Application of Nonuniform Weighted Distribution Method to Enhancing Signal Processing Effect of Subband Spatial-Temproral Adaptive Filter

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Abstract

The very complicated proplem in spatial processing is effects of phading (Multipath and Delay Spread) and co-channel interference (CCI). The phading is one of principal causes, that form inter-symbol interference (ISI). Spatial-Temproral Adaptive Filter (STAF) has been taken as a solution of this problem, because it can suppress both these types of interference. But the performance of STAF exposes some elemental limitations, in which are the slow convergence of adaptive process and computational complexity. The cause of this is that, STAF must treat a large quantity of information in both space and time. The way that master these limitation is a use of Subband Spatial-Temproral Adaptive Filter (SSTAF). SSTAF reduce computational complexity by pruning off samples of signal and thus it lost some information in time. This draw on attennation of output SINR of SSTAF. The article analyse a optimal solution of this problem by introducing SSTAF with nonuniform weighted distribution.

1. INTRODUTION

A complex problem in radar and mobile communication is the effect of fading and co-channel interference [4]. The fading may be in the form of multipath or delay spread. The its consequence is *inter symbol interference* (ISI). In the case of frequency reuse in mobile communications networks or of operating of many radars in certain local area (Shown in Figure 1), the interference is referred as *co-channel interference* (CCI).

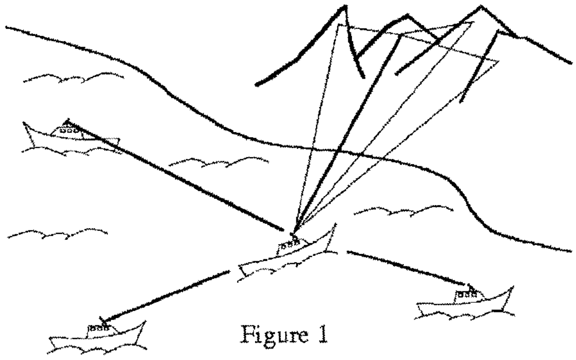

Figure 1

In order to avoiding CCI, the traditional solution is using a spatial adative filter (SAF). The response of this filter has the peak again points as desired signal and nulls points as for co-channel noise sources. Simultaneously by adding to SAF the temproral filter in the form of FIR, inter symbol interference can be decreased. The result is that this filter is called *spatial temproral adaptive filter* (STAF).

According to theory, a STAF with M elements has $(M-1)$ degrees of freedom (DOF) and that can eliminate $(M-1)$ CCI source. On the other hand, the decrease of ISI is inverse with the length K of filter FIR, that is, ISI more decreased when K more increased. In general, the efficiency of STAF to filtrate interference is often measured by parameter *signal to interference plus noice rate* SINR. The SINR is more larger when M and K are more larger. Therefore, STAF requires a large number of operations (particulary number of multip-lications) and this is a fundamental disadvantage of STAF, which is decreasing availability of its performance and of its application.

Recently, in oder to overcome these disadvantages many studied works has been given. In general, their results are sumarized in two ways as follows:

- Solution in technology (know as "hard ware"). For example, the solution Mesh Synchronous Processor.
- Solution in algorithm (know as "soft ware"). The typical example is use multirate filter bank in *subband spatial-temproral adaptive filter* SSTAF.

While STAF processes the input signal on sample-by-sample basic, this is done on block-by-block in SSTAF and as a result, the calculational complexity is decreased many time. In oder to increased SINR, STAF need many informations such as spatial and temproral, so this methode is complex. But with SSTAF, although the SINR is not as hight as it is in STAF, the calculation is decreased much, so it is an effective solution.

The goal of this work is to give the optimal solution: SSTAF with nonuniform weighted distribution (SSTAF-NWD). The SSTAF-NWD has advantage of SSTAF, that is smaller calculational complexity. But its SINR higher than one of SSTAF and can be in comparison with one of STAF.

2. IMPLEMENTATION OF SSTAF-NWD MODEL

Other way to increase SINR of conventional spatial filters is applied in two ways as follows: either changing structure (spatial sample shape) or applying nonuni-form weighted distribution (NWD) . Both of solutions are generaly called synthesis pattern. This is a traditional solution in antenna technology. The first solution is usually applied to fixed systems and the NWD with Taylor method is often applied to systems, which processed complexity signal operating environment (for example radar, mobile communication, ...). By combining SSTAF with method of NWD, we have given SSTAF-NWD, the optimal solution that said above.

