GPS 응용을 위한 새로운 RAIM 알고리즘 개발 I

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Development of A New RAIM Algorithm for GPS Applications I

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Abstract

RAIM techniques based on TLS have rarely been addressed because TLS requires a great number of computations. In this paper, the special form of the observation matrix $\mathbf{H}$, that is, one column is exactly known, is exploited so as to develop a new TLS-based sequential algorithm to identify an errant satellite. The algorithm makes us enjoy the advantages of TLS with less computational burden. The proposed algorithm is verified through a numerical simulation.

1. Introduction

Recently Juang[1] reformulated a linear measurement model and proposed a positioning and integrity monitoring scheme based on total least squares (TLS) instead of least squares. He took advantage of TLS at the expense of more numerical computations by proposing a new integrity monitoring metric. The new metric employs minimum singular value from SVD that is almost indispensable to solve the TLS problem. The Juang's scheme is good for failure detection.

In this paper, a new TLS-based sequential algorithm focused on failure isolation, even though it can be used for failure detection, is proposed. This algorithm utilizes the fact that a linear matrix equation has an exactly known column. The algorithm takes a sequential form so as to reduce amount of computations. It makes use of previous results without repeating the whole process. Therefore one can enjoy, with less computational burden, the advantages for integrity monitoring provided by Juang who employed TLS as a tool for positioning and integrity monitoring.

2. Algorithm Technical Background

2.1 Linear Measurement Model

A linear model is generally employed for proper positioning and integrity monitoring. In [1], linear measurement model was reinvestigated considering errors in observation matrix H. In this model, error due to a failed satellite is included in observation matrix H. Therefore the observation matrix H is not exactly known any more. Naturally

TLS is employed to solve this problem. In this paper, the linear model in [1] will be used. Therefore a brief description of the model is given. Suppose n satellites are visible. The measurement model is

$$\rho^i = \left\| \mathbf{u} - \mathbf{s}^i - \Delta\mathbf{s}^i \right\| + c + e^i \qquad (1)$$

where ρ^i is pseudo-range measurement with respect to the i-th GPS satellite, $\mathbf{u}$ is the user's position, c is clock offset, $\mathbf{s}^i$ is the broadcast position of the i-th GPS satellite, $\Delta\mathbf{s}^i$ is the difference between the broadcast position and true position of the i-th GPS satellite, and e^i accounts for the other errors. e^i is treated as zero mean noise. Both the pseudo-range ρ^i and the broadcast position $\mathbf{s}^i$ are subject to errors due to ephemeris errors, SA effects if it exists, environment effects, satellite failure, interferences, noises, and so on. Let

$$\mathbf{u} = \begin{bmatrix} x \\ y \\ z \end{bmatrix},\ \mathbf{s}^i = \begin{bmatrix} x^i \\ y^i \\ z^i \end{bmatrix},\ \text{and}\ \Delta\mathbf{s}^i = \begin{bmatrix} \Delta x^i \\ \Delta y^i \\ \Delta z^i \end{bmatrix}. \qquad (2)$$

Suppose that the linearization point is at

$$\mathbf{u} = \mathbf{u}_0 = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \ \text{and}\ c = c_0 \qquad (3)$$

then the estimated of the psudo-range measurement is given by

$$\rho_0^i = \left\| \mathbf{u}_0 - \mathbf{s}^i - \Delta\mathbf{s}^i \right\| + c_0$$

$$= \sqrt{(x^i + \Delta x^i - x_0)^2 + (y^i + \Delta y^i - y_0)^2 + (z^i + \Delta z^i - z_0)^2} + c_0 \qquad (4)$$

Define

$$\mathbf{r} = \begin{bmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{bmatrix},\ \delta = -c + c_0,\ \text{and}\ \mathbf{p} = \begin{bmatrix} \mathbf{r} \\ \delta \end{bmatrix}. \qquad (5)$$

Then, the linearized matrix equation of (1) with respect to n observable satellites becomes

$$\mathbf{Hp} = \mathbf{q} + \mathbf{e} \qquad (6)$$

where

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & h_{13} & 1 \\ h_{21} & h_{22} & h_{23} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ h_{n1} & h_{n2} & h_{n3} & 1 \end{bmatrix},\ \mathbf{q} = \begin{bmatrix} \rho_0^1 - \rho^1 \\ \rho_0^2 - \rho^2 \\ \vdots \\ \rho_0^n - \rho^n \end{bmatrix},\ \mathbf{e} = \begin{bmatrix} e^1 \\ e^2 \\ \vdots \\ e^n \end{bmatrix},$$

