

# Design of High Stability Space Tube

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**Abstract:** Laminate Design of a composite tube for a space telescope(Fig. 1) under hygrothermal load is studied. Carpet plots for laminate effective engineering constants are generated and used for selecting the best tube lay-ups satisfying the opto-mechanical requirements for a space telescope being dimensionally stable under orbital thermal loading. Despace of the tubes constructed with the selected lay-ups are calculated with a Zig-Zag Triangular Element which accurately represents through thickness stress variations for laminated structures. The effects of moisture absorption when exposed to humidity environment and moisture desorption through outgassing on the dimensional stability are also investigated.

**Keywords:** HygroThermal, Composite Tube, Zig-Zag Theory.

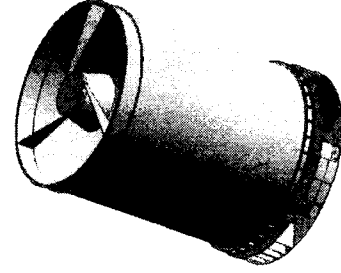


Fig. 1. Super Resolution Space Camera.

## 1. Introduction

Composite materials have been widely used for aerospace, automobile and civil structures due to high specific strength(strength to density ratio) and high specific stiffness(modulus to density ratio) and these superior mechanical properties along with excellent hygrothermal features make the composite materials engineer's favorite choice for the space instruments which require highly dimensional stability under orbital environmental conditions.

There need certain mechanical requirements for a space instrument to be dimensionally stable in orbital operational conditions to produce high quality scientific data, which are attained by selecting the proper laminated configurations. Carpet plot, graphical representations showing the range of properties, is particularly helpful for the designer to determine the best laminated configurations satisfying the requirements[1].

Structural analysis of the laminated structures demands an accurate laminated theory to account for layerwise property variations. Equivalent single layer representations of the laminated structures lead to inaccurate predictions of through thickness stress variations while the zig-zag displacement theory based on the refined higher order displacement field that satisfies *priori* traction free boundary conditions superimposed by the zig-zag displacement field gives very precise stress distribution through thickness compared to the three dimensional elasticity solutions([2], [3], [4]).

In the present work, laminate design of a dimensionally stable composite tube for a space telescope under hygrothermal environment was carried out with Carpet plots. Despace of the tubes constructed with the selected lay-ups were calculated with a Zig-Zag Triangular Element developed which accurately represents through thickness stress variations for laminated structures. The effects of moisture absorption and desorption on the dimensional stability are also investigated

## 2. Theoretical Development

Consider a plate composed of multiple layers. The improved layerwise zig-zag displacement fields for the  $k^{th}$  layer are defined as in [5],

$$\begin{aligned} u^{(k)} &= u_0 - z \frac{\partial w_0}{\partial x} + h_{11}^{(k)} \phi_x + h_{12}^{(k)} \phi_y \\ v^{(k)} &= v_0 - z \frac{\partial w_0}{\partial y} + h_{21}^{(k)} \phi_x + h_{22}^{(k)} \phi_y \\ w^{(k)} &= w_0 \end{aligned} \quad (1)$$

where  $u_0, v_0$  are the in-plane displacements and  $w_0$  is a transverse displacement and  $\phi_x, \phi_y$  are the higher order shear rotations and  $h_{11}^{(k)}, h_{12}^{(k)}, h_{21}^{(k)}, h_{22}^{(k)}$  are defined in [5].

The linear strain-displacement relations for the  $k^{th}$  layer in a Cartesian coordinate system are defined as follows,

$$\begin{Bmatrix} e_x \\ e_y \\ \gamma_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} 1 & 0 & 0 & -z & 0 & 0 & h_{11}^{(k)} & h_{12}^{(k)} & 0 & 0 \\ 0 & 1 & 0 & 0 & -z & 0 & 0 & 0 & h_{21}^{(k)} & h_{22}^{(k)} \\ 0 & 0 & 1 & 0 & 0 & -z & h_{21}^{(k)} & h_{22}^{(k)} & h_{11}^{(k)} & h_{12}^{(k)} \end{bmatrix} \{\varepsilon\} \quad (2)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}^{(k)} = \begin{bmatrix} h_{21,z}^{(k)} & h_{22,z}^{(k)} \\ h_{11,z}^{(k)} & h_{12,z}^{(k)} \end{bmatrix} \{\gamma\} \quad (3)$$

