

# The Extraction of End-Pixels in Feature Space for Remote Sensing Data and Its Applications

Lu YUAN

Department of Electronic Engineering, Tsinghua University, Beijing, China  
yuanl02@mails.tsinghua.edu.cn

Wei-dong SUN

Department of Electronic Engineering, Tsinghua University, Beijing, China  
wdsun@tsinghua.edu.cn

**Abstract:** The extraction of “end-pixels” (i.e. end-members) aims to quantify the abundance of different materials in a single pixel, which becomes popular in the subpixel analysis for hyperspectral dataset. In this paper, we present a new concept called “End-Pixel of Features (EPF)” to extend the concept of end-pixels for multispectral data and even panchromatic data. The algorithm combines the advantages of previous simplex and clustering methods to search the EPFs in the feature space and reduce the effects of noise. Some experimental results show that, the proposed methodology can be successfully used to hyperspectral data and other remote sensing data.

**Key words:** End-Pixel of Features, Multispectral data, Feature space, Geometric similarity

## 1. Introduction

In recent years, hyperspectral imaging technique has become one of the most powerful tools in many application areas, such as environmental monitoring, natural resources exploiting and agriculture. Now, spectral information can be obtained through a great number of spectral bands, from a few to several hundreds. How to extract the interesting features from the multi-dimensional hyperspectral data more efficiently, is very important for the identification and detection of the individual materials.

Linear spectral unmixing is one of the most important approaches for the analysis and classification of multi/hyperspectral datasets. This approach involves two steps: to find spectrally unique signatures of pure ground components (usually referred to as end-pixels or end-members [10]) and to express individual pixels as a linear combination of end-pixels [5]. Let  $s(x, y)$  be a spectrum of values obtained at the sensor for a certain pixel with spatial coordinated as an  $N$ -dimensional vector (where  $N$  is the number of spectral bands) and may be modeled in terms of a linear combination of several end-pixel vectors  $e_i$ ,  $i = 1, \dots, k$ , according to the equations and constraints [2].

$$s(x, y) = \sum_{i=1}^k c_i \cdot e_i + \boldsymbol{\varepsilon} \quad (1)$$

$$\sum_{i=1}^k c_i = 1, \quad 0 \leq c_i \leq 1 \quad (2)$$

where  $k$  is the number of end-pixels needs to accurately model the original spectrum,  $c_i$  is the scalar value representing the fractional coverage of end-pixel vector  $e_i$  in pixel  $(x, y)$ , and  $\boldsymbol{\varepsilon}$  is Gaussian random error. The ideal case is that the coefficients in the linear combination are

nonnegative and sum to 1, being, therefore, interpretable as cover functions or abundances. Once these end-pixels are extracted, they provide a basis set, in whose terms the rest of the data can be described, and moreover the basis set gives a more “physical” description of data without the orthogonalization restriction compared with the principle components.

Several autonomous techniques for finding these end-pixels in the hyperspectral data are currently proposed, such as PPI [7], Orasis [6], MEST [3], MESMA [13] and IEA [14]. In these approaches, there are two fundamentally different models. (1) The N-FINDR algorithm [15] finds the simplex of maximum volume that can be enclosed within all of the pixels. (2) The SEM algorithm [11] uses a stochastic technique for characterizing spectral classes of all the multidimensional data. Although the two methods can achieve very similar qualities in a global sense [4], both still exist differences in applicability of each model. The simplex approaches can generally find the purest end-pixels even including abnormal end-pixels from image, but the results are sensitive to the noise. The stochastic approaches are robust to noise, but the extracted end-pixels are usually combined with mixture pixels. Therefore, a hybrid solution is required to extract the purest end-pixels and reduce the effects of noise. In addition, many of previous works are only limited for hyperspectral data and focus exclusively on the spectral nature of the data. Available analysis techniques do not usually take into account the information related to the spatial context, which is useful to improve the quality of the extracted end-pixels.

In this paper, in order to extending the concept of end-pixels for multispectral data and even for panchromatic data, a new concept called “End-Pixel of Features (EPF)” which is defined in the feature space is given, and an extraction method of EPFs based on the geometric characteristics of feature points is proposed to find a set of more pure EPFs in the feature space. In this method, all pixels in the image space are mapped into the feature space at first, then some candidates of EPFs will be selected which are centers of each group of extreme points with higher similarity in the feature space, finally a similar searching approach as N-FINDR is adopted to find the EPFs consisting a convex set with the largest volume. Some experimental results using real data sets show that, this new method is robust to the abnormal noise, and can be used not only in the analysis of hyperspectral data but also other kinds of remote sensing data.

