E_N^n상의 비선형 퍼지 Integro 미분방정식에 대한 제어가능성

Controllability of the nonlinear Fuzzy Integro-Differential Equation on E_N^n

권영철, 박동근, 손기도, 정두환

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Abstract

In this paper we study the controllability for the nonlinear fuzzy integro-differential equations on E_N^n by using the concept of fuzzy number of dimension n whose values are normal, convex, upper semicontinuous and compactly supported surface in R^n. E_N^n be the set of all fuzzy numbers in R^n with edges having bases parallel to axis $X_1, X_2, \cdots, X_n$.

Key Words : fuzzy number of dimension n, fuzzy control, nonlinear fuzzy integro-differential equation

1. Introduction

Many authors have studied several concepts of fuzzy systems. Kaleva[2] studied the existence and uniqueness of solution for the fuzzy differential equation on E^n where E^n is normal, convex, upper semicontinuous and compactly supported surface in R^n. Seikkala[9] proved the existence and uniqueness of fuzzy solution for the initial value problem on E^1. Subrahmanyam and Sudarsanam[10] studied fuzzy Volterra-integral equation. Park etal.[8] are proved the existence and uniqueness of fuzzy solution for the nonlinear fuzzy differential equation on E_N^n with nonlocal initial condition, Kwun etal.[6] are studied controllability for the nonlinear fuzzy control system on E_N^n, where E_N^n be the set of all fuzzy numbers in R^n with edges having bases parallel to axis $X_1, X_2, \cdots, X_n$. For example E_N^2 be the set of all fuzzy pyramidal numbers in R^2 with edges having rectangular bases parallel to the axis

1) This paper was supported by Dong-A University Research Fund in 2002.

X_1 and X_2.

Recently, Kwun etal [] are studied the existence and uniqueness of fuzzy solutions for the following nonlinear integrodifferential equations:

$$(1.1)\quad \begin{cases} \frac{dx(t)}{dt} = a(t)x(t) + f(t, x(t), \\ \int_0^t k(t,s,x(s))ds)) + u(t), \ t\in[0,T], \\ x(0) = x_0, \end{cases}$$

where $a:[0,T]\times \to E_N^n$ is fuzzy coefficient, init -ial value $x_0 \in E_N^n$ and $f:[0,T]\times E_N^n \times E_N^n \to E_N^n$ and $k:[0,T]\times[0,T]\times E_N^n \to E_N^n$ are nonlinear regular fuzzy functions.

In this paper, we consider the controllability of fuzzy nonlinear integro-differential equation (1.1).

2. Properties of n-dimensional fuzzy numbers and metric

In this section, we give some definitions, properties and notations of the fuzzy number of dimension n.

Definition 2.1.([6, 8]) We consider a fuzzy graph $G\subset R^n$ that is functional fuzzy relation in R^n such that its membership function $m_G(x_1,x_2,\cdots,x_n)\in[0,1]$, $(x_1,x_2,\cdots,x_n)\in R^n$ has the following properties :

(1) For all $x_i\in R (i=1,2,\cdots,n)$, $m_G(x_1,x_2,\cdots,x_n)\in[0,1]$ is a convex membership function.

(2) For all $\alpha\in[0,1]$, $\{(x_1,x_2,\cdots,x_n)\in R^n | m_G(x_1,x_2,\cdots,x_n)\geq\alpha$ is convex set.

(3) There exists $(x_1,x_2,\cdots,x_n)\in R^n$, $m_G(x_1,x_2,\cdots,x_n)=1$.

If the above conditions are satisfied, the fuzzy subset G is called a fuzzy number of dimension n.

We denote by fuzzy number in E_N^n, $A=\{(a_1, a_2,\cdots,a_n)$ where a_i is projection of A to axis $X_i (i=1,2,\cdots,n)$, respectively. And $a_i (i=1,2,\cdots,n)$ is fuzzy number in R

Definition 2.2. The α-level set of fuzzy number in E_N^n is defined by

$$(2.2)\quad [A]^\alpha = \{(x_1,x_2,\cdots,x_n)\in R^n | (x_1,x_2,\cdots, x_n)\in \prod_{i=1}^n [a_i]^\alpha,\ 0<\alpha\leq 1\}$$

where

$$(2.3)\quad [a_i]^\alpha = \{x_i\in R | m_{a_i}(x_i)\geq\alpha,\ 0<\alpha\leq 1\}$$

and $\prod$ is the Cartesian product of sets.