and the general membrane-bending strain( $\{\varepsilon\}$ ) is defined as,

$$\{\varepsilon\} = \left\{ \frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}, \frac{\partial^2 w_0}{\partial x^2}, \frac{\partial^2 w_0}{\partial y^2}, 2 \frac{\partial^2 w_0}{\partial x \partial y}, \frac{\partial \phi_x}{\partial x}, \frac{\partial \phi_x}{\partial x}, \frac{\partial \phi_x}{\partial y}, \frac{\partial \phi_x}{\partial y} \right\} \quad (4)$$

the general shear strain ( $\{\gamma\}$ ) is defined as,

$$\{\gamma\} = \{\phi_x, \phi_y\}^T \quad (5)$$

The reduced-transformed orthotropic stress-strain relation under the plane stress assumption ( $\sigma_z = 0$ ) for any individual layer ( $k^{th}$  layer) is defined as,

$$\{\sigma\}^{(k)} = [C]^{(k)} \{e\}^{(k)} \quad (6)$$

$$\{e\}^{(k)} = \begin{Bmatrix} e_x \\ e_y \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}^{(k)} = -\Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ 0 \\ 2\alpha_{xy} \end{Bmatrix}^{(k)} - \Delta C \begin{Bmatrix} \beta_x \\ \beta_y \\ 0 \\ 2\beta_{xy} \end{Bmatrix}^{(k)} \quad (7)$$

where  $\Delta T$  and  $\Delta C$  are the changes in temperature and moisture concentration, respectively, and  $\alpha_x, \alpha_y, \alpha_{xy}$ : thermal expansion coefficients are defined as,

$$\begin{aligned} \alpha_x &= \cos^2 \theta_k \alpha_1 + \sin^2 \theta_k \alpha_2 \\ \alpha_y &= \sin^2 \theta_k \alpha_1 + \cos^2 \theta_k \alpha_2 \\ \alpha_{xy} &= \sin \theta_k \cos \theta_k (\alpha_1 - \alpha_2) \end{aligned} \quad (8)$$

above,  $\alpha_1, \alpha_2$  are thermal expansion coefficients in material principal directions and  $\beta_x, \beta_y, \beta_{xy}$ : moisture expansion coefficients are defined in the same manner as thermal expansion coefficients.

A three node triangular element (Fig. 2) based on the Zig-Zag displacement field (1) is developed. The elemental degrees of freedom ( $\{d_e\}$ ) are defined as,

$$\{d_e\} = \{\{d_e\}_1, \{d_e\}_2, \{d_e\}_3\} \quad (9)$$

where  $\{d_e\}_i = \{u_0^i, v_0^i, w_0^i, \theta_x^i, \theta_y^i, \theta_z^i, \phi_x^i, \phi_y^i, \phi_z^i\}, i=1,2,3$ .

and  $\theta_x^i = \left(\frac{\partial w_0}{\partial y}\right)_i, \theta_y^i = -\left(\frac{\partial w_0}{\partial x}\right)_i$ .

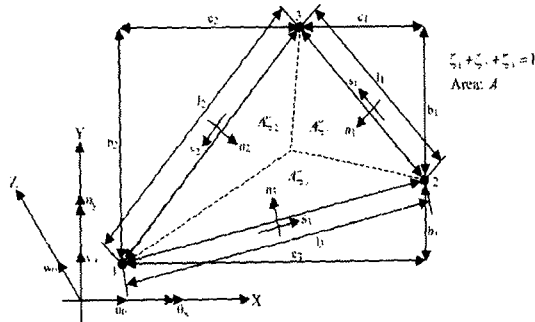


Fig. 2. 3-node Zig-Zag Triangular Element.