## 2. Definition, Searching and Unmixing Algorithms of EPFs

This section is organized as follows. Subsection 1) introduces the concept of end-pixel of features which extends the definition of end-pixels for hyperspectral data. Subsection 2) focuses on the operations to project each pixel in original images into the determined feature space. Subsection 3) demonstrates the searching procedure of end-pixels of features in the feature space. Finally in subsection 4), a similar unmixing algorithm is proposed.

### 1) Definition of End-pixel of Features (EPF)

The analysis of hyperspectral data sets requires the determination of certain basis spectra called “end-pixels”, which represent spectrally unique signatures of pure ground components. Moreover, it is very important for other kinds of remote sensing data sets, including multispectral and panchromatic image, to identify a set of reference signatures to model the spectral nature at each pixel. In fact, previous simplex approaches based on the originally spectral space (or image space) can hardly be used to other remote sensing data beyond hyperspectral data directly, for the small number of their spectral bands is not often sufficient to the computation of end-pixels extraction.

A very useful technique comes from spectral clustering algorithm [1][15], which relies on the eigenstructure of a similarity matrix to partition points into disjoint clusters, with point in the same cluster having high similarity and points in different clusters having low similarity. In fact, this approach is available to obtain features of each pixel through the spatially spectrum information from its neighbor pixels. Therefore, each pixel in the originally spectral space would correspond to a feature point. Here, the space consisting of all these feature points is called “feature space”, where the dimensions of feature space can be set to be larger or smaller than original ones. This advantage would make it possible to search the end-pixels in the feature space through original simplex methods. As a rule, those feature points are defined as “end-pixel of features (EPF)”.

### 2) Feature Space Projection

Our algorithm begins with the procedure of achieving the feature space for the input remote sensing datasets. Then each pixel in original spectral space will be mapped to a point in the determined feature space. As we known, the procedure requires to calculate the similarity matrix for all samples, which costs much computation. Here, we propose an accelerated implementation based on the downsampling method described as follows:

Given original dataset  $S = \{s_1, L, s_n\}$ , and set the number of clusters to  $K$ :

1. A training subset  $S' = \{s'_1, L, s'_m\}$  can be obtained by downsampling the dataset  $S$  and the rest pixels compose the test subset  $P = \{p_1, L, p_r\}$ , where  $S' \cup P = S$ ,  $S' \cap P = \Omega$ .

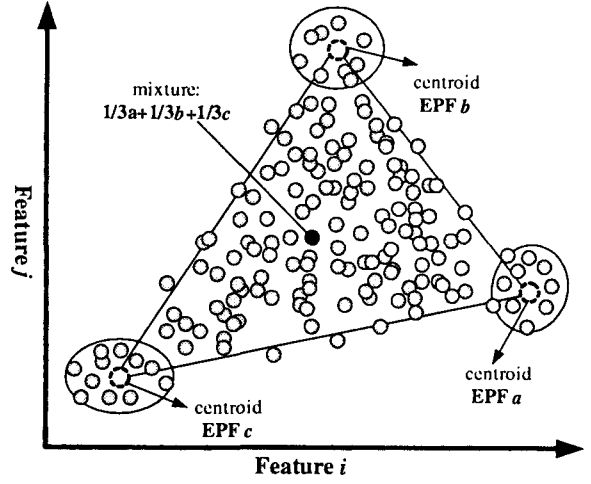


Fig.1 Geometrical interpolation of a mixture model based on EPFs in two-dimensional feature space

2. The training similarity matrix  $A \in {}^{m \times m}$  can be calculated by  $A_{ij} = \exp\{-\|s'_i - s'_j\|^2 / 2\sigma^2\}$  and the test similarity matrix  $T \in {}^{r \times m}$  can be defined as  $T_{ij} = \exp\{-\|p_i - s'_j\|^2 / 2\sigma^2\}$ . When two spatially distant pixels are computed,  $A_{ij}$  and  $T_{ij}$  are both set to zero.

