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Kernel-Based Fuzzy Regression Machine For Predicting Turbulent Flows

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Abstract

The turbulent flow is of fundamental interest because the conservation equations for thermodynamics, mass and momentum are linked together. This turbulent flow consists of some coherent time- and space-organized vortical structures. Research has already shown that some dynamic systems and experimental models still cannot provide a good nonlinear analysis of turbulent time series. In the real turbulent flow, very complicated nonlinear behaviors, which are affected by many vague factors are present. In this paper, a kernel-based machine for fuzzy nonlinear regression analysis is proposed to predict the nonlinear time series of turbulent flows. In order to show the practicality and usefulness of this model, we present an example of predicting the near-wall turbulence time series as a verifiable model and compare with fuzzy piecewise regression. The results of practical applications show that the proposed method is appropriate and appears to be useful in nonlinear analysis and in fuzzy environments to predict the turbulence time series.

Keywords : Fuzzy regression, near wall turbulent, necessity, possibility, time series, kernel-based machine.

1. Introduction to Near-Wall Turbulence

Most turbulent flow research has focused on understanding the characteristics of turbulence and using semi-empirical theories to fit the experimental data, as evidenced by the volumes of publications involving experimental data, mathematical analysis, and computational modeling. These methods can be used to estimate the characteristics by using statistical analysis and fitting the parameters from the experimental data.

In near-wall regions, a large production occurs and the presence of nonlinearity becomes significant. Many studies have concentrated on the mechanism of near-wall turbulence and the coherent vortical structure of bursting. Aubry et al.(1988) employed a dynamical systems

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approach to study the behavior of streamwise vortices in the near-wall region of turbulent boundary layers. Hamilton et al.(1995) used direct numerical simulations to study the regeneration dynamics of turbulent structure found in the near-wall region. These studies can identify many main features of the near-wall dynamics and provide important information concerning the physical basis of turbulent generation mechanisms.

One important feature in the near-wall turbulence is that these instantaneous characteristics of the velocity, intensity and location (relative distance) are all dependent on time series and are interrelated. Time series analysis with these correlations then becomes an important clue about how to approach the study of the turbulence. However, in this field only a few papers that appear in publications about turbulent flow depend on time-series. Porporato and Ridolfi(1997) applied the nonlinear time series analysis to a near wall turbulence signal in a hydraulically smooth pipe. Tseng et al.(2001) used fuzzy piecewise regression analysis to the nonlinear time series of turbulent flows. In practical circumstances, it is difficult to grasp rules for predicting the nonlinear turbulent behavior and to forecast the velocity and intensities of the turbulent flow at different times. In this paper we propose a kernel-based machine for fuzzy nonlinear regression analysis to predict the nonlinear time series of near-wall turbulent flows.

2. Some Results in Interval Regression Analysis

Tanaka et al.(1982) introduced a linear programming(LP) based regression method using a linear model with symmetrical triangular fuzzy parameters and then defined the possibility and necessity regression analyses. However, two weaknesses involving the fuzzy regression model have arisen. First, in possibility analysis, it turned out that Tanaka's methodologies were extremely sensitive to outliers. Furthermore, the fuzzy predictive interval tends to become fuzzier as more data are collected and has no operational definition or interpretation. Second, in necessity analysis, the necessity area would not be obtained owing to the large variation in data. Tanaka and Lee(1998) proposed quadratic programming(QP) approach to interval regression analysis. In this section, we illustrate how to get solutions for interval regression models using QP approach proposed by Tanaka and Lee(1998).

Suppose that we are given training data $\{(\boldsymbol{x}_i, y_i),\ i=1,\cdots,n\} \subset \mathrm{X} \times R$, where X denotes the space of the input patterns. we begin by describing the case of interval linear regression functions $Y(\boldsymbol{x})$, taking the form

$$Y(\boldsymbol{x}) = A_0 + A_1 x_1 + \cdots + A_m x_m = \boldsymbol{A}^t \boldsymbol{x}, \tag{1}$$

where $\boldsymbol{x} = (1, x_1, \cdots, x_m)^t$ is a real input vector, $\boldsymbol{A} = (A_0, A_1, \cdots, A_m)^t$ is an interval coefficient vector, and $Y(\boldsymbol{x})$ is the corresponding estimated interval. An interval coefficient A_i is denoted as $A_i=(a_i, c_i)$ where a_i is a center and c_i is a radius. By interval arithmetic, the regression model (1) can be expressed as