Definition 2.3. Let $A,B\in E_N^n$, for all $\alpha\in(0,1]$,

$$A=B \Leftrightarrow [A]^\alpha = [B]^\alpha.$$

Definition 2.4. Let $A,B\in E_N^n$, for all $\alpha\in(0,1]$,

$$(2.4)\quad [A *_n B]^\alpha = \prod_{i=1}^n [a_i * b_i]^\alpha$$

where $*_n$ is operation in E_N^n and $*$ is operation in E_N. And $(A *_n B)_i^\alpha = A_i^\alpha * B_i^\alpha$.

Definition 2.5. The drivative $\frac{dx(t)}{dt}\in E_N^n$ of fuzzy process $x(t)\in E_N^n$ is defined by

(2.5) $\left[\frac{dx(t)}{dt}\right]^{\alpha} = \prod_{i=1}^{n} \left[\frac{d}{dt} x^{\alpha}_{il}(t),\ \frac{d}{dt} x^{\alpha}_{ir}(t)\right],\ 0 < \alpha \le 1.$

Definition 2.6. The fuzzy integral $\int_a^b x(t)\,dt$ is defined by

(2.6)

$$\left[\int_a^b x(t)\,dt\right]^{\alpha} = \prod_{i=1}^{n}\left[\int_a^b x^{\alpha}_{il}(t)\,dt,\ \int_a^b x^{\alpha}_{ir}(t)\,dt\right],\quad 0<\alpha\le 1,$$

where $x(t) \in E^n_N, a, b \in R$.

Let $\prod_{i=1}^{n}[a_i]^{\alpha}$, $0 < \alpha \le 1$, be a given family of nonempty areas. If

(2.7) $\prod_{i=1}^{n}[a_i]^{\beta} \subset \prod_{i=1}^{n}[a_i]^{\alpha},\ 0 < \alpha < \beta \le 1$

and

(2.8) $\prod_{i=1}^{n} \lim_{k\to\infty}[a_i]^{\alpha_k} = \prod_{i=1}^{n}[a_i]^{\alpha}$

whenever (α_k) is a nondecreasing sequence converging to $\alpha \in (0,1]$, then the family $\prod_{i=1}^{n}[a_i]^{\alpha}$, $0 < \alpha \le 1$, represents the α-level sets of fuzzy number $A \in E^n_N$.

Conversly, if $\prod_{i=1}^{n}[a_i]^{\alpha}$, $0 < \alpha \le 1$, are the α-level sets of fuzzy number R^n, then the condition (2.7) and (2.8) hold true.

We define the metric d_{∞} on E^n_N and the suprimum metric H on $C([0, T]: E^n_N)$.

Definition 2.7. Let $A, B \in E^n_N$,

$$d_{\infty} = \sup\{d_H([A]^{\alpha},[B]^{\alpha}) \mid \alpha\in(0,1]\}$$
$$= \sup\left\{\left(\sum_{i=1}^{n}(d_H([a_i]^{\alpha},[b_i]^{\alpha}))^2\right)^{\frac{1}{2}} \mid \alpha\in(0,1]\right\}$$

where d_H is Hausdorff distance and $a_i,\ b_i \in E_N$.

Definition 2.8. The supremum metric H on $C([0, T]: E^n_N)$ is defined by

$$H(x,y) = \sup\{d_{\infty}(x(t), y(t)) \mid t\in[0,T]\}$$

where $x, y \in C([0, T]: E^n_N)$.

Definition 2.9. Nonlinear regular fuzzy function $f:[0,T]\times E^n_N\times E^n_N \to E^n_N$ is satisfied, $x, y \in E^n_N$,

$$\begin{aligned} f(t,[x]^{\alpha},[y]^{\alpha}) &= f\left(t, \prod_{m=1}^{n}[x_m]^{\alpha}, \prod_{m=1}^{n}[y_m]^{\alpha}\right) \\ &= \prod_{m=1}^{n} f_m(t,[x_m]^{\alpha},[y_m]^{\alpha}) \\ &= \prod_{m=1}^{n} f^{\alpha}_m(t,x,y) \\ &= f^{\alpha}(t,x,y). \end{aligned}$$

3. Controllability

In this section, we show the exact controllability for (1.1).

