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Fuzzy Puse-Width-Modulated Feedback Control

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Abstract

This paper discusses an intelligent digital redesign technique for designing a fuzzy pulse-width-modulated (PWM) control. First when we are given a well-designed fuzzy analog control, the equivalent digital control is intelligently redesigned. Using the similar technique we intelligently redesign the fuzzy PWM control from the intelligently redesigned fuzzy digital control. A stabilizability of the intelligently redesigned PWM control is rigorously analyzed.

Key Words : 지능형 디지털 재설계, TS 퍼지 시스템, 관측기, PWM 제어, 안정도

1. Introduction

Pulse-width-modulation has extensively been used in electronic and electrical systems including attitude control of artificial satellite systems. The classical example of pulse-width-modulated (PWM) control is the constant temperature oven suggested by Gouy [14].

One advantage of PWM control is the simplicity of its realization: the control assumes only two or three values, and hence the control action is realized through the switch operation. One of the reasons for its wide applicability is that PWM operation makes it possible to process large signals with high efficiency and low sensitivity to noise. However, its rather unique and inherent nonlinear and hybrid characteristics give rise to challenges in the synthesis and analysis especially for the Takagi--Sugeno (T--S) fuzzy systems.

This paper concerns an intelligent digital redesign technique for designing a fuzzy PWM control in global state-matching sense [1,9,11]. First when we are given a well-designed fuzzy analog control, the equivalent digital control is intelligently redesigned. That is, we convert the well-designed analog control into the equivalent digital one maintaining the properties of the original analog control system in the sense of state-matching [1,5,6,9--12]. Using the similar technique we then again intelligently redesign the fuzzy PWM control from the intelligently redesigned fuzzy digital control by considering a global state-matching sense. A stabilizability of the intelligently redesigned PWM control is rigorously analyzed.

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2. Intelligent Digital Redesign

2.1 Analog T-S Fuzzy Systems

Consider a single-input-single-output T--S fuzzy system

$$\dot{x}_c(t)=\sum_{i=1}^{r}\theta_i(z(t))(A_i x_c(t)+B_i u_c(t)). \quad (1)$$

Throughout this paper, a well-constructed fuzzy analog control is assumed to be pre-designed, which will be used in intelligently redesigning the digital control of the following form:

$$u_c(t)=\sum_{i=1}^{r}\theta_i(z(t))K_c^i x_c(t). \quad (2)$$

The overall closed-loop T--S fuzzy system is

$$\dot{x}_c(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(t))\theta_j(z(t))(A_i + B_i K_c^j)x_c(t). \quad (3)$$

2.2 Discretization of Analog T--S Fuzzy Systems

This subsection discusses the discretization of the T--S fuzzy systems. Consider a class of T--S fuzzy systems governed by

$$\dot{x}_d(t) = \sum_{i=1}^{r}\theta_i(z(t))(A_i x_d(t) + B_i u_d(t)) \quad (4)$$

where $u_d(t) = u_d(kT)$ is the piecewise-constant control input vector to be determined in the time interval $[kT, kT+T)$, taking the following for

$$u_d(t) = \sum_{i=1}^{r}\theta_i(z(kT))K_d^i x_d(kT) \quad (5)$$

where K_d^i is the digital control gain matrix to be redesigned.

The intelligent digital redesign problem (IDR) is to find digital control gains in (5) from the analog gains in {eq: fuzzy control}, so that the closed-loop state $x_d(t)$ in (4) with (5) can closely match the closed-loop state $x_c(t)$ in (3) at all sampling time instants $t = kT$, $k \in Z_{\geq 0}$. Thus it is more convenient to convert the T--S fuzzy system into discrete-time version for derivation of the state matching condition.

Assumption 1: Assume that the firing strength of the i *th rule,* $\theta_i(z(t))$ *is approximated by their values at time* kT*, that is,* $\theta_i(z(t)) \approx \theta_i(z(kT))$ *for* $t \in [kT, kT+T)$ *[7]. Consequently, the nonlinear matrices* $\theta_i(z(t))A_i$ *and* $\sum_{i=1}^{r}\theta_i(z(t))B_i$ *can be approximated as constant matrices* $\sum_{i=1}^{r}\theta_i(z(kT))A_i$ *and* $\sum_{i=1}^{r}\theta_i(z(kT))B_i$*, respectively, over any interval* $[kT, kT+T)$*.*

Theorem 1: The pointwise dynamical behavior of the T--S fuzzy system (4) can be efficiently approximated by

$$x_d(kT+T) \approx \sum_{i=1}^{r}\theta_i(z(kT))(G_i x_d(kT) + H_i u_d(kT)) \quad (6)$$

where $G_i = \exp(A_i T)$ *and* $H_i = (G_i - I)A_i^{-1}B_i$.

Proof: See [13].