$$\begin{aligned} Y(\boldsymbol{x}_i) &= (a_0, c_0)+(a_1, c_1)x_{i1} + \cdots + (a_m, c_m)x_{im} \\ &= (a_0 + a_1 x_{i1} + \cdots + a_m x_{im}, c_0 + c_1|x_{i1}| + \cdots + c_m|x_{im}|) \\ &= (\boldsymbol{a}^t\boldsymbol{x}_i, \boldsymbol{c}^t|\boldsymbol{x}_i|), \end{aligned} \tag{2}$$

where $\boldsymbol{a} = (a_0, a_1, \cdots, a_m)^t$, $\boldsymbol{c} = (c_0, c_1, \cdots, c_m)^t$, and $|\boldsymbol{x}_i|=(1, |x_{i1}|, \cdots, |x_{im}|)^t$.

2.1 Integrating Central Tendency and Possibilistic Property

We now illustrate the formulation integrating central tendency and possibilistic property in Tanaka and Lee(1998). We consider a new objective function which reflects both properties of least squares and possibilistic approaches

$$J = k_1 \sum_{i=1}^{n}(y_i - \boldsymbol{a}^t\boldsymbol{x}_i)^2 + k_2 \sum_{i=1}^{n} \boldsymbol{c}^t|\boldsymbol{x}_i||\boldsymbol{x}_i|^t\boldsymbol{c} \tag{3}$$

where $\sum_{i=1}^{n}|\boldsymbol{x}_i||\boldsymbol{x}_i|^t$ is a symmetric positive definite matrix and k_1 and k_2 are weight coefficients. Interval regression analysis using this new objective function (3) is to determine the interval coefficients $A_i=(a_i, c_i)$, $i = 0,1,\cdots, m$ by solving the following QP problem:

$$\min_{\boldsymbol{a},\boldsymbol{c}} \; J = k_1 \sum_{i=1}^{n}(y_i - \boldsymbol{a}^t\boldsymbol{x}_i)^2 + k_2 \sum_{i=1}^{n} \boldsymbol{c}^t|\boldsymbol{x}_i||\boldsymbol{x}_i|^t\boldsymbol{c} \tag{4}$$

subject to

$$\begin{aligned} &\boldsymbol{a}^t\boldsymbol{x}_i + \boldsymbol{c}^t|\boldsymbol{x}_i| \geq y_i, \;\; \boldsymbol{a}^t\boldsymbol{x}_i - \boldsymbol{c}^t|\boldsymbol{x}_i| \leq y_i, \; i=1,\cdots,n \\ &c_i \geq 0, \; i=0,1,\cdots,m \end{aligned}$$

The weight coefficients k_1 and k_2 in (4) have an important role in formulating fuzzy regression models. These coefficients can be assigned by considering a tardeoff between two terms in (4).

2.2 Unifying the Possibility and Necessity Models

For a data set with crisp inputs and interval outputs, we can consider two interval regression models, i.e., the possibility and necessity models. In this section, we review the unified QP approach to obtain the possibility and necessity models simultaneously. In this unified approach, we assume for simplicity that the center coefficients of the possibility regression model and necessity regression model are same.

Suppose that we are given training data $\{(\boldsymbol{x}_i, Y_i),\ i=1,\cdots,n\}$, where $\boldsymbol{x}_i=(1,x_{i1},\cdots,x_{im})^t$ is the ith input vector, $Y_i=(y_i, e_i)$ is the corresponding interval output that consists of a center y_i and a radius e_i. For this data set, the possibility and necessity estimation models are denoted as

$$Y^*(\boldsymbol{x}_i) = A^*_0 + A^*_1 x_{i1} + \cdots + A^*_m x_{im},\quad i=1,\cdots,n$$
$$Y_*(\boldsymbol{x}_i) = A_{*0} + A_{*1} x_{i1} + \cdots + A_{*m} x_{im},\quad i=1,\cdots,n$$

where the interval coefficients A^*_i and A_{*i} are denoted as $A^*_i=(a^*_i, c^*_i)$ and $A_{*i}=(a_{*i}, c_{*i})$, respectively. The estimated interval $Y^*(\boldsymbol{x}_i)$ by the possibility model always includes the observed interval Y_i, whereas the estimated interval $Y_*(\boldsymbol{x}_i)$ by the necessity model should be included in the observed interval Y_i. In fact, we can denote the interval coefficients A^*_i and A_{*i} as

