$$\Delta p = \begin{bmatrix} 1 & 0 & 0 & 0 & RL'_z & -RL'_y & Ru_x \\ 0 & 1 & 0 & -RL'_z & 0 & RL'_x & Ru_y \\ 0 & 0 & 1 & RL'_y & -RL'_x & 0 & Ru_z \end{bmatrix} \cdot \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta \omega \\ \Delta \phi \\ \Delta \kappa \\ \Delta l \end{bmatrix}, \quad (11)$$

where

$$L' \equiv u \cdot l', \quad \begin{bmatrix} RL'_x \\ RL'_y \\ RL'_z \end{bmatrix} \equiv R' \cdot L', \quad \begin{bmatrix} Ru_x \\ Ru_y \\ Ru_z \end{bmatrix} \equiv R' \cdot u \quad (12)$$

If we substitute  $\Delta p$  in Eq.(10) with Eq.(11), we acquire an observation equation expressed as

$$y_k = A_k \xi + e_k, \quad e_i \sim (0, \sigma_0^2), \quad (13)$$

where  $y_k$ ,  $A_k$ ,  $\xi$  are defined as Eq.(14).  $e_k$  is a random error associated with the equation where its mean and variance are assumed as 0 and  $\sigma_0^2$ , respectively. Subscript  $k$  indicates that the equation is constructed from the  $k$ -th laser point. Each point usually a different observation equation of a similar structure to Eq.(13).

$$y_k \equiv z' - f(x', y')$$

$$A_k \equiv \left[ \frac{\partial f}{\partial x} \quad \frac{\partial f}{\partial y} \quad -1 \right] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & RL'_z & -RL'_y & Ru_x \\ 0 & 1 & 0 & -RL'_z & 0 & RL'_x & Ru_y \\ 0 & 0 & 1 & RL'_y & -RL'_x & 0 & Ru_z \end{bmatrix} \quad (14)$$

$$\xi \equiv [\Delta X \quad \Delta Y \quad \Delta Z \quad \Delta \omega \quad \Delta \phi \quad \Delta \kappa \quad \Delta l]^T$$

If we have  $n$  number of laser points, we construct a set of  $n$  number of observation equations,

$$y = A \xi + e, \quad e \sim (0, \sigma_0^2 I_n), \quad (15)$$

where  $y$ ,  $A$ ,  $e$  are defined as Eq.(16) and  $e$  denote the random errors associated with all equations and they are assumed to be independent with the same variance.

$$y \equiv \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \quad A \equiv \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix}, \quad e \equiv \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \quad (16)$$

### 3) Recursive Parameter Estimation

Parameter estimation is performed using a recursive estimation process based on the linearized observation equations in Eq.(15). The initial values for all the parameters are first assigned to zeroes and the parameters are then estimated using the observation equations constructed with the initial values. These estimated parameters are then used as the initial values for the next iteration. This process continues if the updates for the parameters are negligible.

The main concerns of such a recursive estimation process are dependence on the initial values, the accuracy of the converged values and the convergence rate. These are all closely related to the validity of the linearization performed from Eq.(9) to produce the linearized model in Eq.(10). The smaller the error associated with this linearization, the better results from the recursive process produced. Theoretically, these linearization errors are then eliminated if the reference surface is perfectly planar. It is because the surface function in Eq.(10) is then linear if the surface is a plane. However, natural surfaces never be represented by an infinitely large plane. We should consider the area and boundary of the surface that can be represented as a plane with a reasonably small roughness. The area should at least larger than an uncertainty ellipse centering a laser point with the size derived from a rough error budget about the coordinates of the point. The roughness can also be used to a weight for an observation equation. That is, the rougher surface, the less weight is assigned. Consequently, only a portion of laser points locating within the boundary of surfaces reasonably approximated to planes are carefully selected to construct the observation equations. Otherwise, this recursive estimation process easily results in wrong converged values with long iteration steps and heavy dependence on initial values.

## 3. Experiment and Results

We applied the proposed method to various sets of simulated data. To assess the accuracy of the recovered parameters, we used simulated data instead of real data. This section introduces a representative experiment by describing the test data and the results from the application of the proposed recovery method to the data.

### 1) Test Data

The test data were generated by simulating airborne LIDAR survey over an area of natural surfaces [4]. The topography of this area is given by a DEM, which retains the grid interval of 10 m and covers an area of 1200 m by 1200 m. As shown in Fig. 1, this area includes natural surfaces of various slopes and roughness. A small portion of this DEM is later used as reference data for calibration, so called a control DEM. Table 1 summarizes the operational parameters and Table 2 shows the systematic biases used for the simulation. Five data sets (or

strips) were generated with the given trajectories described in Table 3. In total, the simulation generated more than five million points as shown in Table 4. The boundaries and trajectories of the data sets are visualized with the control DEM in Fig. 2.

