

Computational Fluid Dynamics Based on the Unified Coordinates

Professor Grafton W.H. Hui (許爲厚)

Department of Mathematics & Center for Scientific Computation
Hong Kong University of Science and Technology

Computational Fluid Dynamics (CFD) uses large scale computation to solve problems of fluid motion. There exist two general methods. In Eulerian method, the coordinates (hence the grid) are fixed in space; therefore it is simple but smears contact lines badly and forever. In Lagrangian method, the coordinates move with the fluid; consequently it resolves contact lines sharply but breaks down due to grid deformation with the fluid. Most CFD use the Eulerian method, which also requires generating a body-fitted grid prior to flow computation; grid generation is a major bottleneck in CFD research.

We have introduced a unified coordinate system that moves with velocity hq , q being fluid velocity. It includes the Eulerian system as a special case when $h = 0$, and the Lagrangian when $h = 1$, and it unifies them. By choosing the arbitrary function h to preserve grid orthogonality, it combines the advantages of both Eulerian and Lagrangian methods. In particular, we have achieved the following:

- (a) resolving contact lines sharply without break down of computation;
- (b) computing flow without grid generation, thus greatly saving computing time;
- (c) deriving, for the first time, the Lagrangian gas dynamics equations in conservation form, thus substantially facilitating Lagrangian computation.

Examples include: diamond-shape airfoil, hypersonic flow past a cone, 2-D steady Riemann problem, Mach reflection in a channel, and gridless computation of flow past an airfoil.

References

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- (2) Hui, W.H. and Kudriakov, S. (2001) "A unified coordinate system for solving the three-dimensional Euler equations", *Journal of Computational Physics*, **172**, 235-260 (2001).
- (3) Hui, W. H. and Kudriakov, S. (2002) "Computation of shallow water waves using the unified coordinates", *SIAM Journal on Scientific Computing*, **23**, 1615-1654.
- (4) Hui, W.H. and Tang, L. (2003) "A unified coordinates approach to computational fluid dynamics", AIAA paper 2003-4239, presented at the 16th AIAA CFD Conference, Orlando, FL, 23-26 June, 2003.