$$A^*_i = (a_i,\ c_i+d_i)$$
$$A_{*i} = (a_i,\ c_i)$$

which satisfies the condition $A_{*i}\subseteq A^*_i,\ i=0,1,\cdots,m$ since c_i and d_i are assumed to be nonnegative. Therefore, by interval arithmetic the possibility model $Y^*(\boldsymbol{x}_i)$ and the necessity model $Y_*(\boldsymbol{x}_i)$ can be written as

$$Y^*(\boldsymbol{x}_i) = (\boldsymbol{a}^t\boldsymbol{x}_i,\ \boldsymbol{c}^t|\boldsymbol{x}_i| + \boldsymbol{d}^t|\boldsymbol{x}_i|)$$
$$Y_*(\boldsymbol{x}_i) = (\boldsymbol{a}^t\boldsymbol{x}_i,\ \boldsymbol{c}^t|\boldsymbol{x}_i|).$$

The two objective functions in the possibility and the necessity models are assumed as follows:

$$\min\ J_P = \sum_{i=1}^{n}(\boldsymbol{c}^t|\boldsymbol{x}_i| + \boldsymbol{d}^t|\boldsymbol{x}_i|):\ \text{Possibility model}$$

$$\max\ J_N = \sum_{i=1}^{n}\boldsymbol{c}^t|\boldsymbol{x}_i|:\ \text{Necessity model}$$

The objective function of the unified model can be obtained by the combination of these two objective functions. The combination of two functions yields the following:

$$\min\ J_P - J_N = \sum_{i=1}^{n} d^t |x_i|.$$

Then, the objective function in the unified approach by QP can be assumed as the following quadratic function:

$$J = \sum_{i=1}^{n} (d^t |x_i|)^2 = d^t \left(\sum_{i=1}^{n} |x_i| |x_i|^t \right) d.$$

Therefore, interval regression analysis is to determine the interval coefficients A_i^* and $A_{*i},\ i=0,1,\cdots,m$ that minimize the above objective function J and satisfy inclusion relations $Y_*(x_i) \subseteq Y_i \subseteq Y^*(x_i)$. This problem can be described as the following QP problem:

$$\min_{a,c,d}\ d^t \left(\sum_{i=1}^{n} |x_i| |x_i|^t \right) d + \xi (a^t a + c^t c)$$

subject to

$$a^t x_i + c^t |x_i| + d^t |x_i| \ge y_i + e_i,\quad a^t x_i - c^t |x_i| - d^t |x_i| \le y_i - e_i$$
$$a^t x_i + c^t |x_i| \le y_i + e_i,\quad a^t x_i - c^t |x_i| \ge y_i - e_i,\ i=1,2,\cdots,n$$
$$c_i \ge 0,\ d_i \ge 0,\ i=0,1,\cdots,m$$

where ξ is a significantly small positive number, and so leads the influence of $\xi(a^t a + c^t c)$ to be negligible.

3. Kernel-Based Machine for Interval Regression

In this section, we propose a new method to evaluate interval linear and nonlinear regression models combining the possibility estimation formulation integrating the property of central tendency with the principle of support vector machine(SVM) of Vapnik(1995). We first need to look at how to get solutions for interval linear regression models by implementing quadratic loss SVM approach. We follow the way of constructing objective function in quadratic loss SVM regression. Then, the objective function can be assumed as the following quadratic function:

$$\min_{a,c,d}\ \frac{1}{2}(\| a \|^2 + \| c \|^2 + \| d \|^2) + \frac{C}{2}\left(\sum_{i=1}^{n} \xi_{1i}^2 + \sum_{i=1}^{n} (\xi_{2i}^2 + \xi_{2i}^{*2}) \right) \qquad (5)$$

subject to

$$
\begin{aligned}
& \boldsymbol{d}^t|\boldsymbol{x}_i| \le \xi_{1i} \\
& y_i - \boldsymbol{a}^t\boldsymbol{x}_i \le \xi_{2i}, \quad \boldsymbol{a}^t\boldsymbol{x}_i - y_i \le \xi^*_{2i} \\
& \boldsymbol{a}^t\boldsymbol{x}_i + \boldsymbol{c}^t|\boldsymbol{x}_i| + \boldsymbol{d}^t|\boldsymbol{x}_i| \ge y_i + e_i, \quad \boldsymbol{a}^t\boldsymbol{x}_i - \boldsymbol{c}^t|\boldsymbol{x}_i| - \boldsymbol{d}^t|\boldsymbol{x}_i| \le y_i - e_i \\
& \boldsymbol{a}^t\boldsymbol{x}_i + \boldsymbol{c}^t|\boldsymbol{x}_i| \le y_i + e_i, \quad \boldsymbol{a}^t\boldsymbol{x}_i - \boldsymbol{c}^t|\boldsymbol{x}_i| \ge y_i - e_i, \ i=1,2,\cdots,n
\end{aligned}
$$

