An Efficient Discrete Bit-loading Algorithm for VDSL Channels

Minho Choi*, Sangseob Song* and Jaejin Lee**
* Dept of Electronic Engineering, Chonbuk National University, Jeonju, Korea
Tel : +81-63-251-2413 Fax : +81-63-251-2413 E-mail: mhchoi@mail.chonbuk.ac.kr
**Access Network Research Team, Technology Laboratory. KT, Korea
Tel : +81-42-870-8715 Fax : +81-42-870-8749 E-mail: jaejin@kt.co.kr

Abstract: In this paper we present a linear discrete bit-loading algorithm that maximizes the transmit bit rate using the channel informations to optimize the performance of the very high-speed digital subscriber line(VDSL) system. It will be useful under the constraint of a maximum transmit power for each user. When the level of crosstalk is high, the power allocation of a user changes the noise experienced by the other users in the same binder. In this case, the performance of DSL modems can be improved by jointly considering the bit and power allocation of all users.

VDSL, DMT, Bit-loading, Water filling

1. INTRODUCTION

This paper considers the discrete bit-loading problem for the multi-user case with the constraint of minimizing the total power. In this scheme, each modem should estimate the channel and crosstalk transfer functions, and then transmit the channel information to the spectrum management center. With this channel information, the center determines the bit and power allocation for every modem using the multi-user discrete bit-loading algorithm. Finally, the center transmits the allocation result back to each modem.

For the case of single user, the theoretical solution for achieving the channel capacity for spectrally-shaped channels with Gaussian noise has been found some time ago[1][2]. For multiple access channels, the information theoretical foundations have been laid by Cheng and Verdu[3], who studied the capacity regions for multiple access channels and found a generalization of the single-user waterfilling theorem.

The number of bits in each subchannel should be an integer or a multiple of some base unit for the practical use. Several optimal methods for discrete loading problem have been developed[3][4][5][6]. When the crosstalk noise is high, the power allocation of a user changes the crosstalk noise of the other users in the same binder. In this case, the performance of DSL modems can be improved by jointly considering the bit and power allocation of all users in the binder. The iterative water-filling algorithm may be applied autonomously without any coordination between users[7][8]. So in this paper we consider the centralized power-control problem in a multi-user interference channel. Since increasing the data rate of one user may decrease that of other users, data-rate control is also considered with maximizing the achievable rate region.

The rest of this paper is organized as follows. section 2 models digital subscriber loops as interference channels, section 3 describes the derivation of bit-loading problem and section 4 proposes a multi-user discrete bit-loading algorithm . The simulation results for very high-speed digital subscriber line (VDSL) upstream transmission are presented in section 5, and concluding remarks are given in section 6.

2. CHANNEL MODEL

A DSL cable may consist of up to 50 subscriber lines bundled together. The lines in the same bundle are electromagnetically coupled with each other, and this causes crosstalk noise. Near-end crosstalk refers to the crosstalk(NEXT) induced by transmitters located on the same side as the receiver Far-end crosstalk(FEXT) refers to the crosstalk induced by transmitters located on the opposite side. Frequency division multiplexing scheme is used to make the NEXT negligible.

Because of the FEXT, the DSL channel with M users is an interference channel with inter-symbol interference(ISI). By employing the discrete multi-tone(DMT) technique, the channel can be modeled as N independent ISI-free subchannels, each of which is an interference channel of M users. G_{ij} represents the crosstalk transfer function from user j to user i.

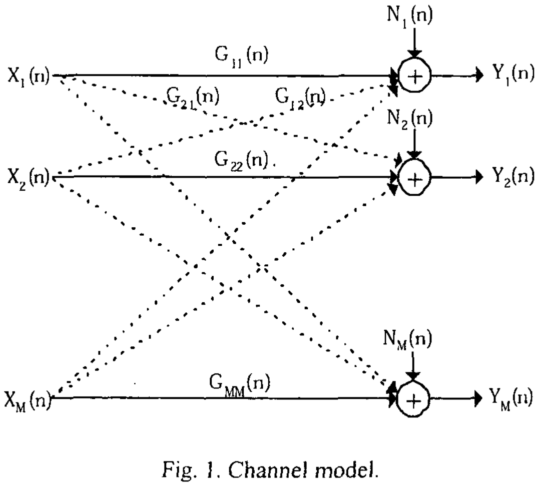

Fig. 1. Channel model.