By combining structures of SSTAF and structure of method NWD, we have given the SSTAF-NWD model can shown in Fig.2

where : - M is number of sensors.

- K is number of delay taps.

- T is sampling period (or delaying time of 1 tap)

In nonuniform weighted distribute method, weights are calculated individually with signal and processed data. But in adaptive filter method, the

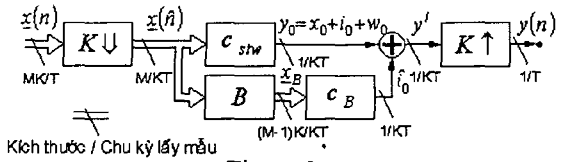

Figure 2

calculating weights require deterministic spatial statistics of signal and noise, that is, weights must associate of processing data. There fore SSTAF-NWD model is seprated 2 branches

- The upper branch is a part, which is realize method of non uniform weighted distribution (NWD).

- Lower branche is a part, which is realize adaptive algorithm.

Upper branch don't realize adaptive algorithm (knowns as "constrained branch"). Nonadaptive spatial-temproral filter has weighted vector $\underline{c}_{stw}$. From principle of spatial-temproral matched filter, we have:

$$\underline{c}_{stw}(\phi_s, f_s) = \underline{v}_{stw}(\phi_s, f_s) \qquad (2.1)$$

where: $\underline{v}_{stw}(\phi_s, f_s)$ - Spatial-temproral steering vector.

ϕ_s - Angle of arrival.

f_s - Frequency of desired signal.

With $\underline{v}_{stw}(\phi_s, f_s)$ is calculated as:

$$\underline{v}_{stw}(\phi_s, f_s) = \underline{v}_t(f_s) \otimes (\underline{v}_w \circ \underline{v}_s(\phi_s)) \quad (2.2)$$

where: $\circ$ - Hadamart product of 2 vector.

$\otimes$ - Kronecker product of 2 vector.

$\underline{v}_w$ - Vector weighted distribution.

$\underline{v}_s(\phi_s)$ - Spatial steering vector (or spatial response).

$\underline{v}_t(f_s)$ - Temproral steering vector.

According to concept about multiplication of Kronecker, Vector $\underline{v}_{stw}(\phi_s, f_s)$ has dimention $KM \times 1$.

Therefore, the output of upper branch is :

$$y_0(\hat{n}) = \underline{c}_{stw}^H . \underline{x}(\hat{n}) = \underline{v}_{stw}^H(\hat{n}) . \underline{x}(\hat{n}) =$$
$$= s_0(\hat{n}) + i_0(\hat{n}) + w_0(\hat{n}) \qquad (2.3)$$

(Note that, as $\underline{x}(\hat{n})$ has dimension $KM \times 1$ so the output $y_0(\hat{n})$ is 1 sample).

The lower branche realize adaptive algorithm and can call being "nonconstrai-ned optimum branch". It contains following blocks.

* $\underline{\underline{B}}$ is blocking matrix, the output signal of which is maked from M-1 sensors that is unstored any signal from angle of arrival ϕ_s. With that poposed, $\underline{\underline{B}}$ has dimension $M \times (M-1)$ and orthogoral with spatial steered vector $\underline{v}_s(\phi_s)$, also we has another method that can choose $\underline{\underline{B}}$ satisfaction conditional:

$$\underline{\underline{B}}^H_{(M-1)\times M} . \underline{v}_s(\phi_s)_{M\times 1} = \underline{0}_{(M-1)\times 1} \qquad (2.4)$$

The most simplicity is that we can choose matrix $\underline{\underline{B}}$ by [1]:

$$\underline{\underline{B}} = [\underline{v}_s(u_1) \quad \underline{v}_s(u_2) \quad \ldots \quad \underline{v}_s(u_{M-1})] \quad (2.5)$$

where u_m is spatial frequency and calculated by:

$$u_m = u_s + \frac{m}{M} \qquad (2.6)$$

with $m = 1, 2, \ldots, (M-1)$

$$u_s = \frac{d}{\lambda} \sin \phi_s$$

where: d - Distance between sesors.

λ - Wavelength of propagating signal.

At the end, out put of lower branches of $\underline{\underline{B}}$ is $\underline{x}_B(\hat{n})$ and it is calculated as:

$$\underline{x}_B(\hat{n}) = \underline{\underline{B}}^H . \underline{x}(\hat{n}) \qquad (2.7)$$

Desiring vector $\underline{x}_B(\hat{n})$ has length $(M-1)K$ that must be doing as this order: Vector $\underline{x}_B(\hat{n})$ is separated to K element vectors with length M for periodic time T. Next step, one by one it is multiplicated with $\underline{\underline{B}}$ matrix with period KT and combines all element vector to make up vector $\underline{x}_B(\hat{n})$ with length $(M-1)K$.