The weight coefficient $C>0$ determines not only the trade-off between $\sum_{i=1}^n (y_i - \boldsymbol{a}^t\boldsymbol{x}_i)^2$ and $\sum_{i=1}^n \boldsymbol{c}^t|\boldsymbol{x}_i||\boldsymbol{x}_i|^t\boldsymbol{c}$, but also the trade-off between the flatness of $Y(\boldsymbol{x})$. Although it is possible to use two weight coefficients like Tanaka and Lee(1998), we use one weight coefficient. Here, ξ_{1i} represents spreads of the estimated outputs, and ξ_{2i}, ξ^*_{2i} are slack variables representing upper and lower constraints on the outputs of the model. Hence, we can construct a Lagrange function as follows:

$$
\begin{aligned}
L = & \frac{1}{2}(\|\boldsymbol{a}\|^2 + \|\boldsymbol{c}\|^2) + \frac{C}{2}\left(\sum_{i=1}^n \xi^2_{1i} + \sum_{i=1}^n (\xi^2_{2i} + \xi^{*2}_{2i})\right) \\
& - \sum_{i=1}^n \alpha_{1i}(\xi_{1i} - \boldsymbol{d}^t|\boldsymbol{x}_i|) \\
& - \sum_{i=1}^n \alpha_{2i}(\xi_{2i} - y_i + \boldsymbol{a}^t\boldsymbol{x}_i) - \sum_{i=1}^n \alpha^*_{2i}(\xi_{2i} - \boldsymbol{a}^t\boldsymbol{x}_i + y_i) \qquad (6) \\
& - \sum_{i=1}^n \alpha_{3i}(\boldsymbol{a}^t\boldsymbol{x}_i + \boldsymbol{c}^t|\boldsymbol{x}_i| + \boldsymbol{d}^t|\boldsymbol{x}_i| - y_i - e_i) - \sum_{i=1}^n \alpha^*_{3i}(y_i - e_i - \boldsymbol{a}^t\boldsymbol{x}_i + \boldsymbol{c}^t|\boldsymbol{x}_i| + \boldsymbol{d}^t|\boldsymbol{x}_i|) \\
& - \sum_{i=1}^n \alpha_{4i}(y_i + e_i - \boldsymbol{a}^t\boldsymbol{x}_i + \boldsymbol{c}^t|\boldsymbol{x}_i|) - \sum_{i=1}^n \alpha^*_{4i}(\boldsymbol{a}^t\boldsymbol{x}_i - \boldsymbol{c}^t|\boldsymbol{x}_i| - y_i + e_i)
\end{aligned}
$$

Here, $\alpha_{1i}, \alpha_{2i}, \alpha^*_{2i}, \alpha_{3i}, \alpha^*_{3i}, \alpha_{4i}, \alpha^*_{4i}$ are Lagrange multipliers. It follows from the saddle point condition that the partial derivatives of L with respect to the primal variables $(\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}, \xi_{1i}, \xi_{2i}, \xi^*_{2i})$ have to vanish for optimality.

$$
\frac{\partial L}{\partial \boldsymbol{a}} = 0 \rightarrow \boldsymbol{a} = \sum_{i=1}^n (\alpha_{2i} - \alpha^*_{2i})\boldsymbol{x}_i + \sum_{i=1}^n (\alpha_{3i} - \alpha^*_{3i})\boldsymbol{x}_i - \sum_{i=1}^n (\alpha_{4i} - \alpha^*_{4i})\boldsymbol{x}_i \qquad (7)
$$

$$
\frac{\partial L}{\partial \boldsymbol{c}} = 0 \rightarrow \boldsymbol{c} = \sum_{i=1}^n (\alpha_{3i} + \alpha^*_{3i})|\boldsymbol{x}_i| - \sum_{i=1}^n (\alpha_{4i} + \alpha^*_{4i})|\boldsymbol{x}_i| \qquad (8)
$$

