

다중 질량 시스템을 이용한 자이로스코프 설계

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Design of MEMS Gyroscope Using Multi Mass System

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**Abstract** - In this paper, new design concept of MEMS gyroscope using multi mass system is proposed. The gyroscope having wide bandwidth was designed utilizing the multi mass system in order to reduce the degradation of the performance by resonance variation. The multi mass system has more than two masses and separates the resonant peak of each mass. Using MATLAB, the variation of bandwidth and driving displacement according to mass ratio of the multi mass system was analyzed. This result was compared with that of current single mass system gyroscope. In the 7 kHz resonant frequency design, the multi mass system has 395.3 Hz bandwidth, which is six times larger than single mass system bandwidth, 58.5 Hz.

1. Introduction

Gyroscopes have been important devices in aviation, automotive and military application. With the development of Micro Electro Mechanical System (MEMS), gyroscopes and other inertial measurement devices can now be produced cheaply and in very small packages in the micro domain [1,2]. Especially, the single crystalline silicon gyroscopes based on Silicon on Glass(SiOG) process were intensively researched for high sensitivity, high resolution and simple process [3]. However, due to unfavorable effects of scaling, the current state of the art MEMS gyroscopes require an order of magnitude improvement in performance, stability, and robustness[4,5]. In the current single mass system, to achieve high sensitivity, the resonant frequencies of driving and sensing part are typically designed and tuned to match, and the device is controlled to operate at or near the peak of the response curve. Thus extensive researches have been focused on design of symmetric suspensions and geometries [6]. But the reliability and the robustness of the device can be deteriorated by the mode matching problem and the device imperfection. For lightly-damped devices, the requirement for mode matching is beyond fabrication tolerances, and none of the symmetric designs can provide the required degree of mode matching without active tuning and feedback control. In this paper, we propose the multi mass system gyroscope design utilizing dynamic amplification to reduce the mode match problem and improve the reliability and robustness of the device.

2. Theory of multi mass system

The multi mass system of the gyroscope can be modeled as a two degree of freedom mass-spring-damp system, as shown in figure 1. Assuming linear springs and damping, the basic equations of motion of the multi mass system are given by

$$m_1 \ddot{x}_1 = -c_1 \dot{x}_1 + c_2 \dot{x}_2 - k_1 x_1 - k_2 (x_1 - x_2) + F \cos \omega t \quad (1)$$

$$m_2 \ddot{x}_2 = -c_2 \dot{x}_2 - k_2 (x_2 - x_1) \quad (2)$$

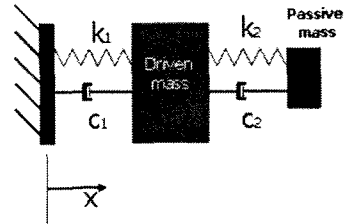


Fig 1. The model of the multi mass system gyroscope

Rearranging these equations, we have

$$m_1 \ddot{x}_1 + c_1 \dot{x}_1 - c_2 \dot{x}_2 + (k_1 + k_2)x_1 - k_2 x_2 = F \cos \omega t \quad (3)$$

$$-k_2 x_1 + m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 = 0 \quad (4)$$

The steady state solution can be obtained readily by the mechanical impedance method. Let us substitute  $Re\{F e^{j\omega t}\}$  for  $F \cos \omega t$ ,  $Re\{\tilde{x}_1 e^{j\omega t}\}$  for  $x_1(t)$ , and  $Re\{\tilde{x}_2 e^{j\omega t}\}$  for  $x_2(t)$ .

Rearranging and factoring out  $e^{j\omega t}$ , we obtain

$$(k_1 + k_2 + j\omega c_1 - \omega^2 m_1)\tilde{x}_1 - (j\omega c_2 + k_2)\tilde{x}_2 = F \quad (5)$$

$$-k_2 \tilde{x}_1 + (k_2 - \omega^2 m_2 + j\omega c_2)\tilde{x}_2 = 0 \quad (6)$$

$\tilde{x}_1$  and  $\tilde{x}_2$  can be obtained from this equation by Cramer's rule and can be expressed as

$$\tilde{x}_1 = \frac{(k_2 - m_2 \omega^2 + j\omega c_2)F}{\Delta(\omega)} \quad (7)$$

$$\tilde{x}_2 = \frac{k_2 F}{\Delta(\omega)} \quad (8)$$

where

$$\Delta(\omega) = (k_1 + k_2 - m_1 \omega^2 + j\omega c_1)(k_2 - m_2 \omega^2 + j\omega c_2) - (k_2^2 + j\omega k_2 c_2) \quad (9)$$

Thus, the steady state response of the masses is

$$x_1(t) = \text{Re}[\bar{X}_1 e^{i\omega t}] = |\bar{X}_1| \cos(\omega t \phi_1) \quad (10)$$

$$x_2(t) = \text{Re}[\bar{X}_2 e^{i\omega t}] = |\bar{X}_2| \cos(\omega t \phi_2) \quad (11)$$

