Three phase flow simulations using the fractional flow based approach with general initial and boundary conditions

Heejun Suk

Korea Institute of Water and Environment, Korea Water Resources Corporation, Daejon, Korea, 305-730
(e-mail: sxh60@kowaco.or.kr)

<Abstract>

The multiphase flow simulator, MPS, is developed based on the fractional flow approach considering the fully three phase flow with general initial and boundary condition. Most existing fractional flow-based models are limited to two-phase flow and specific boundary conditions. Although there appears a number of three-phase flow models, they were mostly developed using pressure based approaches. As a result, these models require cumbersome variable-switch techniques to deal with phase appearance and disappearance. The use of fractional flow based approach in MPS makes it unnecessary to use variable-switch to handle the change of phase configurations. Also most existing fractional flow based models consider only specific boundary conditions. However, the present model considers general boundary conditions of most possible and plausible cases which consists of ten cases.

Key words : Multiple phase flow; Fractional flow approach; General boundary conditions

1. Introduction

The fractional flow approach originated in the petroleum engineering literature employs water saturation, total liquid saturation, and a total pressure as the primary variables. In hydrology boundary conditions are often posed in terms of individual fluid fluxes or pressures. While pressure-based approaches can easily handle these types of boundary conditions [1], fractional flow-based approaches cannot directly use them to solve the multiphase problem, because they cannot be explicitly transformed into boundary conditions for such approaches. In this papar, a two-dimensional finite element model to study the simultaneous movement of nonaqueous phase liquid, water, and gas with general boundary condition is developed. The general boundary condition consists of ten cases (Table 1). The first eight cases are the combinations of two types of boundaries of individual phases, flux type and Dirichlet type boundaries. The other two cases are the variable boundary conditions. Also the general initial conditions are made of eight combinations of two types of initial condition of individual phases, saturation and pressure (Table 2).

Table 1. Eight types of initial condtion for multiphase flow simualtion

Types	P_1	P_2	P_3	S_1	S_2	S_3
1	X	X	X			
2	X	X				X
3	X		X		X	
4		X	X	X		
5			X	X	X	
6		X		X		X
7	X				X	X
8				X	X	X

X indicates chosen items for initial condition

Table 2. Ten types of boundary conditions for multiphase flow simulation

Types	P_1	P_2	P_3	$n \cdot V_1$	$n \cdot V_2$	$n \cdot V_3$
1				X	X	X
2			X	X	X	
3		X		X		X
4	X				X	X
5	X	X	X			
6	X	X				X
7	X		X		X	
8		X	X	X		
9	P_1 is specified $N_2=0$ if $n \cdot V_2 > 0$ or $n \cdot V_2 = 0$ if $n \cdot V_2 \leq 0$ $N_3=0$ if $n \cdot V_3 > 0$ or $n \cdot V_3 = 0$ if $n \cdot V_3 \leq 0$					
10	P3 is specified $N_1=0$ if $n \cdot V_1 > 0$ or $n \cdot V_1 = 0$ if $n \cdot V_1 \leq 0$ $N_2=0$ if $n \cdot V_2 > 0$ or $n \cdot V_2 = 0$ if $n \cdot V_2 \leq 0$					

X indicates chosen items for initial condition

2. Governing Equations

Through manipulation, rearranging, and summation of Darcy velocites of all three phases, the following governing equations can be obtained as follows [2].

$$-\nabla \cdot x(\nabla P_t + \bar{\rho} g \nabla z) = \frac{Q_1}{\rho_1} + \frac{Q_2}{\rho_2} + \frac{Q_3}{\rho_3} \qquad (1)$$

$$\frac{\partial \phi S_1}{\partial t} + \boldsymbol{V}_t \cdot \frac{dx_1}{dS_1} \nabla S_1 = \nabla \cdot x_1 \boldsymbol{x}\left(x_2 \frac{\partial P_{C12}}{\partial S_1} + x_3 \frac{\partial P_{C13}}{\partial S_1} \right) \nabla S_1$$

$$+ \nabla \cdot x_1 \boldsymbol{x}\left(x_2 \frac{\partial P_{C12}}{\partial S_t} + x_3 \frac{\partial P_{c13}}{\partial S_t} \right) \nabla S_t - x_1 \nabla \cdot \boldsymbol{V}_t + \nabla \cdot x_1 \boldsymbol{x}\left(\rho_1 g \nabla z - \bar{\rho} g \nabla z \right) + \frac{Q_1}{\rho_1} \qquad (2)$$