$$
\frac{\partial L}{\partial \boldsymbol{d}} = 0 \rightarrow \boldsymbol{d} = -\sum_{i=1}^n \alpha_{1i}|\boldsymbol{x}_i| + \sum_{i=1}^n (\alpha_{3i} + \alpha^*_{3i})|\boldsymbol{x}_i| \qquad (9)
$$

$$
\frac{\partial L}{\partial \xi_{1i}} = 0 \rightarrow \xi_{1i} = \frac{1}{C}\alpha_{1i} \qquad (10)
$$

$$\frac{\partial L}{\partial \xi_{2i}^{(*)}} = 0 \rightarrow \xi_{2i}^{(*)} = \frac{1}{C}\alpha_{2i}^{(*)} \tag{11}$$

Substituting (7)-(11) into (6) yields the dual optimization problem.

$$\begin{aligned}
\text{maximize}\Big\{ -\frac{1}{2}\Big(& \sum_{i,j=1}^{n}(\alpha_{2i}-\alpha_{2i}^{*})(\alpha_{2j}-\alpha_{2j}^{*})\boldsymbol{x}_i^t\boldsymbol{x}_j \\
& + \sum_{i,j=1}^{n}(\alpha_{3i}-\alpha_{3i}^{*})(\alpha_{3j}-\alpha_{3j}^{*})\boldsymbol{x}_i^t\boldsymbol{x}_j + \sum_{i,j=1}^{n}(\alpha_{4i}-\alpha_{4i}^{*})(\alpha_{4j}-\alpha_{4j}^{*})\boldsymbol{x}_i^t\boldsymbol{x}_j \\
& + 2\sum_{i,j=1}^{n}(\alpha_{2i}-\alpha_{2i}^{*})(\alpha_{3j}-\alpha_{3j}^{*})\boldsymbol{x}_i^t\boldsymbol{x}_j - 2\sum_{i,j=1}^{n}(\alpha_{2i}-\alpha_{2i}^{*})(\alpha_{4j}-\alpha_{4j}^{*})\boldsymbol{x}_i^t\boldsymbol{x}_j \\
& - 2\sum_{i,j=1}^{n}(\alpha_{3i}-\alpha_{3i}^{*})(\alpha_{4j}-\alpha_{4j}^{*})\boldsymbol{x}_i^t\boldsymbol{x}_j + \sum_{i,j=1}^{n}(\alpha_{3i}+\alpha_{3i}^{*})(\alpha_{3j}+\alpha_{3j}^{*})|\boldsymbol{x}_i|^t|\boldsymbol{x}_j| \\
& + \sum_{i,j=1}^{n}(\alpha_{4i}+\alpha_{4i}^{*})(\alpha_{4j}+\alpha_{4j}^{*})|\boldsymbol{x}_i|^t|\boldsymbol{x}_j| - 2\sum_{i,j=1}^{n}(\alpha_{3j}+\alpha_{3j}^{*})(\alpha_{4j}+\alpha_{4j}^{*})|\boldsymbol{x}_i|^t|\boldsymbol{x}_j| \\
& + \sum_{i,j=1}^{n}\alpha_{1i}\alpha_{1j}|\boldsymbol{x}_i|^t|\boldsymbol{x}_j| - 2\sum_{i,j=1}^{n}\alpha_{1i}(\alpha_{3j}+\alpha_{3j}^{*})|\boldsymbol{x}_i|^t|\boldsymbol{x}_j|\Big) \\
& - \frac{1}{2C}\sum_{i=1}^{n}\alpha_{1i}^2 - \frac{1}{2C}\sum_{i=1}^{n}(\alpha_{2i}^2+\alpha_{2i}^{*2}) \\
& + \sum_{i=1}^{n}(\alpha_{2i}-\alpha_{2i}^{*})y_i + \sum_{i=1}^{n}(\alpha_{3i}-\alpha_{3i}^{*})y_i - \sum_{i=1}^{n}(\alpha_{4i}-\alpha_{4i}^{*})y_i \\
& + \sum_{i=1}^{n}(\alpha_{3i}+\alpha_{3i}^{*})e_i - \sum_{i=1}^{n}(\alpha_{4i}+\alpha_{4i}^{*})e_i \Big\}
\end{aligned} \tag{12}$$