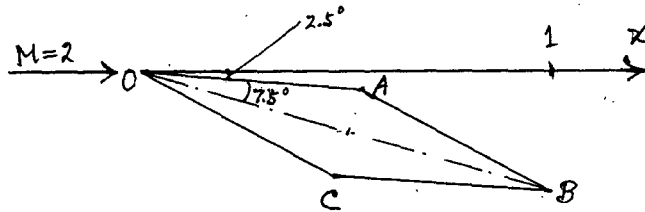


Fig. 1a Sketch

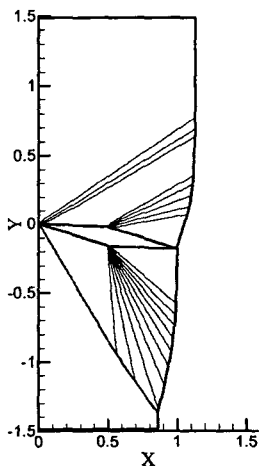


Fig. 1b Computed Pressure Contours

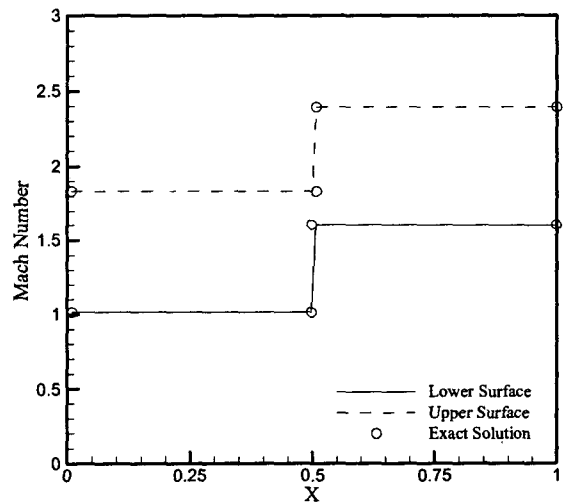