The discretized version of the closed-loop system with (6) and (5) is constructed to yield

$$x_d(kT+T) = \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT))\theta_j(z(kT)) \times (G_i + H_i K_d^j)x_d(kT).$$

Corollary 1: The pointwise dynamical behavior of the closed-loop T--S fuzzy system (3) can also be approximately discretized as

$$x_c(kT+T) \approx \sum_{i=1}^{r}\sum_{j=1}^{r}\theta_i(z(kT))\theta_j(z(kT))\Phi_{ij}x_c(kT)$$

where $\Phi_{ij} = \exp((A_i + B_i K_c^j)T)$.

Proof: It is straightforward from Theorem 1.

2.3 New IDR Based on Global State-Matching Concept

Our goal is to develop an IDR technique for T--S fuzzy systems so that the global dynamical behavior of (4) with the intelligently redesigned digital fuzzy control may retain that of the closed-loop T--S fuzzy system with the existing analog fuzzy control, and the stability of the digitally controlled T--S fuzzy system is secured. Comparing (7) with (8), to obtain $x_c(kT+T) = x_d(kT+T)$ under the assumption of $x_c(kT) = x_d(kT)$, it is necessary to determine the digital control gain matrices K_d^i such that the following matrix equality constraints should be satisfied

$$\Phi_{ij} = G_i + H_i K_d^j, \qquad (i,j) \in I_R \times I_R$$

then, the state $x_d(t)$ closely matches the state $x_c(t)$ globally, provided that their in Itial conditions are the same, that is, $x_c(0) = x_d(0) = x_0$.

To avoid a saturation phenomenon from the principle of equivalent area, we require a sufficient large M such that M= $M = \sup_{k \in Z_{>0}} |u_d(kT)|$.

Theorem 2: If there exist matrices $Q = Q^T > 0$, $O = O^T > 0$ *, matrices* F_i *and a possibly small* $\gamma \in R_{\geq 0}$ *such that the following generalized eigenvalue problem has solutions*

$$\min imize_{Q,O,F_i} \ \gamma \ \text{subject to}$$

$$\begin{bmatrix} -\gamma Q & (\cdot)^T \\ \Phi_{ij}Q - G_i Q - H_i F_j & -\gamma I \end{bmatrix} < 0$$

$$\begin{bmatrix} -Q + (q-1)O & (\cdot)^T \\ G_i Q + H_i F_i & -Q \end{bmatrix} < 0, (i,j) \in I_R \times I_R$$

$$\begin{bmatrix} -Q - O & (\cdot)^T \\ \dfrac{G_i Q + H_i F_j + G_j Q + H_j F_i}{2} & -Q \end{bmatrix} < 0, (i,j) \in I_j \times I_R$$

then, the state $x_d(kT)$ *of the discretized version (7) of the T--S fuzzy system (4) controlled via the redesigned digital fuzzy control (5) closely matches the state* $x_c(kT)$ *of the discretized version of the analogously controlled T--S fuzzy system (8).*

Furthermore, the discretized T--S fuzzy system (7) is globally asymptotically stabilizable in the sense of Lyapunov stability criterion, where $(\cdot)^T$ *denotes the transposed element in symmetric positions.*

Proof: See [11].

3. IDR of Fuzzy PWM Control: Global Approach

The fuzzy PWM control can be developed using the aforementioned digitally redesigned control as follows. Now we consider a PWM controlled T--S fuzzy system be represented as

$$\dot{x}_p(t) = \sum_{i=1}^{r}\theta_i(z(t))\,(A_i x_p(t) + B_i u_p(kT)) \tag{10}$$

Generally, the PWM control is mathematically represented by

$$u_p(t) = \begin{cases} 0, & t\in[kT, kT+\delta_k) \\ \mathrm{sgn}\,(u_d(kT))M, & t\in[kT+\delta_k, kT+T_k+\delta_k) \\ 0, & t\in[kT+T_k+\delta_k, kT+T) \end{cases} \tag{11}$$

where T_k is the firing duration of the predetermined constant control input M in the time interval $[kT, kT+T)$ and δ_k is the firing delay. One easy way to design the PWM control is to determine the firing duration δ_k so that the integration of control input respect to time is the same. This conversion has been widely used in industries for many years. In 1960, R. E. Andeen proved in his paper "The principle of equivalent areas" [3] the validity of this conversion under the assumption that the sampling period is suitably small.

Theorem 3: If T_k *and* δ_k *of the PWM control at the k th sampling is intelligently redesigned by the usual characterization of a PWM control [2], i.e.,*

$$T_k = T\frac{u_d(kT)}{M}, \qquad \delta_k = \frac{1}{2}(T - T_k)$$

then, $x_p(kT)$ *of the pointwise dynamical behavior of (10) controlled by (11) intelligently redesigned digital PWM feedback system closely matches* $x_d(kT)$ *of the pointwise dynamical behavior of (7).*

Proof: The proof begins with the pointwise solutions of (10) and (4)

$$\begin{aligned} x_p(kT+T) &= e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i T} x_p(kT) \\ &\quad + \int_{kT+\delta_k}^{kT+T_k+\delta_k} e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i(kT+T-\tau)} \\ &\quad \times\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)u_p(kT)d\tau \end{aligned}$$

and

$$\begin{aligned} x_d(kT+T) &= e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i T} x_d(kT) \\ &\quad + \int_{kT}^{kT+T} e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i(kT+T-\tau)} \\ &\quad \times\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)u_d(kT)d\tau. \end{aligned}$$