subject to

$$\alpha_{1i},\ \alpha_{ki},\ \alpha_{ki}^{*} \geq 0,\ k=2,3,4.$$

Solving (12) with above constraints determines the Lagrange multipliers, $\alpha_{1i},\ \alpha_{ki}, \alpha_{ki}^{*}$. We take $\boldsymbol{c} = \max\{\boldsymbol{c}, \boldsymbol{0}\}$ and $\boldsymbol{d} = \max\{\boldsymbol{d}, \boldsymbol{0}\}$ since c_i and d_i should be nonnegative. Here, $\boldsymbol{0}$ represents the corresponding zero vector. We use the same $\boldsymbol{c}$ and $\boldsymbol{d}$ to avoid the abuse of notations. Therefore, since $\boldsymbol{c}^t|\boldsymbol{x}| \geq 0$ and $\boldsymbol{d}^t|\boldsymbol{x}| \geq 0$, the interval linear regression function is given by (7), (8) and (9) as follows:

$$Y^{*}(\boldsymbol{x}) = (\boldsymbol{a}^t\boldsymbol{x},\ \boldsymbol{c}^t|\boldsymbol{x}| + \boldsymbol{d}^t|\boldsymbol{x}|) \tag{13}$$

$$Y_{*}(\boldsymbol{x}) = (\boldsymbol{a}^t\boldsymbol{x},\ \boldsymbol{c}^t|\boldsymbol{x}|) \tag{14}$$

Next, we will consider nonlinear interval regression model. In contrast to linear interval regression, there have been no articles on nonlinear interval regression. In this paper we treat nonlinear interval regression, without assuming the underlying model function. In the case where a linear regression function is inappropriate quadratic loss SVM makes algorithm nonlinear. This could be achieved by simply preprocessing input patterns $\boldsymbol{x}_i$ by a map $\Phi: R^d \rightarrow E$ into some feature space E and then applying quadratic loss SVM regression algorithm. This is an astonishingly straightforward way.

First notice that the only way in which the data appears in (12) is in the form of inner products $\boldsymbol{x}_i^t \boldsymbol{x}_j$, $|\boldsymbol{x}_i|^t |\boldsymbol{x}_j|$. The algorithm would only depend on the data through dot products in E, i.e. on functions of the form $K(\boldsymbol{x}_i, \boldsymbol{x}_j) = .\,\Phi(\boldsymbol{x}_i)^t \Phi(\boldsymbol{x}_j)$, $K(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|) = \Phi(|\boldsymbol{x}_i|)^t \Phi(|\boldsymbol{x}_j|)$. The well used kernels for regression problem are given below.

$$K(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x}^t \boldsymbol{y} + 1)^p, \quad K(\boldsymbol{x}, \boldsymbol{y}) = e^{-\frac{|\boldsymbol{x}-\boldsymbol{y}|^2}{2\sigma^2}} .$$

Here, p and σ^2 are kernel parameters. In final, the nonlinear interval regression solution is given by