Additionally, defining the non dimensional parameter as

$$w_1^2 = \frac{k_1}{m_1}, \mu = \frac{m_2}{m_1}, \beta = \frac{w_2}{w_1} \quad (12-14)$$

the two separated frequencies can be obtained from those conditions as

$$\left(\frac{\omega_r}{\omega_1}\right)^2 = \frac{1 + \beta^2(1 + \mu)}{2\beta^2} \pm \frac{1}{2\beta^2} \sqrt{\beta^4(1 + \mu)^2 - 2\beta^2(1 - \mu) + 1} \quad (15)$$

Therefore, the multi mass system gyroscope can be designed quantitatively by determining natural frequency, mass ratio and frequency ratio. Especially, from two separated frequencies, the bandwidth can be designed quantitatively too.

### 3. Design of gyroscope

The proposed design is based on the application of dynamic amplification into the 2 DOF sensing direction resonator which is structurally separated to achieve large oscillation amplitudes without resonance. The dynamic amplification can be implemented successively by matching the resonance frequencies of each mass. Figure 2 shows the frequency response of the passive mass which has two resonant peaks and the flat region between two resonant peaks. The multi mass system gyroscope is operating in the flat region. Thus the small change of the driving frequency makes negligible response gain change. This variation is rather small than gain variation of resonant peak.

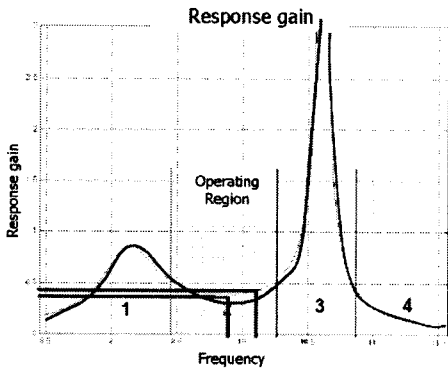


Fig 2. The operating region of the multi mass system based on dynamic amplification

The overall multi mass system gyroscope is composed of three masses, which are the frame mass, the proof mass and the driving mass. The frame mass and the proof mass form the 2 DOF sensing direction resonator. The driving mass forms the 1 DOF driving direction resonator. The driving mass transfers the electrostatic force, generated from comb type actuators, to the frame mass. When angular velocity is applied, a Coriolis force occurs to both the frame mass and the proof mass. By dynamic amplification mechanism, all the displacement is

generated in the proof mass, whereas the frame mass has negligible displacement. Figure 3 shows the layout of the multi mass system gyroscope. In this design, mass ratio( $\mu$ ) is 0.09 and natural frequency is 7001 Hz. To improve sensitivity, the parallel plate type sensing electrodes are implemented. The chip size is 10 mm  $\times$  10 mm.

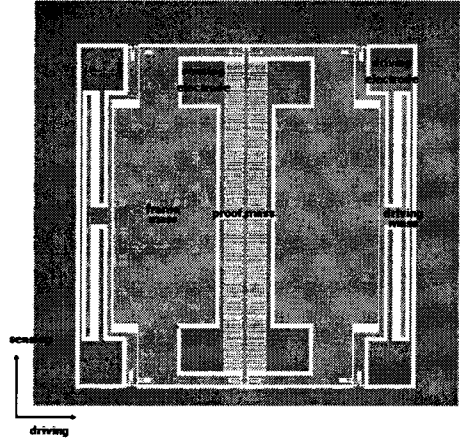


Fig 3. The layout of the multi mass system gyroscope

### 4. Analysis of gyroscope

The characteristics of the multi mass system gyroscope are analyzed quantitatively using MATLAB. The variation of bandwidth and driving displacement according to mass ratio of the multi mass system was analysed. Figure 4 shows the analysis results of bandwidth and driving displacement according to mass ratio of multi mass system. Increase of the mass ratio creates lower displacement and then creates lower resolution. Whereas, according to increase of the mass ratio, the bandwidth is increased from 254 Hz to 677 Hz. Thus it is important to determine proper mass ratio in order to obtain both high resolution and wide bandwidth optimally. In addition, geometric limitation must be considered to determine the mass ratio.

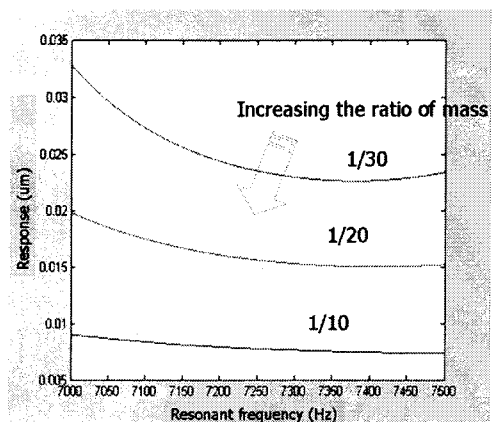


Fig 4. The analysis results of bandwidth and driving displacement according to mass ratio of the multi mass system

Figure 5 shows bandwidth comparison results of single mass and multi mass systems. In the same displacement level, the bandwidth of single mass systems was compared with that of multi mass system.