* $\underline{c}_B$ - Adaptive weighted vector. It is calculated by orginal algorithm of minimum mean square error (MMSE) and is disolves by equation of Wiener - Hopf with the result as:

$$\underline{c}_B = \underline{\underline{R}}_B^{-1} . \underline{r}_B \qquad (2.8)$$

where: $\underline{\underline{R}}_B$ - Correlative matrix of the signal $\underline{x}_B(\hat{n})$, it is calculated:

$$\underline{\underline{R}}_B = E\{\underline{x}_B(\hat{n}) . \underline{x}_B^H(\hat{n})\} \qquad (2.9)$$

($\underline{x}_B(\hat{n})$ is the vector with length $(M-1)K$, so the correlative matrix with $(M-1)K$ x $(M-1)K$).

$\underline{r}_B$ - Cross correlative vector of the signal on upper and lower branches:

$$\underline{r}_B = E\{\underline{x}_B(\hat{n}) . y_0^*(\hat{n})\} \qquad (2.10)$$

(The dimention of $\underline{x}_B(\hat{n})$ is $(M-1)K \times 1$ and the dimention of $y_0(\hat{n})$ is 1×1, so $\underline{r}_B$ is the vector with length $(M-1)K$).

The result, estimated noise of lower branch is:

$$\hat{i}_0(\hat{n}) = \underline{c}_B^H . \underline{x}_B(\hat{n}) \qquad (2.11)$$

After cancelation interference, the output signal is:

$$y(\hat{n}) = y_0(\hat{n}) - \hat{i}_0(\hat{n}) \qquad (2.12)$$

And then, $y(\hat{n})$ is up sampled with $K\uparrow$ and is interpolated to form the output signal $y(n)$ of SSTAF-NWD.

3. ASSUMPTIONS

As the common conditions to esti-mate and to compare effects of diferent solutions, we give the next assumptions, that are practical ability in very complex case. The initial assumptions for our analysis are as follows:

(1) The filter, that consists of M elements, is associted spatially with narrow bandwidth signal (Its' mean: Relative band-width $B = \Delta f / f_c \ll 1$, where f_c is carry frequency and Δf is signal band-width) and cause scatter fading is separated P path. Therefore the received signal in path p ($p = 0,1,..., (P-1)$) has angle of arrival θ_p with T_p delay time and is denoted by $s_p(n)$. If more suppose that these signals have zero mean stationary with average power $\xi_p^2 = E\{|s_p(n)|^2\}$. The p path has time delay $T_p = T.L_p$ (T is sample period or symbol duration), where $L_p \in [0,1,..., K]$. The direct path, that its path index is $p = 0$, due to spatialy matched has $\theta_0 = 0$ and non-delay,that is, has $L_0 = 0$ and $T_0 = 0$. For $a > b$, the assumption is $L_a \geq L_b$ (that is mean, if the path index is more increase then the delay may be more increase).

(2) The noise of each sensor is only thermal $w_m(n)$ ($m = 1,2,..., M$) and is statistically independent and identically distribution (i.i.d) of Gaussian process, that is, it is non-correlative both spatial and temproral with frequency f_m and average power $\sigma_m^2 = E\{|w_m(n)|^2\}$.

(3) The number of CCI sources is N and the assumption in extremely case is $N \geq M$. These interference sources are spatialy statistical independent and are denoted $i_q(n)$ ($q = 1,2,..., N$), with angle of arrival θ_q and average power $\eta_q^2 = E\{|i_q(n)|^2\}$.

4. COMPARING SINR BETWEEN DIFFERENT SOLUTIONS

According to defind, SINR is calculated by formula : (2.11)

$$SINR = \frac{P_S}{P_N + P_I} \qquad (4.1)$$

where: P_S - Power of desired signal.

P_N - Power of noise.

P_I - Power of interference.