The signal-to-interference-plus-noise ratio (SINR) of user i in subchannel n is then expressed as

$$SINR_i(n) = \frac{G_{ii}(n)P(n)_i}{\sum_{j\neq i} G_{ij}(n)P_j(n) + N_i(n)}, \qquad (1)$$
$$i, j \in \{1, 2, \cdots, M\}$$

where $P_i(n)$ and $N_i(n)$ are the signal power and the background noise power of user i in subchannel n. The SINR, γ_i, necessary for user i to transmit bits in subchannel n should satisfy

$$SINR_i(n) \geq \gamma_i(n), \quad i \in \{1, 2, \cdots, M\} \qquad (2)$$
$$\gamma_i(n) = \Gamma(2^{2\cdot b_i(n)} - 1)$$

where

$$\Gamma = \frac{1}{3}\left[Q^{-1}\left(\frac{P_e}{4}\right)\right]^2 \geq 1 \qquad (3)$$

Thus, the SNR gap Γ can be interpreted as a connection between the information theoretic channel capacity and the gap which is achievable with quadrature amplitude modulation(QAM).

Assuming that all transmitted signals, and background noises have Gaussian probability distribution, the number of bits that can be transmitted with QAM is approximated by

$$b_i(n) = \log_2\left(1 + \frac{SINR_i(n)}{\Gamma}\right) \qquad (4)$$

where is signal-to-noise ratio (SNR) gap that depends on the probability of symbol error, the noise margin, and the coding gain.

The data rate of user i is then

$$R_i = \frac{1}{T_s}\sum_{n=1}^{N} b_i(n) \qquad (5)$$

where T_s is a symbol period. A rate region is defined as the union of all the rate sets $\bigcup_{i=1}^{N} R_i$ that can be achieved while satisfying the following power constraint:

$$P_i \leq P_{\max,i} \quad \text{for } i = 1, \cdots M$$

where

$$P_i = \sum_{n=1}^{N} P_i(n), \qquad (6)$$

and is $P_{\max,i}$ a maximum power for user i.

Similarly γ_i depends on the modulation and coding scheme. The goal of the problem is to find the bit and power allocation requiring the least amount of total power under the constraint of

$$\text{minimize} \ \sum_{i=1}^{M}\sum_{n=1}^{N} P_i(n)$$
$$\text{subject to } b_i(n) \in Z, \quad P_i(n) \leq \overline{P(n)} \qquad (7)$$

where $Z = \{0, \cdots, b_{\max}\}$ and $\overline{P(n)}$ is the power mask. In practice, a bit cap is used to limit the number of bits for each user in each subchannel and power spectral density mask constraint is employed to prevent the power from exceeding the maximum power allowed in each subchannel. Another problem is find the achievable rate region of the users under the given power constraint.

3. DERIVATION OF A BIT ALLOCATION

With known SNR gap Γ and the relation $\gamma_i(n) = \Gamma(2^{2\cdot b_i(n)} - 1)$, rewriting the Eq. (2) in matrix form, we get

$$(\mathbf{I} - \mathbf{F})\mathbf{P} \geq \mathbf{u}, \quad \mathbf{P} > 0 \qquad (8)$$

where $\mathbf{P} = [P_1, P_2 \cdots, P_M]^T$ is the column vector of transmitter powers, $\mathbf{u} = [\frac{\gamma_1 N_1}{G_{11}}, \frac{\gamma_2 N_2}{G_{22}} \cdots \frac{\gamma_M N_M}{G_{MM}}]^T$ is the column vector of normalized powers and $\mathbf{F}$ is the matrix with entries

$$F_{ij} = \begin{cases} 0, & \text{if } i = j \\ \frac{\gamma_i G_{ij}}{G_{ii}}, & \text{if } i \neq j \end{cases} \qquad (9)$$