$$\begin{aligned}
\text{maximize}\Bigg\{ &-\frac{1}{2}\Bigg(\sum_{i,j=1}^n (\alpha_{2i}-\alpha_{2i}^*)(\alpha_{2j}-\alpha_{2j}^*)K(\boldsymbol{x}_i, \boldsymbol{x}_j) \\
&+ \sum_{i,j=1}^n (\alpha_{3i}-\alpha_{3i}^*)(\alpha_{3j}-\alpha_{3j}^*)K(\boldsymbol{x}_i, \boldsymbol{x}_j) + \sum_{i,j=1}^n (\alpha_{4i}-\alpha_{4i}^*)(\alpha_{4j}-\alpha_{4j}^*)K(\boldsymbol{x}_i, \boldsymbol{x}_j) \qquad (15)\\
&+ 2\sum_{i,j=1}^n (\alpha_{2i}-\alpha_{2i}^*)(\alpha_{3j}-\alpha_{3j}^*)K(\boldsymbol{x}_i, \boldsymbol{x}_j) - 2\sum_{i,j=1}^n (\alpha_{2i}-\alpha_{2i}^*)(\alpha_{4j}-\alpha_{4j}^*)K(\boldsymbol{x}_i, \boldsymbol{x}_j) \\
&- 2\sum_{i,j=1}^n (\alpha_{3i}-\alpha_{3i}^*)(\alpha_{4j}-\alpha_{4j}^*)K(\boldsymbol{x}_i, \boldsymbol{x}_j) + \sum_{i,j=1}^n (\alpha_{3i}+\alpha_{3i}^*)(\alpha_{3j}+\alpha_{3j}^*)K(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|) \\
&+ \sum_{i,j=1}^n (\alpha_{4i}+\alpha_{4i}^*)(\alpha_{4j}+\alpha_{4j}^*)K(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|) - 2\sum_{i,j=1}^n (\alpha_{3j}+\alpha_{3j}^*)(\alpha_{4j}+\alpha_{4j}^*)K(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|) \\
&+ \sum_{i,j=1}^n \alpha_{1i}\alpha_{1j}K(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|) - 2\sum_{i,j=1}^n \alpha_{1i}(\alpha_{3j}+\alpha_{3j}^*)K(|\boldsymbol{x}_i|, |\boldsymbol{x}_j|)\Bigg) \\
&- \frac{1}{2C}\sum_{i=1}^n \alpha_{1i}^2 - \frac{1}{2C}\sum_{i=1}^n (\alpha_{2i}^2 + \alpha_{2i}^{*2}) \\
&+ \sum_{i=1}^n (\alpha_{2i}-\alpha_{2i}^*)y_i + \sum_{i=1}^n (\alpha_{3i}-\alpha_{3i}^*)y_i - \sum_{i=1}^n (\alpha_{4i}-\alpha_{4i}^*)y_i \\
&+ \sum_{i=1}^n (\alpha_{3i}+\alpha_{3i}^*)e_i - \sum_{i=1}^n (\alpha_{4i}+\alpha_{4i}^*)e_i \Bigg\}
\end{aligned}$$

subject to

$$\alpha_{1i},\ \alpha_{ki},\ \alpha_{ki}^* \geq 0,\ k = 2,3,4.$$

Solving (15) with the above constraints determines the Lagrange multipliers, α_{1i},

$\alpha_{ki}, \alpha^*_{ki}$. Therefore, the interval nonlinear regression function is given as follows:

$$Y^*(\boldsymbol{x}) = \Big(\sum_{i=1}^{n} [(\alpha_{2i}-\alpha^*_{2i})+(\alpha_{3i}-\alpha^*_{3i})-(\alpha_{4i}-\alpha^*_{4i})]K(\boldsymbol{x}_i, \boldsymbol{x}),$$
$$\max\{0, \sum_{i=1}^{n} [(\alpha_{3i}+\alpha^*_{3i})-(\alpha_{4i}+\alpha^*_{4i})]K(|\boldsymbol{x}_i|,|\boldsymbol{x}|\}+ \quad (16)$$
$$\max\{0, \sum_{i=1}^{n} [-\alpha_{1i}+(\alpha_{3i}+\alpha^*_{3i})]K(|\boldsymbol{x}_i|,|\boldsymbol{x}|)\}\Big)$$

$$Y^*(\boldsymbol{x}) = \Big(\sum_{i=1}^{n} [(\alpha_{2i}-\alpha^*_{2i})+(\alpha_{3i}-\alpha^*_{3i})-(\alpha_{4i}-\alpha^*_{4i})]K(\boldsymbol{x}_i, \boldsymbol{x}),$$
$$\max\{0, \sum_{i=1}^{n} [(\alpha_{3i}+\alpha^*_{3i})-(\alpha_{4i}+\alpha^*_{4i})]K(|\boldsymbol{x}_i|,|\boldsymbol{x}|\}\Big) \quad (17)$$

4. Nonlinear Near-Wall Turbulence Time Series

In this section, in order to show the practicality and usefulness of the method described in the previous section, a case for predicting the near-wall turbulence timeseries is taken as a validated model. Porporato and Ridolfi(1997) discovered a phenomenon which the near-wall turbulence time-serial data is nonlinear from the experiments, measured using a Laser Doppler Anemometer. They applied a nonlinear chaotic prediction to a high-dimension system and produced forecasts of rapidly decreasing quality over time, with no consequences for practical applications. They use a trace of the direct prediction with forecast interval = $5\times3\times\Delta t$=160.40 ms: the correspondence with reality is optimum and maintained during the strong and extended velocity gradients. Although the forecast is worse globally and the rapid oscillations largely escape the method, the large-scale behavior is still well captured, and even the strong velocity gradients are forecasted with an accuracy equal to that of the small-scale motions by their research. However, when forecast interval is slightly above the traditional Kolmogorov time-scale, the forecast is rather poor.