and

$$h_{i1} = \frac{x^i + \Delta x^i - x_0}{\sqrt{(x^i + \Delta x^i - x_0)^2 + (y^i + \Delta y^i - y_0)^2 + (z^i + \Delta z^i - z_0)^2}}$$

$$h_{i2} = \frac{y^i + \Delta y^i - y_0}{\sqrt{(x^i + \Delta x^i - x_0)^2 + (y^i + \Delta y^i - y_0)^2 + (z^i + \Delta z^i - z_0)^2}}$$

$$h_{i3} = \frac{z^i + \Delta z^i - z_0}{\sqrt{(x^i + \Delta x^i - x_0)^2 + (y^i + \Delta y^i - y_0)^2 + (z^i + \Delta z^i - z_0)^2}}.$$

Note that the last column of H matrix is exactly known. Therefore solving the linearized matrix equation is a mixed LS-TLS problem described in the next section.

2.2 Mixed LS-TLS Problem

Let $\mathbf{b} = \mathbf{q} + \mathbf{e}$ for simplicity. Then (6) becomes a well-known linear matrix equation,

$$\mathbf{Hp} = \mathbf{b} \qquad (7)$$

In the classical LS approach, all elements of $\mathbf{H}$ are assumed to be free of error; hence, all errors are confined to the observation vector $\mathbf{b}$. This assumption, however, is frequently unrealistic in some applications. The TLS is one method of fitting that is appropriate when there are errors in both the observation vector $\mathbf{b}$ and the data matrix $\mathbf{H}$. Especially when some of the columns, not all, of the data matrix $\mathbf{H}$ are free of error like the case considered in this paper, we call it a mixed LS-TLS problem [2].

3. Derivation of the Algorithm

Because of the limit of space, detail derivation of the algorithm is omitted. The resulting algorithm is summarized as below. The algorithm is expressed via MATLAB grammar, since it is simple and well known.

ALGORITHM

Given $\mathbf{Q}_k, \mathbf{R}_k, \mathbf{B}_k$, $\boldsymbol{\alpha}_k$, β_k

Step 1: Compute Givens rotations $\mathbf{G}_1, \mathbf{G}_2, \cdots, \mathbf{G}_{n-2}$ such that

$$\mathbf{G}_1^T \mathbf{G}_2^T \cdots \mathbf{G}_{n-2}^T \mathbf{q}_1 = \rho \mathbf{e}_1$$

where $\mathbf{q}_1^T$ be the first row of $\mathbf{Q}_k$ and $\rho = \pm 1$.

Step 2: Compute $\overline{\mathbf{R}}_k$, $\overline{\mathbf{Q}}_k$ and $\overline{\mathbf{B}}_k$.

$$\overline{\mathbf{R}}_k = \mathbf{G}_1^T \cdots \mathbf{G}_{n-2}^T \mathbf{R}_k (2:(n-1),:)$$

$$\overline{\mathbf{Q}}_k = \mathbf{Q}_k \mathbf{G}_{n-2} \cdots \mathbf{G}_1 (2:(n-1), 2:(n-1))$$

$$\overline{\mathbf{B}}_k = \mathbf{G}_1^T \cdots \mathbf{G}_{n-2}^T \mathbf{B}_k (2:(n-1),:)$$

Step 3: Compute Givens rotations $\mathbf{J}_1, \mathbf{J}_2, \cdots, \mathbf{J}_p$ such that

$$\mathbf{J}_p^T \mathbf{J}_{p-1}^T \cdots \mathbf{J}_1^T \begin{bmatrix} \boldsymbol{\alpha}_{k,1}^T \\ \overline{\mathbf{R}}_k \end{bmatrix} = \mathbf{R}_{k+1}, \qquad \mathbf{R}_{k+1} \in \mathbf{R}^{(n-1) \times p}$$

is upper triangular.

Step 4: Compute $\mathbf{R}_{k+1,12}$, $\mathbf{R}_{k+1,22}$, $\mathbf{y}_{k+1,1}$, $\mathbf{y}_{k+1,2}$

$$\begin{bmatrix} \mathbf{R}_{k+1,12} & \mathbf{y}_{k+1,1} \\ \mathbf{R}_{k+1,22} & \mathbf{y}_{k+1,2} \end{bmatrix} = \mathbf{J}_p^T \mathbf{J}_{p-1}^T \cdots \mathbf{J}_1^T \begin{bmatrix} \boldsymbol{\alpha}_{k,2}^T ; \beta_k \\ \overline{\mathbf{B}}_k \end{bmatrix}$$

where $\mathbf{R}_{k+1,12} \in \mathbf{R}^{p \times (n-p)}$, $\mathbf{R}_{k+1,22} \in \mathbf{R}^{(n-p-1) \times (n-p)}$, $\mathbf{y}_{k+1,1} \in R^p$ and $\mathbf{y}_{k+1,2} \in R^{n-p-1}$.

