The Hough Transform - A Radon-Like Transform

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Abstract:

The Hough transform has been used as a tool for line detection. The main idea of the Hough transform is to transform each pixel in the image individually into the parameter domain. In this way, the Hough transform converts a difficult global detection problem in the image domain into a more easily solved local peak detection problem in the parameter domain. In this paper, we show that the discrete Hough transform is identical to the discrete Radon transform. Thus, we can use the generalized Radon transform to handle more general parameterized curve types.

Keywords. Curve detection, image processing, Hough transform, Radon transform

1. INTRODUCTION

A current problem in image processing is the curve detection in a given digital image. In the simplest case, the image is formed by black points in a white background. In the case general, the image is represented by a grid of grey levels. Hough [1], Duda and Hart [2], and Giffith [3] have developed methods for detecting straight lines. These methods transform each point in image plane to a curve in a parameter space. The parameter space is defined by the parametric representation used to describe the curves in the image plane. In this paper, we present the Hough transform as a tool for line detection, and we show that this transform is a special case of the Radon transform. Finally, we present a new algorithm for Radon transform of a digital mage.

2. HOUGH TRANSFORM - A MATHEMATICAL TOOL FOR LINE DETECTION

Hough developed a transform for detecting straight line in digital image. The basic idea behind using the Hough transform to identify lines in digital images is quite straightforward and follows directly from the definition.

$$H_g(\rho,\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)\delta(\rho - x\cos\theta - y\sin\theta)dxdy \quad (2.1)$$

Thus the Hough transform of a function concentrated at a point $\delta(x - x_0)\delta(y - y_0)$ yields a sinusoidal curve

$$p = x_0 \cos\theta + y_0 \sin\theta \quad (2.2)$$

in the $\theta\rho$-plane. Furthermore, all collinear point in the xy-plane along the line determined by the fixed values θ_0 and ρ_0 also map to sinusoidal curves in the $\theta\rho$-plane, and they all intersect in the same point: $(\rho,\theta) = (\rho_0,\theta_0)$. Thus, the lines are found from the peaks in the $\theta\rho$-plane as indicated in Figure 1 and Figure 2. In Figure 1, image containing three line pieces is shown. Figure 2 shows the corresponding discrete parameter domain by use of Hough transform. Three peaks in this Figure correspond to three lines in the Figure 1.

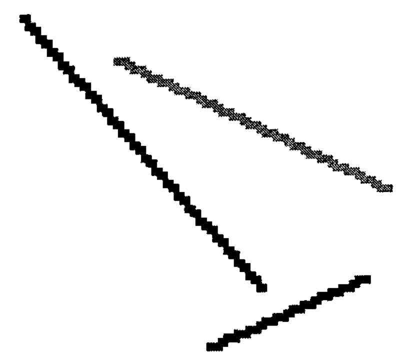

Figure 1 An image with 3 lines pieces

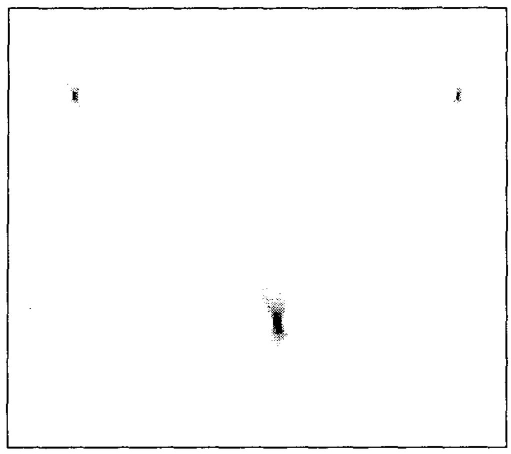

Figure 2 The parameter domain found by Hough transform

3. RADON TRANSFORM

The Radon transform is thoroughly in many publications and will only be conceptually described here.

3.1. Definition

Let bounded function $f : R^2 \rightarrow R$ be a compactly supported piecewise continuous function. The line $L(\theta,\rho)$ is defined by

$$\rho = x\cos\theta + y\sin\theta \qquad (3.1)$$

The line integral of f along the line L is given by

$$R_f(\rho,\theta) = \int_{L(\theta,\rho)} f(x,y)\,ds \qquad (3.2)$$

where ds is an increment of length along L.

If $R_f(\rho,\theta)$ is known for all ρ and θ, then $R_f(\rho,\theta)$ is the two-dimensional Radon transform of $f(x,y)$.

3.2. Relation to the Hough transform

Using a delta function, (3.2) can be rewritten as

$$R_f(\rho,\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(\rho - x\cos\theta - y\sin\theta)dxdy$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\vec{x})\delta(\rho - \vec{x}\cdot\xi)d\vec{x} \qquad (3.3)$$

where $\vec{x} = (x,y)$ and $\xi = (\cos\theta,\sin\theta)$ is a unit vector defines direction in terms of the angle θ.

Substituting $f(x,y) = \delta(x - a)\delta(y - b)$ into equation (3.3) we have

$$R_f(\rho,\theta) = \delta(\rho - \rho_0) \qquad (3.4)$$

where

$$\rho_0 = a\cos\theta + b\sin\theta \qquad (3.5)$$

Equation (3.4) actually expression what Hough meant.

4. A COMPUTATIONAL ALGORITHM FOR THE RADON TRANSFORM OF A DIGITAL IMAGE

Each image is a linear combination of characteristic functions of squares of side 2r. With

$$f(x) = \sum_i c_i \chi_{R_i}(x) \qquad (4.1)$$

we get the Radon transform

$$R_f(\rho,\theta) = \sum_i c_i R_{\chi_{R_i}}(\rho,\theta) \qquad (4.2)$$

When f_0 is the characteristic function of the square $[-r,r]x[-r,r]$, and $0 < \theta \le \pi/4$, we get:

$$R_{f_0}(\rho,\theta) = \begin{cases} \dfrac{2r}{\cos\theta}; & 0 \le |t| \le t_1 \\ \dfrac{t_2 - t}{\sin\theta - \cos\theta}; & t_1 < |t| \le t_2 \\ 0; & |t| > t_2 \end{cases} \qquad (4.3)$$

where

$$t_1 = r(\cos\theta - \sin\theta)\ ;\ t_2 = r(\cos\theta + \sin\theta) \qquad (4.4)$$

And

$$R_{f_0}(\rho,\theta) = \begin{cases} R_{f_0}(\rho,\pi/2-\theta); & 0 \le \theta \le \pi/4 \\ R_{f_0}(\rho,\pi/2+\theta); & \pi/4 < \theta \le \pi/2 \end{cases} \qquad (4.5)$$

Let $f_{a,b}$ be the characteristic function of the square $[a-r,a+r]x[b-a,b+r]$, we get:

$$R_{f_{a,b}}(\rho,\theta) = R_{f_0}(\rho - a\cos\theta - b\sin\theta,\theta) \qquad (4.6)$$

Then, The Radon transform of a digital image is evaluated by (4.2) – (4.6).

5. CONCLUSION

We presented the Hough transform as a line-to-point transformation. A single point in parameter plane contains information about the line in the image plane. Therefor the Hough transform can be used as a tool for line detection. We also showed that Hough transform is a special case of the Radon transform. Thus we can use the generalized Radon transform to handle more general parameterized curve types.

An ongoing effort is being made to apply these ideas in other areas including shape detection, structure detection scene analysis and image compression.

Finally, we proposed an algorithm that evaluates exactly the Radon transform of an arbitrary digital image.

References

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