3. Let each element  $(i, i)$  of a diagonal matrix  $D$  be the sum of the  $i$ -th row of  $A$ , and let each element  $(j, j)$  of a diagonal matrix  $F$  be the sum of the  $j$ -th row of  $T$ . Here, we define a Laplacian matrix  $L = D^{-1/2} A D^{-1/2}$ .

4. Compute the former  $K$  largest eigenvalues and corresponding eigenvectors of  $L$ , which construct an eigenmatrix  $V = [v_1, L, v_K]$  and a diagonal matrix  $\Lambda = \text{diag}(\lambda_1, L, \lambda_K)$ .

5. Calculate the features of all training samples  $Z_1 = D^{-1/2} V$  and calculate the features of all testing samples  $Z_2 = F^{-1/2} T D^{-1/2} V \Lambda$  to obtain the features of all samples  $Z = [Z_1; Z_2]$ .

### 3) Searching EPFs

This procedure is initialized by a random set of feature points as end-pixels of features. In order to refine the estimate of EPFs, the volume must be calculated with each point in place of each EPFs. The volume ( $V$ ) of the simplex formed by using the EPF estimation is proportional to the determinant of  $E$ :

$$V(E) = \frac{1}{(k-1)!} \text{abs} \begin{pmatrix} 1 & 1 & L & 1 \\ e_1 & e_2 & L & e_k \end{pmatrix} \quad (3)$$

A trail volume is calculated for every point in each EPF position by replacing that EPF and recalculating the volume. If the replacement results in an increase in

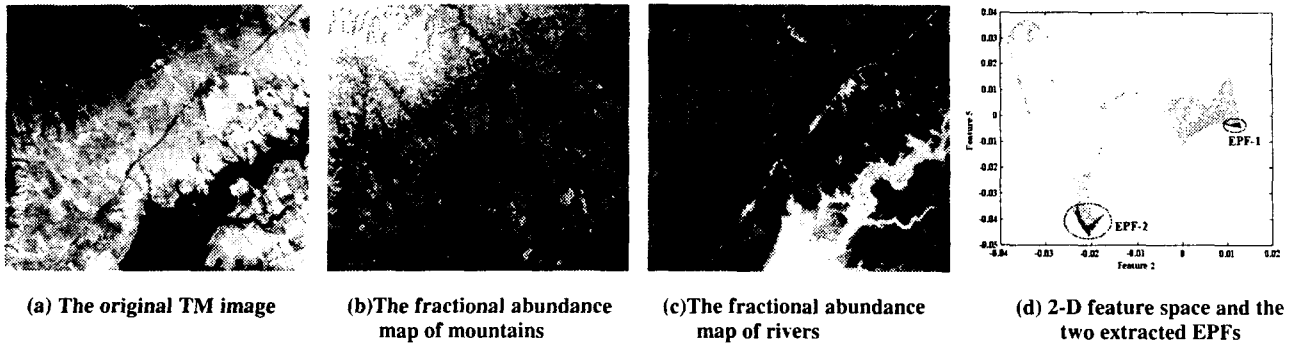


Fig.2 The Experimental results of multi-channels TM image

the point will replace the EPF. This procedure is repeated until there are no more replacements of EPFs. We remove these extracted EPFs from the feature space and continue to find the largest volume among the rest points until  $q$  largest volumes with different vertices are found. For each vertex of simplex, some points very near to the  $q$  candidate EPFs are all used to estimate the centroid of the vertex cluster, which becomes the final EPF at the vertex.

For instance, a simple mixture model based on three end-pixels of features has the geometrical interpretation of a triangle whose vertices are the EPFs, shown in Fig.1. Each EPF is determined by the centroid of all candidate points near the vertex. Cover functions are determined by the position of spectra within the triangle and can be considered relative coordinates in a new reference system given by the EPFs.