Therefore, we try to use a kernel-based machine for fuzzy nonlinear regression analysis to predict the nonlinear time series of turbulent flows, and compare it with fuzzy piecewise regression of Tseng et al.(2001). The testing data used here as an example were taken subjectively from Porporato and Ridolfi(1997). We divided the time serial data into 10 segments from the experimental results and use 11 samples of time series to test this method. Very few data is used in order to demonstrate the practicality of this model. The time series x_t ranges from 0 to 10. This data are given in Table 1.

Table 1. The comparison of observed output with predicted output

Sample	Observed Output	Predicted Output (m/s)		
(time, s)	(raw data, m/s)	Kernel Machine	Possibility(3)	Necessity(5)
0	[0.0248, 0.0260]	[0.0244, 0.0260]	[0.02360, 0.02662]	[0.02480, 0.02530]
1	[0.0203, 0.0210]	[0.0203, 0.0212]	[0.02012, 0.02322]	[0.02050, 0.02100]
2	[0.0185, 0.0208]	[0.0181, 0.0208]	[0.01762, 0.02080]	[0.01850, 0.01900]
3	[0.0161, 0.0173]	[0.0161, 0.0174]	[0.01610, 0.01936]	[0.01645, 0.01695]
4	[0.0167, 0.0189]	[0.0162, 0.0188]	[0.01556, 0.01890]	[0.01670, 0.01720]
5	[0.0160, 0.0165]	[0.0160, 0.0166]	[0.01600, 0.01942]	[0.01600, 0.01650]
6	[0.0235, 0.0270]	[0.0231, 0.0270]	[0.02350, 0.02700]	[0.02647, 0.02697]
7	[0.0270, 0.0282]	[0.0269, 0.0283]	[0.02462, 0.02820]	[0.02749, 0.02799]
8	[0.0227, 0.0245]	[0.0224, 0.0246]	[0.02226, 0.02592]	[0.02390, 0.02440]
9	[0.0225, 0.0238]	[0.0223, 0.0240]	[0.02088, 0.02462]	[0.02262, 0.02312]
10	[0.0232, 0.0243]	[0.0229, 0.0243]	[0.02048, 0.02430]	[0.02364, 0.02414]

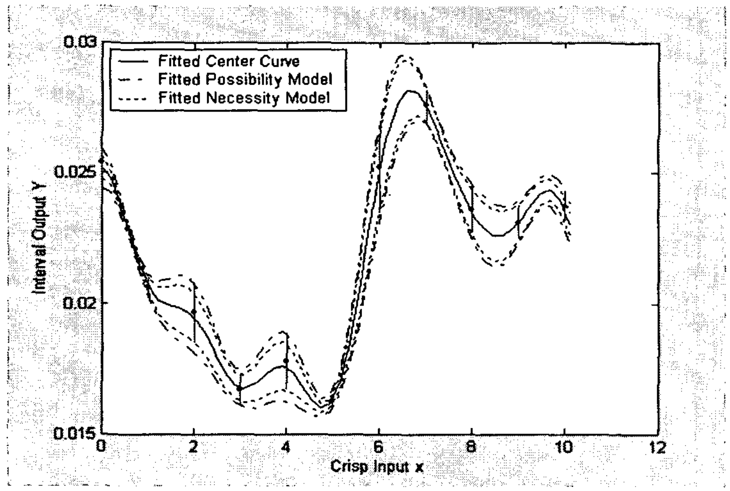

Figure 1. Fuzzy nonlinear regression model

Figure 1 illustrates the results for the proposed nonlinear model. Here, we use Gaussian kernel for fuzzy nonlinear regression model. We have used leave-one-out(LOO) cross-validation based on the sum of squares in the optimization problem (3) to determine an optimal combination of C and σ, which are C=30 and σ=1.0. The solid curve explains the fitted regression curve for center. The two dashdot and dotted curves explain the fitted possibility and necessity models, respectively. Figure 1 depicts the change intervals for the proposed kernel-based machine. From Table 1 and Figure 1 we know that the proposed kernel-based machine performs well. In particular, based on the results of objective value (total/average vagueness) and similarity index from Table 1, we can judge that the proposed kernel-based machine is better than possibility and necessity models by quadratic piecewise model in Tseng et al.(2001).

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