The equation (1.1) is related to the following fuzzy integral equations :

(3.1)

$$\begin{cases} x(t) = S(t)x_0 + \int_0^t S(t-s)\{f(s,x(s)), \\ \qquad \int_0^s k(s,t,x(\tau)))\,d\tau\}ds + \int_0^t S(t-s)\,u(s)\,ds, \\ x(0) = x_0. \end{cases}$$

where $S(t) \in E^n_N$ and

$$[S(t)]^{\alpha} = \prod_{m=1}^{n}[S_m(t)]^{\alpha} = \prod_{m=1}^{n}[S^{\alpha}_{ml}(t),\ S^{\alpha}_{mr}(t)]$$

where

$$S^{\alpha}_{mi}(t) = \exp\left\{\int_0^t a^{\alpha}_{mi}(s)\,ds\right\},\quad i = l, r$$

is continuous. That is, there exists a constant $C>0$ such that $|S^{\alpha}_{mi}(t)|\le C$ for all $t\in[0,T]$.

Definition 3.1. The (1.1) is exact controllable if, there exists $u(t)$ such that the fuzzy solution $x(t)$ of (1.1) satisfies

$$x(T)={}_{\alpha}x^{1}\ \ (\text{i.e.},\ [x(T)]^{\alpha}=\prod_{i=1}^{n}[x_i(T)]^{\alpha}=\prod_{i=1}^{n}[(x^1)_i]^{\alpha}=[x^1]^{\alpha})$$

where x^1 is target set.

Defined the fuzzy mapping $\tilde{g}:\mathcal{P}(R^n)\to E^n_N$ by

$$\tilde{g}^{\alpha}(v)=\begin{cases}\int_0^T S^{\alpha}(T-s)v(s)ds, & v\subset\overline{\Gamma_u},\\ 0, & \text{otherwise}.\end{cases}$$

Then there exists

$\tilde{g}_i:\mathcal{P}(R)\to E_N\,(i=1,2,\cdots,n)$ such that

$$\tilde{g}_i^{\alpha}(v_i)=\begin{cases}\int_0^T S^{\alpha}_i(T-s)v_i(s)ds, & v_i(s)\subset\overline{\Gamma_{u_i}},\\ 0, & \text{otherwise}\end{cases}$$

where u_i is projection of u to axis X_i, $(i=1,\cdots,n)$ respectively and

there exists $\tilde{g}_{ij^{\alpha}}\ (j=l,r)$

$$\tilde{g}_{il}^{\alpha}(v_{il})=\int_0^T S^{\alpha}_{il}(T-s)v_{il}(s)ds,$$
$$v_{il}(s)\in[u^{\alpha}_{il}(s),\,u^1_i(s)],$$
$$\tilde{g}_{ir}^{\alpha}(v_{ir})=\int_0^T S^{\alpha}_{ir}(T-s)v_{ir}(s)ds,$$
$$v_{ir}(s)\in[u^1_i(s),\,u^{\alpha}_{ir}(s)].$$

We assume that $\tilde{g}_{il}^{\alpha}$, $\tilde{g}_{ir}^{\alpha}$ are bijective mappings. We can be introduced $u(s)$ of nonlinear system

$$\begin{aligned}
&[u(s)]^{\alpha}\\
&=\prod_{i=1}^{n}[u_i(s)]^{\alpha}=\prod_{i=1}^{n}[u^{\alpha}_{il}(s),\,u^{\alpha}_{ir}(s)]\\
&=\prod_{i=1}^{n}[(\tilde{g}_{il}^{\alpha})^{-1}((x^1)^{\alpha}_{il}-S^{\alpha}_{il}(T)(x_0)^{\alpha}_{il}\\
&-\int_0^T S^{\alpha}_{il}(T-s)f^{\alpha}_{il}(s,x^{\alpha}_{il}(s),\int_0^s k(s,\tau,x^{\alpha}_{il}(\tau))d\tau)ds),\\
&(\tilde{g}_{ir}^{\alpha})^{-1}((x^1)^{\alpha}_{ir}-S^{\alpha}_{ir}(T)(x_0)^{\alpha}_{ir}\\
&-\int_0^T S^{\alpha}_{ir}(T-s)f^{\alpha}_{ir}(s,x^{\alpha}_{ir}(s),\int_0^s k(s,\tau,x^{\alpha}_{ir}(\tau))d\tau)ds)].
\end{aligned}$$