#### 4) Unmixing with the EPFs

Once the pure pixels are found and original pixels are given, their spectra can be used to unmix the original image. For the linear mixture model, the goal is to find abundances  $\mathbf{c}$  to minimize  $\|\mathbf{s} - \mathbf{E}\mathbf{c}\|^2$ , subject to the sum-to-one and positivity constraints [9]. The QP method offers the fully constrained solution to the estimation problem and can be described as follows:

$$\begin{aligned} & \underset{\mathbf{c}}{\text{minimize}} (\mathbf{c}^T \mathbf{E}^T \mathbf{E} \mathbf{c} - 2 \cdot \mathbf{s}^T \mathbf{E} \mathbf{c}) \\ & \text{subject to } \mathbf{1}^T \mathbf{c} = 1, \text{ and } c_i \geq 0, \quad i = 1, L, k \end{aligned} \quad (4)$$

where  $\mathbf{1} = [1, 1, L, 1]$ . To solve the above problem numerically, firstly, an initial feasible solution is calculated. Then an iterative procedure is taken to generate a series of feasible points that converge to the solution. The implementation of QP algorithm is beyond this paper, more details are described in [8].

### 3. Experiments and Discussions

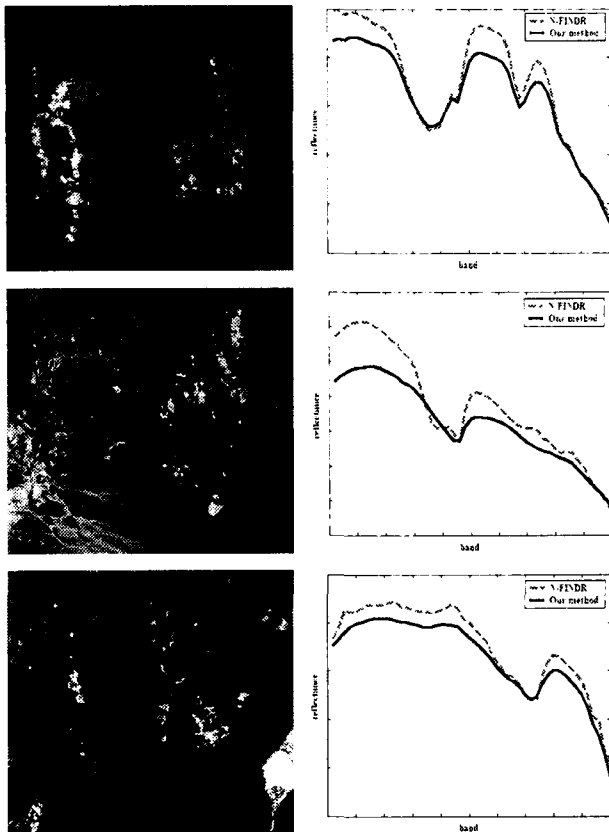
The end-pixel of feature extraction algorithm can provide good results on images with a wide variety of remote sensing dataset. Here as some of the experiments, our method is applied to one real multi-channels satellite image and one hyperspectral image.

The multi-channels image is produced by previous three bands of TM satellite data. Ground surface in this test region is very abundant, however we are interested only in two typical classes: mountain and river. The proposed algorithm is applied to map each pixel in the original 3-D image space into a feature point in the 6-D feature space. Through the algorithm, we find that the feature subspace consisting of "Feature 2" and "Feature 5" is clearly illustrated by two EPFs distributions. Those candidate vertices of the simplex indicated by black points in the ellipse in Fig.2(d), are used to estimate the final position of EPF-1 and EPF-2, which reflects the spectral nature of mountain and river. The corresponding abundance maps of the two classes can be derived by non-negative and sum-to-one constraint, as shown in Fig.2(b) and Fig.2(c).

The hyperspectral image is collected by the AVIRIS sensor over Cuprite, Nevada, which has been used in the previous N-FINDR experiments. For the original dataset, our method maps 50 contiguous SWIR bands (1978 to 2478nm) to a 20-D feature space. Our algorithm can be used to automatically determine the EPFs reflecting three minerals and derive their abundance maps. Fig.4(a) shows the spectral information of the three EPFs extracted by our proposed method are comparable to the N-FINDR method, where the gray dash line indicates N-FINDR's results and the black line indicates ours. In addition, from Fig.4(b) we can see that, corresponding unmixing abundance maps of three minerals derived by our approach are very comparable to the results of [12].