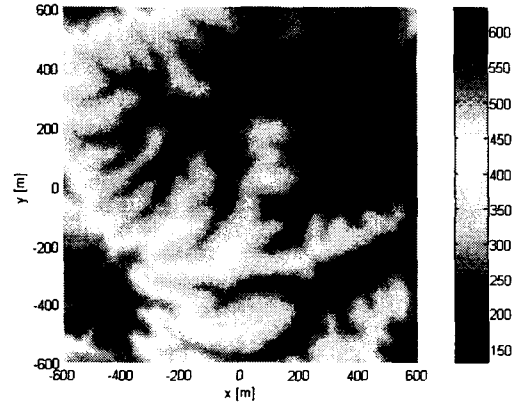


Fig. 1: Area used for LIDAR data simulation.

Table 1: Operational parameters of LIDAR simulation.

Platform velocity [m/s]	30
Pulse rate [kHz]	30
Scan rate [Hz]	50
Scan angle [deg]	20

Table 2: Systematic biases for LIDAR simulation.

GPS bias, x [m]	$\Delta X$	2
GPS bias, y [m]	$\Delta Y$	1
GPS bias, z [m]	$\Delta Z$	0
INS bias, omega [deg]	$\Delta \omega$	0.1
INS bias, phi [deg]	$\Delta \phi$	0.2
INS bias, kappa [deg]	$\Delta \kappa$	0
Range bias [m]	$\Delta l$	0.0

Table 3: Trajectories of simulated data sets.

Set	trajectory begin			trajectory end		
	x	y	z	x	y	z
1	-500	300	1500	500	300	1500
2	500	0	1500	-500	0	1500
3	-500	-300	1500	500	-300	1500
4	150	-550	1500	150	550	1500
5	-150	550	1500	150	-550	1500

Table 4: Simulation duration and number of points.

Set	Duration [s]	No. points [s]
1	33.34	1000200
2	33.34	1000200
3	33.34	1000200
4	36.68	1100400
5	36.68	1100400
<b>Total</b>	<b>173.38</b>	<b>5201400</b>

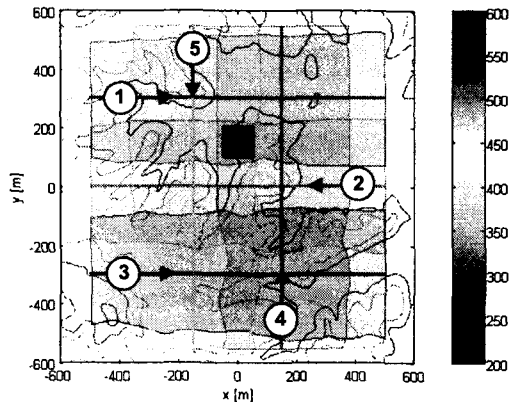


Fig. 2: Simulated data set and a control DEM.

## 2) Test Results

The proposed method is implemented as an object oriented program using C++ with Standard Template Library. This program was applied to estimating the bias parameters using the control DEM.

Among these points within the range of the control DEM, we randomly selected 10 % of the points, that is, 9245 points. At each step, we computed the roughness of the small local area centering each point and selected only the points with the roughness values of less than  $\pm 0.4$  m. In average, 8 to 10 % of the total points were actually selected as shown in Table 5.

Table 5: Number of points used for estimation.

Iteration	Total	1	2	3	4	5
No. points	9245	756	877	873	874	874
Ratio [%]	100	8.2	9.5	9.4	9.5	9.5

The iteration quickly converged. After each iteration step, we updated the points with the estimated parameters. In five steps, the RMS update decreased to  $\pm 8.2e-9$  m. Table 6 shows the bias parameters estimated by each iteration step. It is shown that the parameters quickly converge to the true values with acceptable tolerance.

Table 6: Parameters estimated at each iteration step.

Iteration	$\Delta X$	$\Delta Y$	$\Delta \omega$	$\Delta \phi$
1	2.1791	0.7295	0.1032	0.2109
2	1.9827	0.9711	0.1015	0.1989
3	1.9886	0.9668	0.1010	0.1991
4	1.9886	0.9671	0.1010	0.1991
5	1.9886	0.9671	0.1010	0.1991
True	2.0000	1.0000	0.1000	0.2000

The estimated parameters were then used to calibrate the entire sets of more than five million points. The RMS error that quantifies the difference between the observed point and the corresponding error-free point can be computed since the error-free points are known from the

simulation. The RMS errors were computed from each set before and after calibration, as shown in Table 7. After calibration, the data were refined with about  $\pm 1$  cm in every direction. Particularly, even though set 3 has no overlap with the control DEM, the RMS error of its calibrated set is similar to other sets.

Table 7: RMS errors before and after calibration (unit: m).

Set	Before calibration			After calibration		
	dx	dy	dz	dx	dy	dz
1	2.346	3.171	0.219	0.007	0.010	0.010
2	2.252	3.123	0.213	0.007	0.011	0.010
3	2.027	3.004	0.203	0.006	0.012	0.009
4	2.296	3.146	0.437	0.007	0.011	0.010
5	2.141	3.066	0.418	0.006	0.011	0.010

## 4. Conclusions

We have proposed a robust method to estimating systematic biases for LIDAR data calibration and verified the method by applying it to calibrating the simulated data. From the experimental results, we conclude that the proposed method can recover the bias parameters with acceptable errors within a few iteration steps, and the calibration results based on the recovered parameters shows the superior quality with respect to RMS errors.

## Acknowledgement

The work was supported by grants from The University of Seoul, Korea.

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