The matrix $\mathbf{F}$ has nonnegative elements and is irreducible. By the Perron-Frobenious theorem, the maximum modulus eigenvalue ρ_F of matrix $\mathbf{F}$ is real, positive and simple. If $\rho_F < 1$

$$(\mathbf{I} - \mathbf{F})^{-1} = \sum_{k=0}^{\infty} \mathbf{F}^k \qquad (10)$$

exists and is component-wise positive.

With direct matrix inversion, the power allocation vector $\mathbf{P}^*$ is obtained.

$$\mathbf{P}^* = (\mathbf{I} - \mathbf{F})^{-1}\mathbf{u} \qquad (11)$$

And the vector $\mathbf{P}^*$ is the Pareto optimal solution satisfying Eq. (2), in the sense that any other $\mathbf{P} > 0$ satisfying it would require at least as much power from each transmitter.

The iteration $\mathbf{P}(k+1) = \mathbf{F}\mathbf{P}(k) + \mathbf{u}$, converges to

$$\begin{aligned} \lim_{k\to\infty} \mathbf{P}(k) &= \lim_{k\to\infty} \mathbf{F}^k \mathbf{P}(0) + \lim_{k\to\infty}\left[\sum_{i=0}^{k-1} \mathbf{F}^i\right]\mathbf{u} \\ &= \mathbf{0} + \left[\sum_{i=0}^{\infty} \mathbf{F}^i\right]\mathbf{u} \qquad (12) \\ &= (\mathbf{I} - \mathbf{F})^{-1}\mathbf{u} \\ &= \mathbf{P}^* \end{aligned}$$

when $\rho_F < 1$.

Observing the iteration,

$$P_i(k+1) = \frac{\gamma_i}{SINR_i(k)} P_i(k) \qquad (13)$$

this can be interpreted as autonomous single-user water-filling, where $P_i(k)$ is the transmitter power of the user i. Each user measures autonomously its current SINR and tries to achieve its target γ_i in the next step, by increasing its power when the current SINR is below its target γ_i and lowering it otherwise.

4. PROPOSED BIT ALLOCATION METHOD

The single-user greedy bit loading algorithm assigns one bit in the subchannel that requires the least amount of power increase, in order to transmit this additional bit. Similarly, the multi-user discrete bit-loading algorithm allocate one bit to the user and subchannel where adding one bit requires the minimum cost. However, the cost $C(i,n)$ to allocate one more bit to user i and subchannel n is not simply the power increase of user i. If the power of one user is increased, then induced interference to the other users forces them to increase their power in order to maintain their SINRs. Thus, the cost $C(i,n)$ is chosen as the minimum total incremental power of all users.

The multi-user discrete bit-loading algorithm is described as follows.

Initialization:

1) Initialize user bit allocation $\mathbf{b}(n) = [b_1(n), \cdots, b_M(n)]^T$.

2) Find initial cost $C(i,n)$ of subchannel m for user i to allocate one bit.

Iteration:

3) Add one bit in the user and subchannel (i,n) where adding one bit requires the minimum cost among all available users and subchannels.

4) Update the cost $C(i,n)$ to increase one bit of the user in subchannel.

5) Check the given constraints. If satisfied go to step 3), if not stop.

5. SIMULATION RESULTS

We compare the performance of the multi-user discrete bit-loading algorithm with the discrete version of the iterative water-filling, the iterative discrete bit-loading scheme which applies the single-user greedy algorithm iteratively until the bit and power allocation of every user converges to an equilibrium point.

We used the simulation parameters of 2kft and 3kft AWG26 UTP lines for VDSL upstream transmission. Fig 2 shows the gain and FEXT transfer function of the lines.