Fig. 1c Computed Surface Mach Number

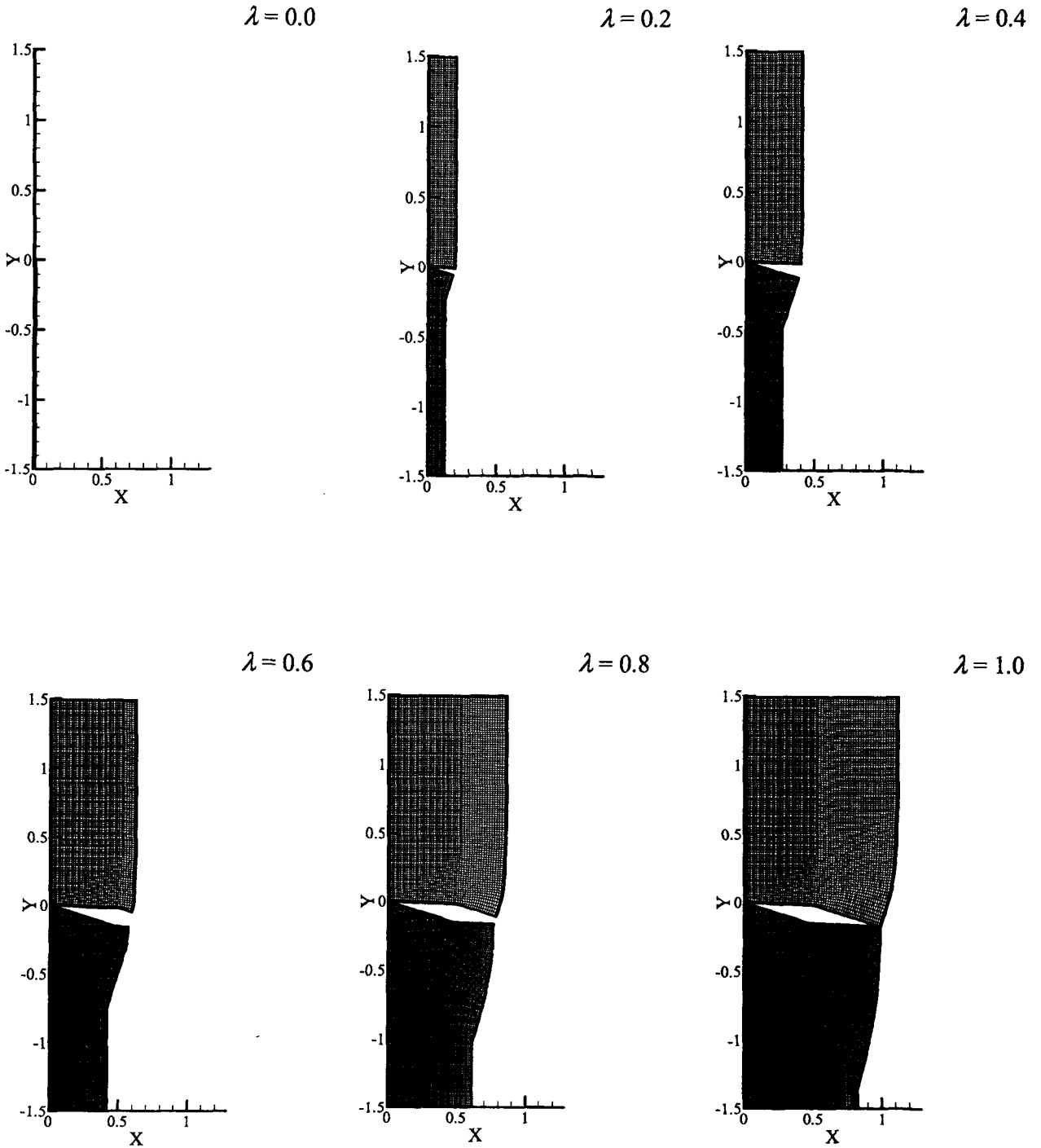


Fig. 2 Flow Generated Grids at Different λ

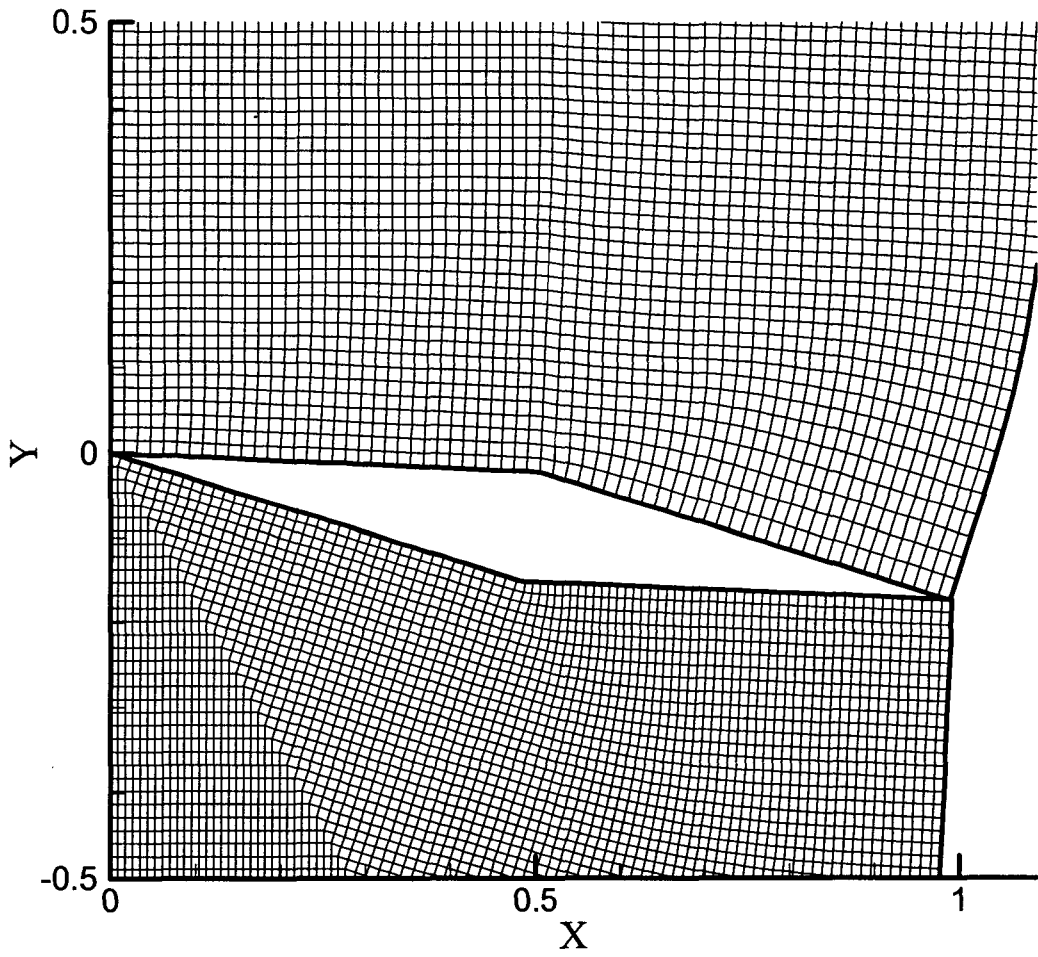


Fig. 3 Flow-Generated Grid (Close View)