4.1. Output SINR of STAF

The power of desired signal consists of 2 composing element paths: direct arrival path and fading path. Direct arrival path is matched with spatial-temproral response of filter and accordingly its maximum (unit) power equal $MK\xi_0^2$. Fading path is matched with maximum temproral response, but with spatial response by angel of arrival θ_p. Therefore:

$$\begin{aligned} P_S &= MK\xi_0^2 + E\left\{ .\left| \underline{c}_s^H(\theta_p) \underline{s}_p(n) \right|^2 \right\} = \\ &= MK\xi_0^2 + \frac{1}{P}\sum_{p=1}^{P} \overline{c}_{s.sl}\xi_p^2 = \\ &= MK\xi_0^2 + \frac{\overline{c}_{s.sl}}{P}\sum_{p=1}^{P}\xi_p^2 = \\ &= MK\xi_0^2 + \overline{c}_{s.sl}.\xi^2 \end{aligned} \qquad (4.2)$$

where: $\overline{c}_{s.sl}$ - Average spatial response of sidelobe.

ξ^2 - Average power of fading path.

The noise power in M spatial sensors $w_m(n)$ is only matched with temproral res-ponse. Therefore it is given form:

$$\begin{aligned} P_N &= E\left\{ .\left| \underline{c}_t^H(f_m) \underline{w}_m(n) \right|^2 \right\} = \\ &= \frac{1}{M}\sum_{m=1}^{M} \overline{c}_t.\sigma_m^2 = \frac{\overline{c}_t}{M}\sum_{m=1}^{M}\sigma_m^2 = \\ &= \overline{c}_t.\sigma^2 \end{aligned} \qquad (4.3)$$

where: $\overline{c}_t$ - Average temproral response.

σ^2 - Average power of noise.

Because optimum property of filter, $M-1$ interference sources have been canceled, therefore N interference sources are remained $N-M+1$ sources. The remaining interfe-rence sources are matched with spatial response by angle of arrival θ_q and them power give:

$$\begin{aligned} P_I &= E\left\{ .\left| \underline{c}_s^H(\theta_q) \underline{i}_q(n) \right|^2 \right\} = \\ &= \frac{1}{N-M+1}\sum_{q=1}^{N-M+1} \overline{c}_{s.sl}.\eta_q^2 = \end{aligned}$$

$$= \frac{\bar{c}_{s.sl}}{N-M+1}\sum_{q=1}^{N-M+1}\eta_q^2 =$$

$$= \bar{c}_{s.sl}.\eta^2 \qquad (4.4)$$

where: η^2 - average power of $N-M+1$ remaining interference sources.

The result, output SINR of STAF is:

$$SINR_{STAF} = \frac{MK\xi_0^2 + \bar{c}_{s.sl}.\xi^2}{\bar{c}_t.\sigma^2 + \bar{c}_{s.sl}.\eta^2} \qquad (4.5)$$

4.2. Output SINR of SSTAF

Because the structure of temproral filter of SSTAF is separated to subbank, the number of temproral sample is reduced N time. Similarly (4.2), the power of desired signal is calculated by:

$$P_S = M\xi_0^2 + E\left\{\left|\underline{c}_s^H(\theta_p)\underline{s}_p(n)\right|^2\right\} =$$

$$= M\xi_0^2 + \bar{c}_{s.sl}\xi^2 \qquad (4.6)$$

Analogy, noise power is:

$$P_N = \bar{c}_t^{/}.\sigma^2 \qquad (4.7\ ')$$

where: $\bar{c}_t^{/}$ is average temproral response in subbank, reality with established and calculated mean value is doing that it can give $\bar{c}_t^{/} \approx \bar{c}_t$. Therefore, (4.7 ') can rewrite to:

$$P_N = \bar{c}_t.\sigma^2 \qquad (4.7)$$

Interference power is independently of temproral response, therefore it may be calculated by (4.4) as before.

The result, output SINR of SSTAF is:

$$SINR_{SSTAF} = \frac{M\xi_0^2 + \bar{c}_{s.sl}.\xi^2}{\bar{c}_t.\sigma^2 + \bar{c}_{s.sl}.\eta^2} \qquad (4.8)$$

4.3. Output SINR of SSTAF-NWD

The purpose of method of nonuni-form weighted distribution is supressed the sidelobe response of spatial filter with G factor and therefore the average response of spatial filter become $\bar{c}_{s.sl}/G$. By deduction that (4.6) and (4.4) is calculated for power of desired signal power and of interference, in SSTAF-NWD case:

$$P_S = M\xi_0^2 + \frac{\bar{c}_{s.sl}}{G}.\xi^2 \qquad (4.9)$$

and

$$P_I = \frac{\bar{c}_{s.sl}}{G}.\eta^2 \qquad (4.10)$$

The result, output SINR of SSTAF-NWD is:

$$SINR_{SSTAF-NWD} = \frac{M\xi_0^2 + \frac{\bar{c}_{s.sl}}{G}.\xi^2}{\bar{c}_t.\sigma^2 + \frac{\bar{c}_{s.sl}}{G}.\eta^2} \qquad (4.11)$$