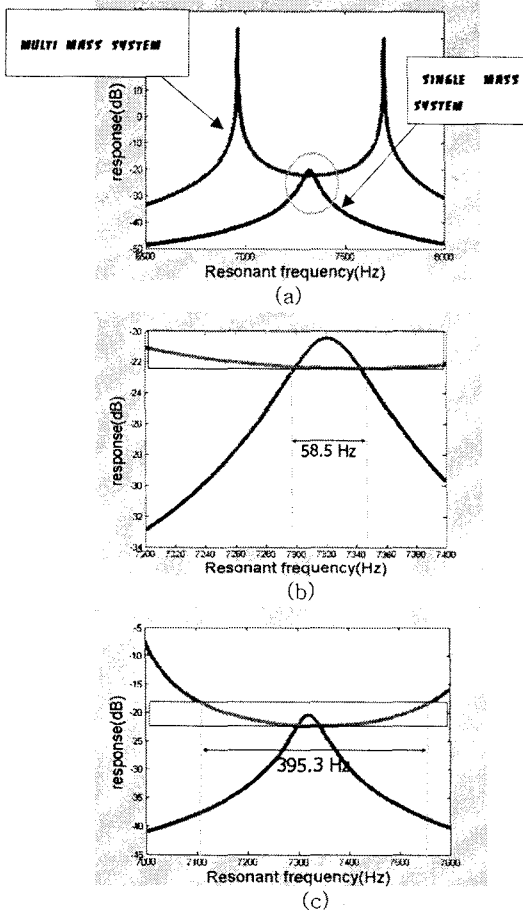


Fig 5. Bandwidth comparison results of single mass and multi mass systems: (a) overall frequency response (b) the single mass system's bandwidth (c) the multi mass system's bandwidth

In the 7 kHz resonant frequency design, the multi mass system has 395.3 Hz bandwidth, which is six times larger than single mass system bandwidth, 58.5 Hz. Therefore by utilizing the multi mass system, the characteristics of gyroscope can be insensitive to mode matching problem and other parametric variations.

## 5. Conclusion

In this paper, the new design concept of MEMS gyroscope using multi mass system was proposed. The gyroscope having wide bandwidth was designed utilizing the multi mass system in order to reduce the degradation of the performance by resonance variation. The multi mass system has more than two masses and separates the resonant peak of each mass. The multi mass system gyroscope is operating in the flat region. Thus the small change of the driving frequency makes

negligible response gain change. In the same displacement level, the bandwidth of single mass systems was compared with that of multi mass system. When analyzed using MATLAB analysis, in the 7 kHz resonant frequency design, the multi mass system has 395.3 Hz bandwidth, which is 6 times larger than that of the single mass system, 58.5 Hz. Therefore, by the multi mass system, the degradation of the device by mode matching problem can be reduced. Thus it is expected that the multi mass system can be applied to the design of other resonant devices.

## [참고 문헌]

- [1] N. Yazdi, F. Ayazi, K. Najafi, "Micromachined inertial sensors," Proceedings of the IEEE, Vol. 86, No. 8, pp. 1640-1659, 1998.
- [2] C. Song, "Commercial vision of silicon based inertial sensors," Proceedings of the Ninth International Conference on Solid State Sensors and Actuators, Transducers'97 (digest of technical papers), Chicago, 1997, pp. 839-842.
- [3] Che-Heung Kim, Hee-Moon Jeong, Jong-Up Jeon and Yong-Kweon Kim, "Silicon Micro XY-Stage With a Large Area Shuttle and No-Etching Holes for SPM-Based Data Storage", Journal of Micro electro mechanical System, Vol.12, No.4, pp.470-478, Aug 2003.
- [4] M. Lutz, W. Goldere, J. Gerstenmeier, J. Marek, B. Maihofer, S. Mahler, H. Munzel, U. Bischof, "A precision yaw rate sensor in silicon micromachining," Proceedings of The Ninth International Conference on Solid State Sensors and Actuators, Transducers'97(digest of technical papers), Chicago, 1997, pp. 847-850.
- [5] X. X. Li, M. H. Bao, H. Yang, S. Q. Shen, D. R. Lu, "A micromachined piezoresistive angular rate sensor with a composite beam structure," Sensors and Actuators A72, pp. 217-223, 1999.
- [6] H. T. Lim, J. W. Song, J. G. Lee, and Y. K. Kim, "A few deg/hr resolvable low noise lateral microgyroscope," Proceedings of the IEEE MEMS Conference, Las Vegas, 2002, pp. 627-630.