Axisymmetric Flow past a Circular Cone
Gridless Computation

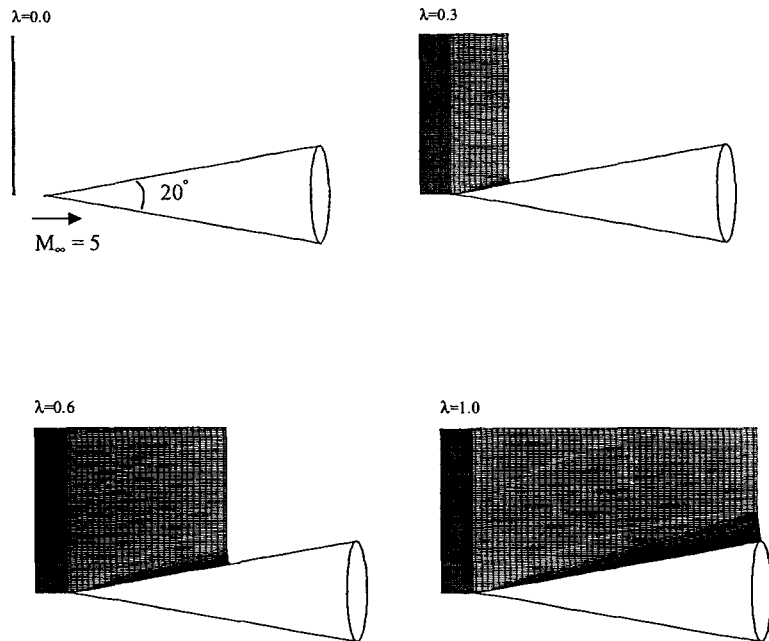


Fig. 1 Flow-generated Orthogonal Grids

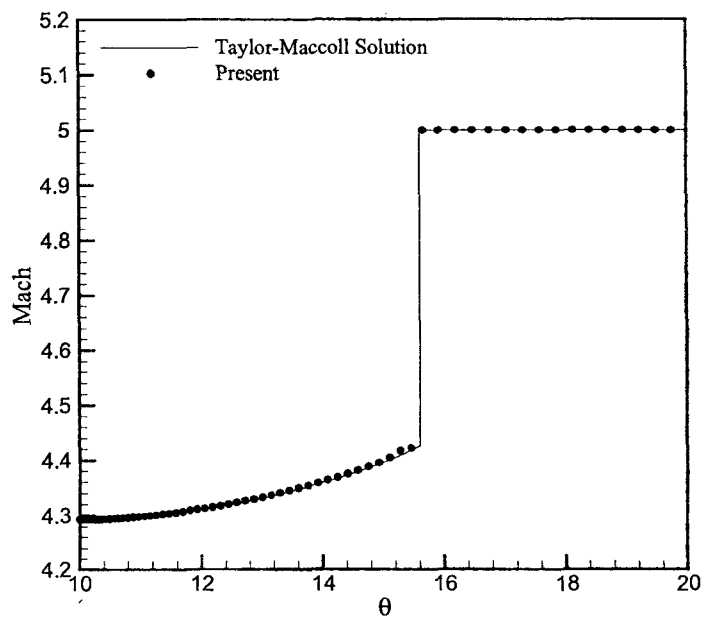


Fig. 2 Mach number distribution. Computing time: 135s (P4, 2.8GHz)