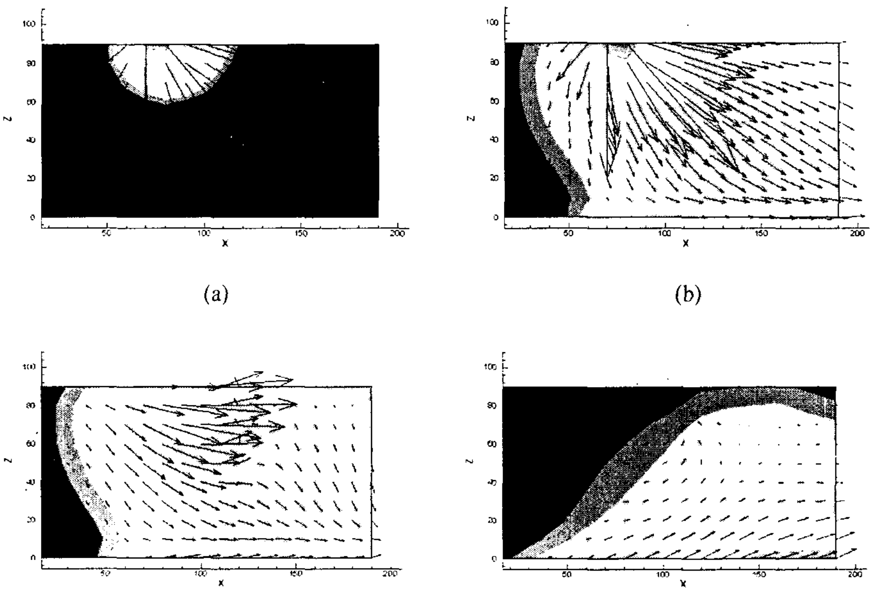
$$- \frac{\partial \phi S_t}{\partial t} + \boldsymbol{V}_t \cdot \frac{dx_3}{dS_t} \nabla S_t = \nabla \cdot x_3 \boldsymbol{x}\left(x_2 \frac{\partial P_{C32}}{\partial S_1} + x_1 \frac{\partial P_{C31}}{\partial S_1} \right) \nabla S_1$$

$$+ \nabla \cdot x_3 \boldsymbol{x}\left(x_2 \frac{\partial P_{C32}}{\partial S_t} + x_1 \frac{\partial P_{c31}}{\partial S_t} \right) \nabla S_t - x_3 \nabla \cdot \boldsymbol{V}_t + \nabla \cdot x_3 \boldsymbol{x}\left(\rho_3 g \nabla z - \bar{\rho} g \nabla z \right) + \frac{Q_3}{\rho_3} \qquad (3)$$

where $S_t = S_1 + S_2$ is a total liquid saturation. The set of partial differential equations given by (2) and (3) are coupled through the several constitutive relationships.

3. Results and Discussions

The DNAPL is infiltrated from a spill area for 3.226 days under the hydraulic gradient in which the ambient water flow direction goes from left to right with a rate of 40 cm/day. After 3.226 days the infiltration of DNAPL is suddenly stopped and a cleanup is started using pumping wells with a pumping rate 1000 day-1 for the remainder of the simulation time. This problem is solved with MPS. The simulaton results are shown in Figure 1.



(a) (b)

(c) (d)

Figure 1. Distributions of DNAPL velocity and DNAPL saturation plots at various times (b) 3.3604e-1 days, (b) 3.226 days, (c) 3.2936 days, and (d) 5.4419 days

4. Conclusions

Robust and efficient strategies were researched to iteratively(using outer iteration loop) transform general types of boundary conditions for pressure based approaches to those for fractional flow based approaches. As a result, the number of outer-loop iterations required for the boundary condition types 2-4, 6, and 8 in Table 2 is usually two or three only. For boundary condition types 1 and 5, there is no need for the outer iteration at all. Furthermore, the number of iterations in the inner loop to solve the coupled nonlinear water and liquid saturation equations is usually small. So MPS is a practial simulator to consider general boundary conditions.

References

[1] Bastian P, Helmig R. Efficient fully-coupled solution techniques for two-phase flow in porous media parallel multigrid solution and large scale computations. Advances in Water Resources 1999;23:199-216.

[2] Heejun Suk, Development of 2- and 3-D simulator for three-phase flow with general initial and boundary conditions on the fractional flow approach. Ph.D dissertation Department of Civil and Environmental Engineering, The Pennsylvania State University, State College, PA, 2003.