### 3. Numerical Results

Laminate Design of a composite tube for a space telescope (Figure 3) under hygrothermal load was carried out using

Carpet plots and despace for the selected lay-ups was calculated with a zig-zag triangular element. Mechanical properties for the UD tape used in this study, temperature ranges and requirements for mechanical properties are listed in Table 1.

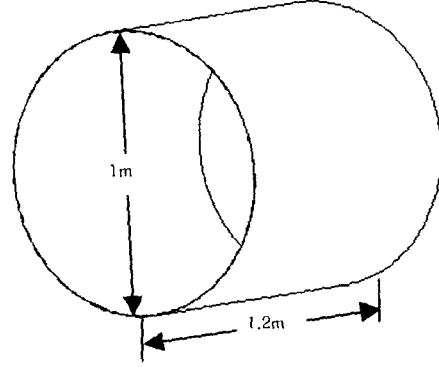


Fig. 3. Composite tube.

Table 1. Parameters for composite tube.

UD: Mechanical properties	$E_1 = 538 \text{ GPa}$
	$E_2 = 5.1 \text{ GPa}$
	$G_{12} = 4.5 \text{ GPa}$
	$\nu_{12} = 0.3$
	$\alpha_1 = -1.08 \text{ ppm}/^\circ \text{C}$
	$\alpha_2 = 30.02 \text{ ppm}/^\circ \text{C}$
	$\beta_1 = 10 \text{ ppm}/\%$
	$\beta_2 = 1500 \text{ ppm}/\%$
	$F_{1t} = 1841 \text{ MPa}$
	$F_{1c} = 366 \text{ MPa}$
$F_{2t} = 20 \text{ MPa}$	
$F_{2c} = 120 \text{ MPa}$	
$F_{16} = 47 \text{ MPa}$	
$\rho = 1796 \text{ kg}/\text{m}^3$	
Temperature range	$-20^\circ \text{C} \sim 55^\circ \text{C}$
Requirements for mechanical properties	$E_x \geq 290 \text{ GPa}$ $\alpha_x \leq 1 \text{ ppm}/^\circ \text{C}$

Carpet plots for laminate effective engineering constants are generated, showing how a given laminate property varies with the percentages of the plies at the various orientations and longitudinal modulus ( $E_x$ ) is depicted in Figure 4, shear modulus ( $G_{xy}$ ) in Figure 5, Poisson's ratio ( $\nu_{xy}$ ) in Figure 6, coefficient of thermal expansion ( $\alpha_x$ ) in Figure 7, coefficient of moisture expansion ( $\beta_x$ ) in Figure 8, longitudinal tensile strength ( $F_{1t}$ ) in Figure 9 and shear strength ( $F_{16}$ ) in Figure 10. Tsai-Wu First-Ply Failure was used to calculate laminate

strengths. The fractions of  $0^\circ$ ,  $90^\circ$  and  $\pm 45^\circ$  plies are defined as,

$$\alpha = \frac{2m}{N}, \quad \beta = \frac{2n}{N}, \quad \gamma = \frac{4p}{N}, \quad \alpha + \beta + \gamma = 1$$

Total Number of plies,  $N = 2(m + n + 2p)$

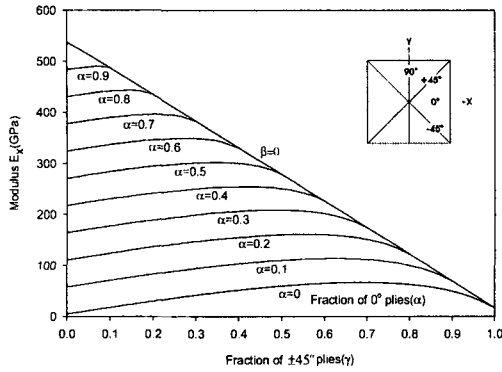


Fig. 4. Carpet plot: longitudinal modulus,  $[0_m/90_n/\pm 45_p]_s$ .