Then substituting this expression into the (1.1) yields α-level of $x(T)$. For each $i=1,\cdots,n$,

$$\begin{aligned}
&[x_i(T)]^{\alpha}\\
&=[S^{\alpha}_{il}(T)(x_0)^{\alpha}_{il}+\int_0^T S^{\alpha}_{il}(T-s)f^{\alpha}_{il}(s,x^{\alpha}_{il}(s),\\
&\qquad\int_0^s k(s,\tau,x^{\alpha}_{il}(\tau))d\tau)ds\\
&+\int_0^T S^{\alpha}_{il}(T-s)(\tilde{g}_{il}^{\alpha})^{-1}((x^1)^{\alpha}_{il}-S^{\alpha}_{il}(T)(x_0)^{\alpha}_{il}\\
&-\int_0^T S^{\alpha}_{il}(T-s)f^{\alpha}_{il}(s,x^{\alpha}_{il}(s),\int_0^s k(s,\tau,x^{\alpha}_{il}(\tau))d\tau)ds)ds,\\
&S^{\alpha}_{ir}(T)(x_0)^{\alpha}_{ir}+\int_0^T S^{\alpha}_{ir}(T-s)f^{\alpha}_{ir}(s,x^{\alpha}_{ir}(s),\\
&\qquad\int_0^s k(s,\tau,x^{\alpha}_{ir}(\tau))d\tau)ds\\
&+\int_0^T S^{\alpha}_{ir}(T-s)(\tilde{g}_{ir^{\alpha}})^{-1}((x^1)^{\alpha}_{ir}-S^{\alpha}_{ir}(T)(x_0)^{\alpha}_{ir}\\
&-\int_0^T S^{\alpha}_{ir}(T-s)f^{\alpha}_{ir}(s,x^{\alpha}_{ir}(s),\int_0^s k(s,\tau,x^{\alpha}_{ir}(\tau))d\tau)ds)ds]\\
&=[(x^1)^{\alpha}_{il},\,(x^1)^{\alpha}_{ir}]=[(x^1)_i]^{\alpha}
\end{aligned}$$

Therefore

$$[x(T)]^{\alpha}=\prod_{i=1}^{n}[x_i(T)]^{\alpha}=\prod_{i=1}^{n}[(x^1)_i]^{\alpha}=[x^1]^{\alpha}.$$

We now set

$$\begin{aligned}
(\Phi x)(t)&={}_{\alpha}S(t)x_0+\int_0^t S(t-s)f(s,x(s),\\
&\qquad\int_0^s k(s,\tau,x(\tau))d\tau)ds\\
&+\int_0^t S(t-s)\,\tilde{g}^{-1}(x^1-S(T))x_0\\
&-\int_0^T S(T-s)f(s,x(s),\int_0^s k(s,\tau,x(\tau))d\tau)ds)ds.
\end{aligned}$$

where the fuzzy mappings $\tilde{g}^{-1}$ satisfied above statements.

Notice that $(\Phi x)(T)={}_{\alpha}x^1$, which means that the control $u(t)$ steers the (F.C.S.) from the origine to x^1 in time T provided we can obtain a fixed point of the operator Φ.

Assume that the following hypotheses :

(H1) Linear system of (1.1) $f\equiv 0$ is exact ntrollable.

H2) nonlinear reular function $[0, T]\times E_N^n\times E_N^n \to E_N^n$ and $:[0, T]\times[0, T]\times E_N^n \to E_N^n$ are satisfy a global pschitz condition, there exist $K>0$ and >0 such that

ere $x_i,\ y_i \in E_N$ $(i=1,\ 2)$.

Theorem 3.1. Suppose that hypotheses (H1), (H2) are satisfied. Then the state of the (1.1) can be steered from the initial value x_0 to any final state x^1 in time T.

Proof. The continuous function from $C([0,T]:E_N^n)$ to itself defined by

$$(\Phi x)(t) = {}_a S(t)x_0 + \int_0^t S(t-s)f(s,x(s),\int_0^s k(s,\tau,x(\tau))d\tau)ds$$
$$+\int_0^t S(t-s)\,\tilde{g}^{-1}(x^1 - S(T)x_0$$
$$-\int_0^T S(T-s)f(s,x(s),\int_0^s k(s,\tau,x(\tau))d\tau)ds)ds.$$

There exist $\Phi_i(i=1,\cdots,n)$ is continuous function from $C([0,T]:E_N)$ to itself.