Comparing (4.5), (4.8) and (4.11), we has:

$$SINR_{STAF} = \frac{MK\xi_0^2 + \bar{c}_{s.sl}.\xi^2}{\bar{c}_t.\sigma^2 + \bar{c}_{s.sl}.\eta^2} \sim MK\xi_0^2$$

$$SINR_{SSTAF} = \frac{M\xi_0^2 + \bar{c}_{s.sl}.\xi^2}{\bar{c}_t.\sigma^2 + \bar{c}_{s.sl}.\eta^2} \sim M\xi_0^2$$

$$SINR_{SSTAF-NWD} = \frac{M\xi_0^2 + \frac{\bar{c}_{s.sl}}{G}.\xi^2}{\bar{c}_t.\sigma^2 + \frac{\bar{c}_{s.sl}}{G}.\eta^2} \sim M$$

Thus clearly we can recognize that:

$$SINR_{STAF} > SINR_{SSTAF-NWD} > SINR_{SSTAF}$$

As conclusion, by the application of method of nonuniform weighted distribution, the SINR, what is one of very important para-meters for estimating effect of the spatial-temproral filter, has been improved. In order to comparing these methods above, we shown Fig.3. These are the graphs of SINR as a function depending on number of sensors M.

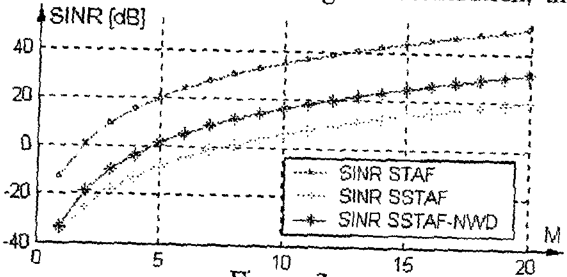

Figure 3

5. BENEFITS IN CALCULATIONS

In the mentioned solutions above, the complexity of calculation is given in adaptive algorithm, specially in applying sample matrix inversion (SMI) algorithm. This is a fastest algorithm to estimate weights, but it also is an algorithm with largest numbers of calculations (depending on the dimension of correlation matrix). So that, the time to implementation of adaptive process is the one to do all operations, in which mainly are multiplications.

In order to compare the benefits of solutions in calculation, we considers them with the same condition: The adaptive algorithm is SMI; The array of sensors consist of M elements; The delay line consist of K taps.

+ The STAF solution need $(KM)^3$ multiplications [6].

+ The SSTAF solution with bank filter FFT/IFFT need $K(M^3 + 2\log_2 K)$ multiplica-tions [6].

+ The SSTAF-NWD solution is more complex than SSTAF one. Because it has "constrained branch", so the number of multiplications is needed:

- M multiplication are needed for Hadamart products of two vectors which have length M. And then it's result is multiplicatied by tensor with a vector which has length K, so the number of multiplications need to calculate spatial-temproral weighted vector $\underline{c}_{s/tv}$ is $M + KM$.

- The multiplication of two vectors having same long KM need $(KM)^2$ multiplications.

So, total multiplications need:

$$K\left(M^3 + M + KM^2 + 2\log_2 K\right) + M \quad (5.1)$$

In case, there is multipath fading with long line delayer and need a large number of multiplications. Then the advantages of diference solutions is given in table 5.1 of Appendix A and

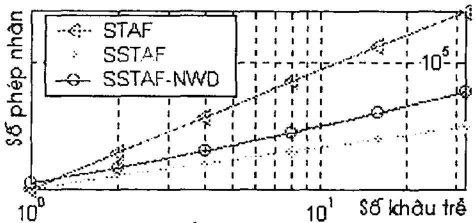

Figure 4

shown in fig 4. Looking fig.4 the most complex in calculating is belong to STAF solution and least complex is belong to SSTAF one. In SSTAF-NWD, the SINR is better than, it is in SSTAF but the calculating is more complex. Any where, this is an efficient choose between effect and complexcity.