Fig. 3 Observation area of AVIRIS hyperspectral image



(a) Abundance maps of alunite, kaolinite and calcite expressed using the derived EPFs  
 (b) The realistic spectra of the 3 minerals compared with N-FINDR method

Fig.4 The Experimental results of AVIRIS hyperspectral image

#### 4. Conclusion

This paper presents a new concept called “End-Pixel of Features (EPF)” to extend the concept of end-pixels for a variety of remote sensing data, including not only hyper/multispectral data but also panchromatic data. The algorithm considers the spatial spectrum information of neighbor pixels to constitute a feature space, where we can achieve purest end-pixels by finding a simplex with largest volume and can reduce the effects of noise by selecting candidate points near to vertex to estimate the final EPFs. Some experimental results show that, the proposed methodology can be efficiently used to realistic hyperspectral data and other remote sensing data.

#### Acknowledgement

The authors would like to express their gratitude to M. E. Winter for providing results of the N-FINDR algorithm for Cuprite dataset. This project is supported by the national “863” project (No.2001AA135010) and National Natural Science Foundation of China (No.60472029).

#### References

- [1] F. R. Bach and M. I. Jordan, 2003, “Learning Spectral Clustering”, *Technical report*, UC Berkeley.
- [2] C. A. Bateson, G. P. Asner, and C. A. Wessman, 2000, “Endmember bundles: A new approach to incorporating endmember variability into spectral mixture analysis”, in *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 38, pp. 1083-1094.
- [3] C. A. Bateson and B. Curtiss, 1993, “A tool for manual endmember selection and spectral unmixing”, in *Summaries of the V JPL Airborne Earth Science Workshop*, Pasadena, CA.
- [4] S. G. Beaven, L. E. Hoff, and E. M. Winter, “Comparison of SEM and Linear Unmixing Approaches for Classification of Spectral Data”, in *Proc. of SPIE*.
- [5] J. W. Boardman, F. A. Kruse, and R. O. Green, 1995, “Mapping target signatures via partial unmixing of AVIRIS data”, in *Summaries of the VII JPL Airborne Earth Science Workshop*, Pasadena, CA.
- [6] J. Bowles, P. J. Palmadesso, J. A. Antoniadis, M. M. Baumbach, and L. J. Rickard, 1995, “Use of filter vectors in hyperspectral data analysis”, in *Proc. SPIE Infrared Spaceborne Remote Sensing III*, pp. 148-157.
- [7] M. D. Craig, 1994, “Minimum Volume Transforms for Remotely Sensed Data”, in *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 32, pp. 542-552.
- [8] R. Fletcher, 1980, “Practical methods of optimization”, in *Constrained Optimization*, Vol. 2. New York: Wiley.
- [9] Y. H. Hu, H. B. Lee, and F. L. Scarpace, 1999, “Optimal Linear Spectral Unmixing”, in *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 37, No. 1, pp. 639-644.
- [10] F. A. Kruse, 1998, “Spectral identification of image endmembers determined from AVIRIS data”, in *Summaries of the VII JPL Airborne Earth Science Workshop*, Pasadena, CA.
- [11] P. Masson and W. Pieczynski, 1993, “SEM Algorithm and Unsupervised Statistical Segmentation of Satellite Images”, in *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 31, No. 3.
- [12] R. G. Resmini, M. E. Karpus, W. S. Aldrich, J. C. Harsanyi, and M. Anderson, 1997, “Mineral mapping with hyperspectral digital imagery collection experiment (HYDICE) sensor data at Cuprite, Nevada, U. S. A.”, in *Int. J. Remote Sensing*, Vol. 18, pp. 1553-1570.
- [13] D. Roberts, M. Gardener, J. Regelbrugge, D. Pedreros, and S. Ustin, 1998, “Mapping the distribution of wildfire fuels using AVIRIS in the Santa Monica Mountains”, in *Summaries of the V JPL Airborne Earth Science Workshop*, Pasadena, CA.
- [14] K. Staenz, T. Szeredi, and J. Schwarz, 1998, “ISDAS-A system for processing/analyzing hyperspectral data”, *Can. J. Remote Sensing*, Vol.24, pp. 99-113.
- [15] M. E. Winter, 1999, “N-FINDR: An algorithm for fast autonomous spectral end-member determination in hyperspectral data”, in *Proc. SPIE Imaging Spectrometry*, pp. 266-275.
- [16] Stella X. Yu, and Jianbo Shi, 2003, “Multiclass Spectral Clustering”, in *Proc. of International Conference on Computer Vision*, Nice, France.