Table 1 Simulation parameters

Number of tones	4096
Tone width	4.3125kHz
Symbol rate	4kHz
Coding gain	3.8dB
Noise margin	6dB
Required BER	$< 10^{-7}$
Power per user	10 dBm
Cable type	AWG26(0.4mm)
Frequency Plan	Plan 998
SNR gap	12dB

The proposed bit-loading algorithm performs better than the iterative water-filling. The disadvantage is that the bit and power allocation should be determined in a spectrum-management center using the complete channel information, which is not required by the autonomous iterative water-filling algorithm. The simulation results validates that the proposed algorithm extends the rate region compared with that of the iterative water-filling.

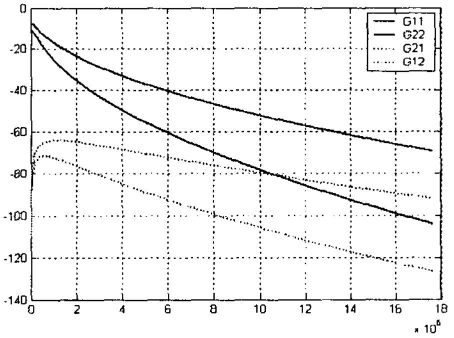

Fig. 2. Channel gain and FEXT transfer function.

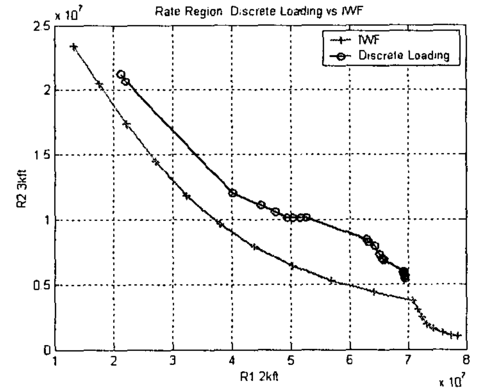

Fig. 3. Rate region of proposed method vs. iterative water-filling

6. CONCLUSION

This paper has considered the multi-user case of the rate maximization problem which was formulated as minimizing the total power. Use of the multi-user bit-loading scheme can provide additional improvements in binder spectrum design over autonomous method with iterative water-filling methods. With knowledge of channel information can lead to better performance. In the case of large number of users, the efficient and stable inversion of the matrix should be considered

References

[1] R. G. Gallager, *Information theory and reliable communications.* New York, Wiley, 1968.

[2] T. M. Cover and J. A. Thomas, *Elements of information theory,* Wiley, 1991.

[3] R. S. Cheng and S. Verdu, "Gaussian multi-access channels with ISI: Capacity region and multiuser water-filling" IEEE Trans. Inform. Theory, vol. 39, no. 3, pp. 773-785, May 1993.

[4] C. Y. Wong, R. S. Cheng, K. B. Letaief, and R. D. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," IEEE JSAC, vol. 17, no. 10, pp. 1747-1758, Oct. 1999.

[5] S. N. Diggavi, "Multiuser DMT: A multiple access modulation scheme," in Globecom, 1996.

[6] G. Munz, S. Pfletschinger, J. Speidel, "An efficient waterfilling algorithm for multiple access OFDM ," *Global Telecommunications Conference,* 2002.GLOBECOM '02. IEEE, vol. 1, 17-21, pp. 681 – 685, Nov. 2002.

[7] Wei Yu, Wonjong Rhee, Stephen Boyd, John Cioffi, "Iterative Water-filling for Vector Multiple Access Channels," *IEEE International Symposium on Information Theory, (ISIT),* 2001.

[8] Wei Yu, G. Ginis, J. M. Cioffi, "Distributed multiuser power control for digital subscriber lines," Selected Areas in Communications, IEEE Journal, vol. 20, Issue: 5, pp. 1105 – 1115, Jun. 2002.

[9] J. M. Cioffi, "A multicarrier primer," ANSI T1E1.4 Committee Contribution, Nov. 1991.