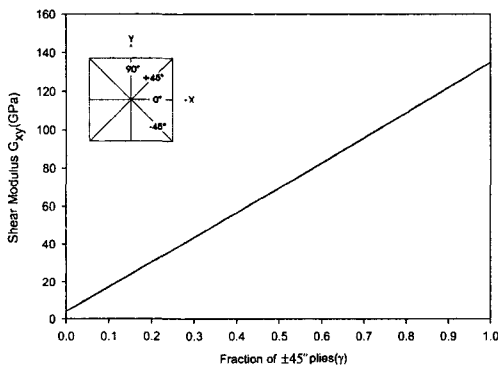


Fig. 5. Carpet plot: shear modulus,  $[0_m/90_n/\pm 45_p]_s$ .

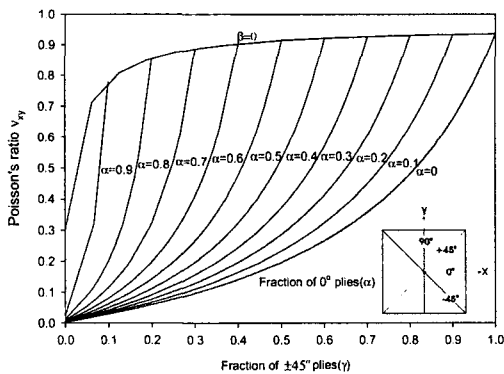


Fig. 6. Carpet plot: Poisson's ratio,  $[0_m/90_n/\pm 45_p]_s$ .

Table 2 shows laminate effective engineering constants for the selected lay-ups satisfying the opto-mechanical requirements with the least number of layers for a space telescope to be dimensionally stable under orbital thermal loading.

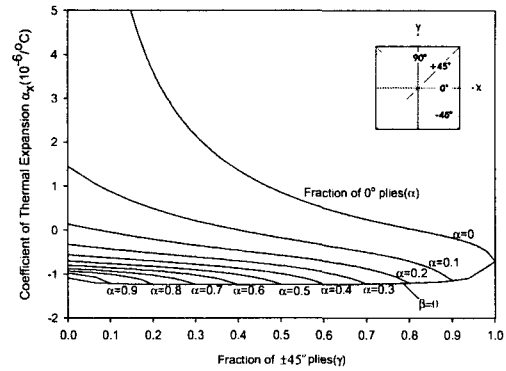


Fig. 7. Carpet plot: Coefficient of thermal expansion,  $[0_m/90_n/\pm 45_p]_s$ .

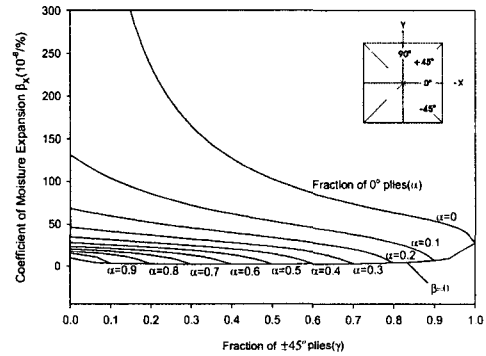


Fig. 8. Carpet plot: Coefficient of moisture expansion,  $[0_m/90_n/\pm 45_p]_s$ .

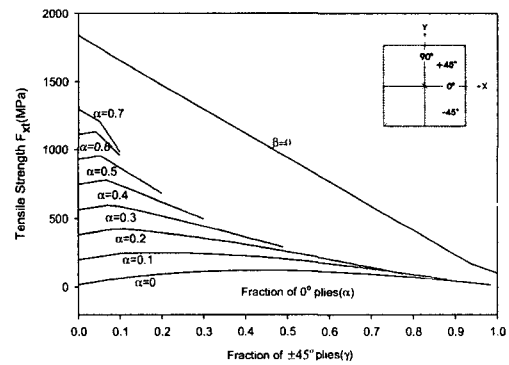


Fig. 9. Carpet plot: Uniaxial tensile strength,  $[0_m/90_n/\pm 45_p]_s$ .

Table 3 shows despace for the selected lay-ups for the survival temperature ranges with no moisture concentration. Cyanate matrix consisting UD tape has a very low moisture uptake and low outgassing, ideal for space environments where thermal and hygroscopic stability are critical. Space tube got through

high temperature curing process and outgassing before entering

the orbit and even further bake-out action once in orbit, which lead to a very low enough moisture change not producing significant dimensional stability. Also low temperature range for the operation is critical for dimensional stability of the tube, which was confirmed by analysis in Table 3.