6. SIMULATION RESULT

The beampattern of spatial filter aplied with NWD according to window – method is

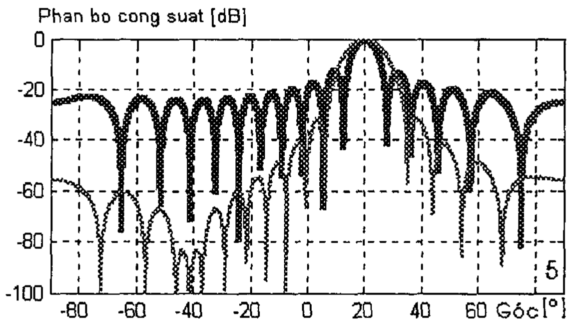

Figure 5

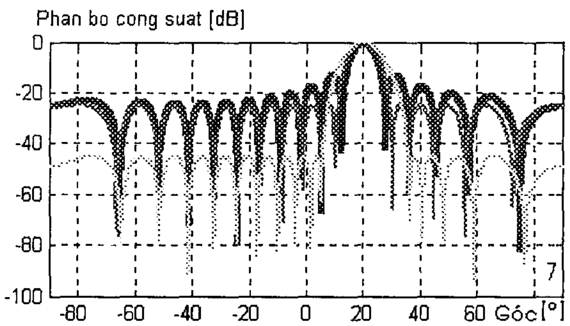

Figure 6

shown in figures 5 and 6 (another method of nonuniform weighted distribution will be present by another text). These graphs are equivalent rules with (ULA) wich consists of 16 element ($M = 16$), distant of half wavelength $d = 0{,}5.\lambda$ and direction arrival of angle of signal 20° ($\phi_s = 20^o$). The distribution of weighs having the shape of Hamming window is shown in fig.5. The distribution of weighs having the shape of Dolph-Chebyshev window, with two pressed levels 25[db] and 45[db] is shown in fig.6.

Looking fig.7 is an example about the effect of detection signal to interference. This is an example making the effect of applying NWD. A mixed signal consisting is shown in fig.7a and its SINR is equal to $SNR = 20[dB]$ (Measure on input of sensor – No. 10). The number of strong noise source is equal to number of polars $M = N = 20$ so there is bigest power of a noise equal 40[dB] and direction arrival of angle

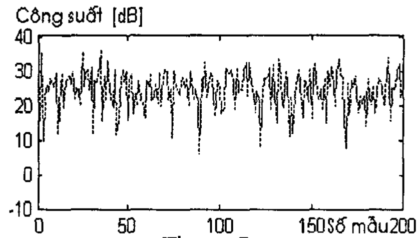

Figure 7.a

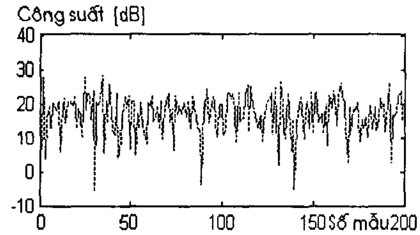

Figure 7.b

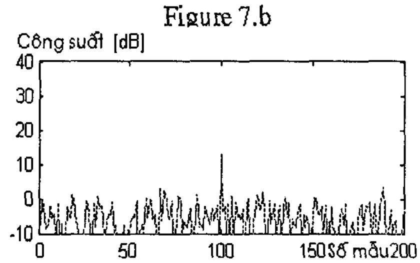

Figure 7.c

$\phi_i = 20^o$ passing filter. The output of SSTAF is shown in fig.7b. Cause of not applying NWD so that the radar signal can't detected from noise. The output of SSTAF-NWD is shown in fig.7c. Which signal is supressed to 50[dB] by distri-bution of weighs having the shape of Dolph-Chebyshev and radar signal can sparate clearly although the leve of signal receive below 0[dB]).

7. CONCLUSION

In spatial – temproral adaptive filter, cause of using statistics characterist of sigal and noise according to temproral and spatial by gathing spatial information, its SINR is high but its base disadvantage is a much calculation.

A subband solution is used to improve this problem but infact, this solution make time sample decrease so processing effect is decreased clearly. The solution mentioned above is method used to emphasize spatial processing effect the growing general trend of this subject is mentioned to other solution in which there are both spatial and temproral problems which are used to emphasize the processing effect of temproral and spatial filter.

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Appendix A.

Table 5.1. *Number of multiplications for different solutions with* $M = 4$.

K	1	2	4	8	16	32
STAF	64	512	4096	32768	262144	2097152
SSTAF	64	132	272	560	1152	2368
SSTAF-NWD	88	208	548	1620	5316	18884