Let $x, y\in C([0,T]:E_N^n)$ there exist $(i=1,\cdots,n)$

$x_i, y_i \in C([0,T]:E_N)$.

$$d_H([\Phi_i x_i(t)]^\alpha, [\Phi_i y_i(t)]^\alpha)$$
$$d_H([\int_0^t S_i(t-s)f_i(s,x_i(s),\int_0^s k(s,\tau,x_i(\tau))d\tau)ds]^\alpha,$$
$$[\int_0^t S_i(t-s)f_i(s,y_i(s),\int_0^s k(s,\tau,y_i(\tau))d\tau)ds]^\alpha)$$
$$d_H([\int_0^T S_i(T-s)\ \tilde{g}_i^{-1}(\int_0^T S_i(T-s)$$
$$f_i(s,x_i(s),\int_0^s k(s,\tau,x_i(\tau))d\tau)ds)ds]^\alpha, [\int_0^T S_i(T-s)\ \tilde{g}_i^{-1}$$
$$(\int_0^T S_i(T-s)f_i(s,y_i(s),\int_0^s k(s,\tau,y_i(\tau))d\tau)ds)ds]^\alpha)$$

$$\le d_H(\int_0^t [S_i(t-s)f_i(s,x_i(s),\int_0^s k(s,\tau,x_i(\tau))d\tau)]^\alpha ds,$$
$$\int_0^t [S_i(t-s)f_i(s,y_i(s),\int_0^s k(s,\tau,y_i(\tau))d\tau)]^\alpha ds)$$
$$+\ d_H([\tilde{g}_i(\ \tilde{g}_i^{-1}(\int_0^T S_i(T-s)f_i(s,x_i(s),\int_0^s k(s,\tau,x_i(\tau))d\tau)ds))]$$
$$[\tilde{g}_i(\ \tilde{g}_i^{-1}(\int_0^T S_i(T-s)f_i(s,y_i(s),\int_0^s k(s,\tau,y_i(\tau))d\tau)ds))]^\alpha)$$
$$\le\ CK(t+\frac{pt^2}{2})d_H([x_i(t)]^\alpha,[y_i(t)]^\alpha)$$
$$+\ CK(T+\frac{pT^2}{2})d_H([x_i(t)]^\alpha,[y_i(t)]^\alpha)$$

$$=\ CK\{(t+\frac{pt^2}{2})+(T+\frac{pT^2}{2})\}d_H([x_i(t)]^\alpha,[y_i(t)]^\alpha),$$

Thus

$$d_\infty(\Phi x, \Phi y) = \sup_{\alpha\in[0,1]} d_H([\Phi x)]^\alpha, [\Phi y]^\alpha)$$

$$=\sup_{\alpha\in[0,1]}\left\{\sum_{i=1}^n (d_H([\Phi_i x_i(t)]^\alpha,[\Phi_i y_i(t)]^\alpha))^2\right\}^{\frac{1}{2}}$$

$$=\ CK\{(t+\frac{pt^2}{2})+(T+\frac{pT^2}{2})\}$$
$$\sup_{\alpha\in[0,1]}\left\{\sum_{i=1}^n (d_H([x_i(t)]^\alpha,[y_i(t)]^\alpha))^2\right\}^{\frac{1}{2}}$$

$$= CK\{(t+\frac{pt^2}{2})+(T+\frac{pT^2}{2})\}\sup_{\alpha\in[0,1]} d_\infty(x,y).$$

Hence

$$H(\Phi x,\Phi y) = \sup_{t\in[0,T]} d_\infty(\Phi x,\Phi y)$$
$$\le\ \sup_{t\in[0,T]} CK\{(t+\frac{pt^2}{2})+(T+\frac{pT^2}{2})\}\ d_\infty(x,y)$$
$$=\ 2CK(T+\frac{pT^2}{2})H(x,y).$$

We take sufficiently small T, $2CK(T+\frac{pT^2}{2})<1$. Hence Φ is a contraction mapping. By the Banach fixed point theorem, Φ has fixed point.

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