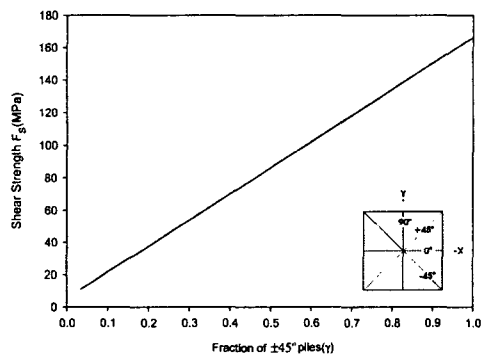


Fig. 10. Carpet plot: Shear strength,  $[0_m/90_n/\pm 45_p]_s$ .

Table 2. Laminate effective engineering constants for selected lay-ups.

$[0_m/90_n/(\pm 45)_p]_s$	$E_x$	$G_{xy}$	$\nu_{xy}$	$\alpha_x$	$\beta_x$
m=3,n=1,p=1	302.6	48.0	0.3	-0.93	17.17
m=5,n=1,p=1	361.4	37.2	0.3	-0.99	14.39
m=4,n=2,p=1	299.0	37.2	0.2	-0.86	20.59
m=5,n=2,p=1	325.6	33.5	0.2	-0.89	18.9
m=5,n=3,p=1	295.0	30.6	0.1	-0.82	22.28
m=6,n=2,p=1	346.8	30.6	0.2	-0.92	17.74
m=6,n=3,p=1	317.1	28.3	0.1	-0.86	20.72

Unit:  $E_x = GPa$ ,  $G_{xy} = GPa$ ,  $\alpha_x = ppm/^\circ C$ ,  $\beta_x = ppm/^\circ C$

Table 3. Despace ( $\mu m$ ) for the selected lay-ups,  $[0_m/90_n/\pm 45_p]_s$ : p=1, ( $\Delta C = 0$ ).

	-20°C	-10°C	0°C	10°C	20°C	55°C
m=3,n=1	18.1	9.1	0	-9.05	-18.1	-49.8
m=5,n=1	17.4	8.7	0	-8.7	-17.4	-47.8
m=4,n=2	18.2	9.1	0	-9.12	-18.2	-50.2
m=5,n=2	18.3	9.1	0	-9.14	-18.3	-50.3
m=5,n=3	18.2	9.1	0	-9.08	-18.2	-50.0
m=6,n=2	18.2	9.1	0	-9.1	-18.2	-50.0
m=6,n=3	18.3	9.2	0	-9.17	-18.3	-50.4

In Figure 11, despace as a function of temperature and moisture absorption/desorption for the lay-up  $[0_m/90_n/(\pm 45)_p]_s$  is depicted.

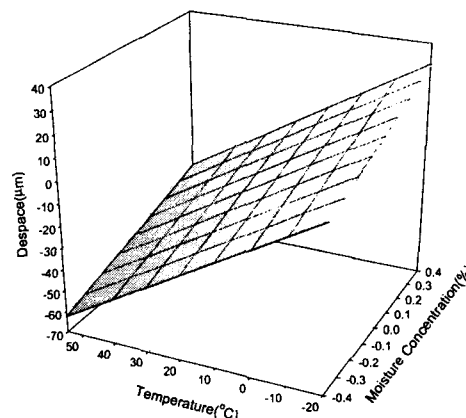


Fig. 11. Despace as a function of temperature and moisture concentration for the lay-up  $[0_m/90_n/\pm 45_p]_s$ .

### 3. Conclusions

Laminate Design of a composite tube for a space telescope under hygrothermal load was performed. Carpet plots for laminate effective engineering constants were generated for the selection of the best tube lay-ups satisfying the optomechanical requirements for a dimensionally stable space telescope under orbital thermal loading.

Despace of the tubes constructed with the selected lay-ups was calculated with a Zig-Zag Triangular Element which accurately represents laminated structures. The effects of moisture absorption and desorption on the dimensional stability were also investigated.

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