To realize $x_d(kT+T) = x_p(kT+T)$ *under the assumption* $x_d(kT) = x_p(kT)$ *it suffices to satisfy*

$$\begin{aligned} &\int_{kT+\delta_k}^{kT+T_k+\delta_k} e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i(kT+T-\tau)} \\ &\quad \times\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)u_p(kT)d\tau \\ &= \int_{kT}^{kT+T} e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i(kT+T-\tau)}\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big) \\ &\quad \times u_d(kT)d\tau. \end{aligned}$$

Further computation with Assumption 1 yields

$$\begin{aligned} &\Big(\exp\Big(\sum_{i=1}^{r}\theta_i(z(kT))A_i T\Big) - I\Big)\Big(\sum_{i=1}^{r}\theta_i(z(kT))A_i\Big)^{-1} \\ &\quad \times\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)u_d(kT) \\ &= \Big(\exp\Big(\sum_{i=1}^{r}\theta_i(z(kT))A_i(T-\delta_k)\Big) \\ &\quad - \exp\Big(\sum_{i=1}^{r}\theta_i(z(kT))A_i(T-T_k-\delta_k)\Big)\Big) \\ &\quad \times\Big(\sum_{i=1}^{r}\theta_i(z(kT))A_i\Big)^{-1}\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)M. \end{aligned}$$

Taking the second-order Taylor series expansion, we have

$$\begin{aligned} &\Big(I + \frac{1}{2!}\Big(\sum_{i=1}^{r}\theta_i(z(kT)A_i\Big)T\Big)\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)u_d(kT) \\ &= T_k\Big(I + \frac{1}{2!}\Big(\sum_{i=1}^{r}\theta_i(z(kT)A_i\Big)(2T - 2\delta_k - T_k)\Big) \\ &\quad \times\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)M. \end{aligned}$$

Solving above with respect to T_k *and* δ_k *we arrive at the claim.*

Theorem 4: Suppose (7) is asymptotically stable; then the zero equilibrium point $x_{p_{eq}} = [0]_{n\times 1}$ *of the hybrid fuzzy PWM control system (10) with (11) is also asymptotically stable. From the supposition of the asymptotic stability of (7), any closed-loop solution of (10) with (11) is majorized by*

$$\begin{aligned} \| x_p(t) \| &\le \Big\| e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i(t-kT)} x_p(kT) \Big\| \\ &\quad + \Big\| \int_{kT}^{t} e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i(t-\tau)} \\ &\quad \times\Big(\sum_{i=1}^{r}\theta_i(z(kT))B_i\Big)u_p(t)d\tau \Big\| \\ &\le \sup_{z(kT)\in U_z}\Big\{ \Big\| e^{\sum_{i=1}^{r}\theta_i(z(kT))A_i(t-kT)} \\ &\quad \times x_p(kT) \Big\| \end{aligned}$$

$$
\begin{aligned}
&+ \int_{kT+\delta_k}^{kT+T_k+\delta_k} \left\| e^{\sum_{i=1}^{r} \theta_i(z(kT))A_i(t-\tau)} \right. \\
&\quad \left. \times \left(\sum_{i=1}^{r} \theta_i(z(kT))B_i \right) \operatorname{sgn}(u_p(t)) M d\tau \right\| \Bigg\} \\
&\le \sup_{(i,j,h)\in I_R\times I_R\times I_R} \{ e^{\|A_i\|T} \\
&\quad \times (1 + Te^{\|A_i\|T} \| B_j \| \, \| K_d^h \|) \\
&\quad \times \| x_p(kT) \| \}
\end{aligned}
$$

when $t \in [kT, kT+T_k]$ *, and that*

$$
\begin{aligned}
\| x_p(t) \| &\le \left\| e^{\sum_{i=1}^{r} \theta_i(z(kT))A_i(t-kT)} x_p(kT+T_k) \right\| \\
&\le \sup_{i\in I_R} e^{\|A_i T\|} \| x_p(kT+T_k) \| \\
&\le \sup_{(i,j,h)\in I_R\times I_R\times I_R} e^{2\|A_i\|T} \\
&\quad \times (1 + Te^{\|A_i\|T} \| B_j \| \, \| K_d^h \|) \\
&\quad \times \| x_p(kT) \| \\
&= \mu \| x_p(kT) \|
\end{aligned}
$$

when $t \in (kT+T_k, kT+T)$, *where* μ *is independent of* k. *This implies* $x_d(t)(t)$ *over the interval* $[kT, kT+T)$ *is bounded by* $\mu \| x_d(kT) \|$. *Hence* $x_p(t)$ *converges to the origin simultaneously with* $x_p(kT)$ *and* $x_d(kT)$, *and we conclude that the trivial solution is asymptotically stable.*

4. Conclusions

This paper presented a fuzzy PWM control design method. The principle of equivalent area was utilized together with the IDR technique under the assumption of sufficient large PWM control action to avoid some saturation phenomenon. The future research effort will be devoted to deliberating the control saturation.

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