2-D Steady Riemann Problem

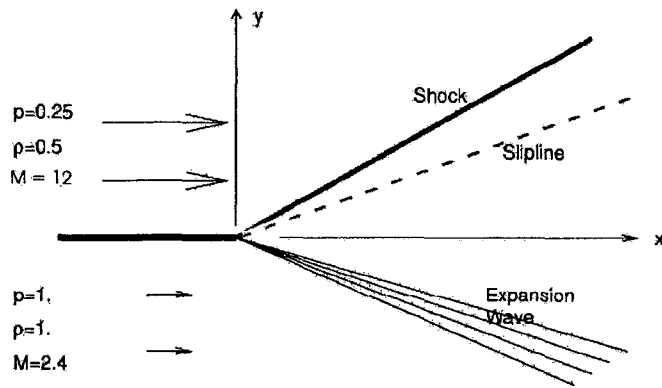


Fig. 1 Sketch

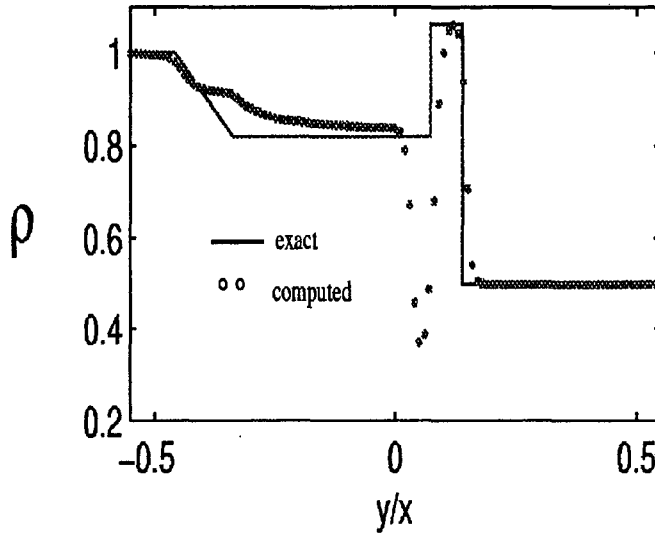


Fig. 2 Eulerian Computation

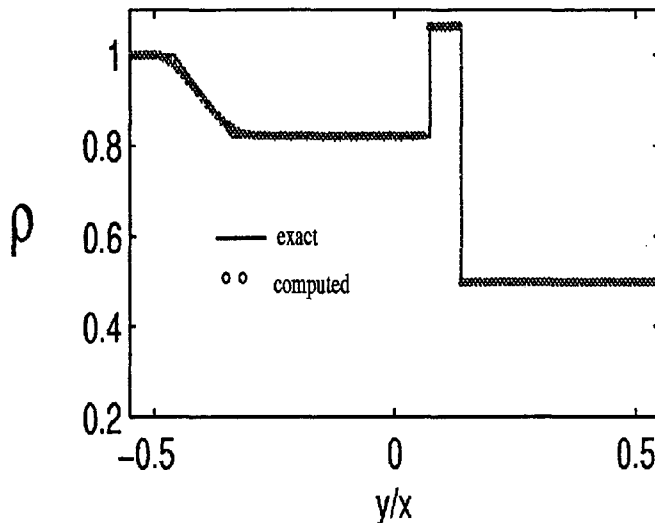
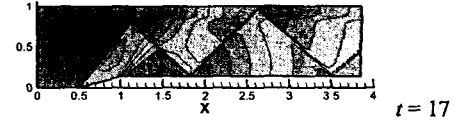
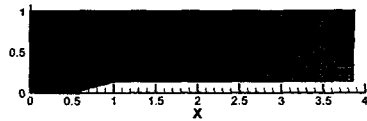
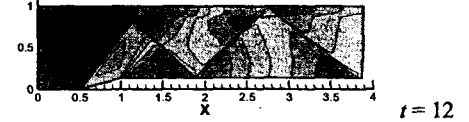
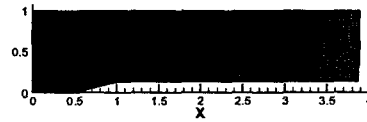
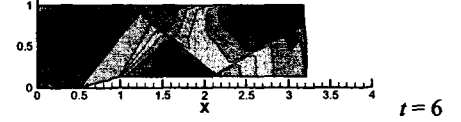
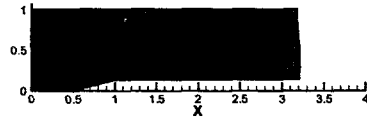
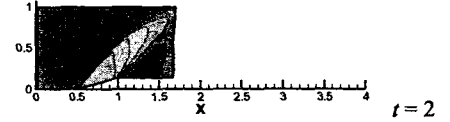
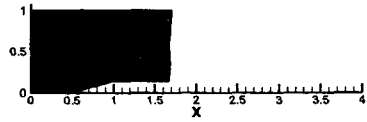
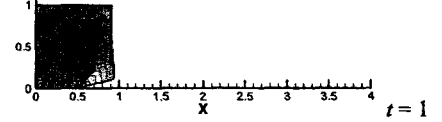
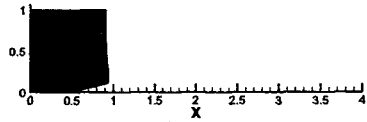
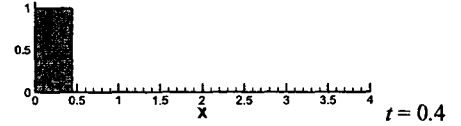
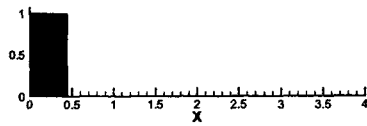
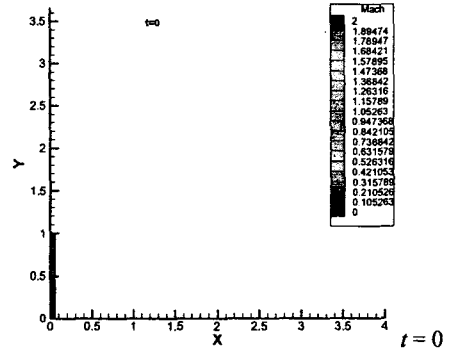
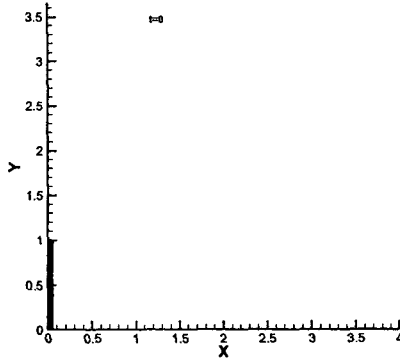


Fig. 3 Unified Coordinates Computation

Channel Flow with Mach Reflection, $M_\infty = 1.8$



Flow-generated grids and computed Mach contours at different times