Step 5: Construct the matrix $\mathbf{D}_{k+1}$ and compute the minimum eigenvector $\mathbf{v}$ of the matrix $\mathbf{D}_{k+1}$ using the FALM[3-4]. Compute, then, the TLS solution $\mathbf{x}_{k+1,2}$

$$\mathbf{x}_{k+1,2} = -\frac{1}{v_{k-p+1}} [v_1, v_2, \cdots, v_{k-p}]^T$$

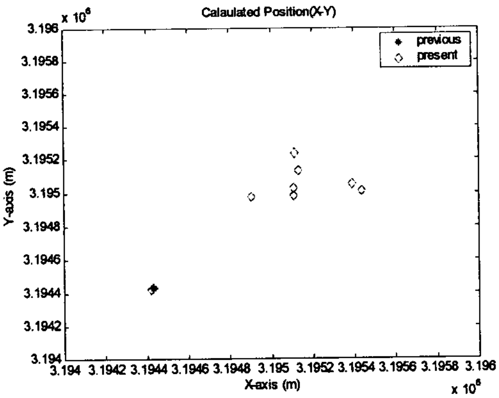
Step 6: Compute the least squares solution $\mathbf{x}_{k+1,1}$. Then the overall solution is

$$\mathbf{x}_{k+1} = [\mathbf{x}_{k+1,1}^T ; \mathbf{x}_{k+1,2}^T]^T.$$

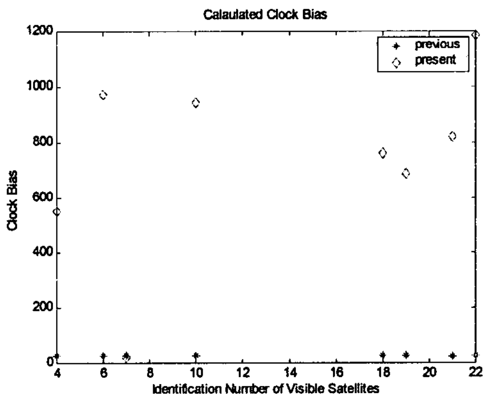
Step 7: Compute $\mathbf{Q}_{k+1}$. If every satellite is excluded one by one, stop. If not, go to *Step* 1.

4. Simulation Results

In this section, a simulation result is discussed The simulation is focused on how the proposed algorithm working well under a satellite failed circumstance. The satellites data were generated using MATLAB toolbox. Thermal noise, tropospheric error, multipath error, and ionospheric error were considered in generating the satellite data. A simulated pseudo-range error was injected to a satellite at time t. What is the time t is not important because the proposed algorithm runs between adjacent epochs. In this simulation, we assume that there is no failure until time t-1 and a satellite (PRN #7 in this case) fails between t-1 and t. Then, the satellite date at time t is incorrect. We examine how the algorithm is working in this case. The following figures describe the results. Figure 1 shows the calculated positions. The star denotes previous positions at time t-1. Since the algorithm calculates a position with 7 satellites, 8 positions are calculated at each time. The 8 positions are almost coincided with each other because there is no failure in 8 visible satellites.



[그림 1] Calculated position



[그림 2] Calculated clock bias

The diamond denotes the present 8 positions. 7 present positions are apart from the previous position, which implies there is an errant satellite. Only one position is located the near previous positions. This means the excluded satellite is failed.

Figure 2 shows calculated clock bias. We assumed zero clock bias in this simulation. The 8 previous clock biases (star mark) are near zero. Diamond stands for present clock biases. The figure only one present clock bias (PRN #7) is near zero, which means the 7th satellite is errant.

5. Conclusions

In this paper, a new TLS-based sequential algorithm to identify an errant satellite is proposed. A major contribution of this paper might be the fact the algorithm is new and it makes us enjoy the advantages of TLS with less computational burden since it takes sequential form. With a proper measure and threshold that is extensively studied until now, it can provide performance for failure detection and identification.

[References]

[1] Jyh-Ching Juang, “On GPS Position and Integrity Monitoring”, IEEE Transactions on Aerospace and Electronics Systems, Vol. 36, No. 1, Jan. 2000, pp. 327-336.

[2] Van Huffel, S. and Vandewalle, J., *The Total Least Squares Problem Computational Aspects and Analysis*, SIAM, 1991.

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