Novel High-Rate High-Performance Space-Time Codes

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Abstract— **In this paper, we propose two novel high-rate high-performance space-time codes for multiple-input multiple-output (MIMO) systems. When n_T transmit antennas and $n_R = n_T$ receive antennas are deployed, the two proposed codes respectively offer transmission rates of $(n_T - 1)$ and $(n_T - 2)$ symbols per channel use and diversity orders of 3 and 5. As a consequence, our proposed codes allow the MIMO systems to employ a simple detection technique based on QR decomposition. Moreover, for equal or even higher spectral efficiencies, our proposed codes always provide much better bit error rate (BER) performances than V-BLAST architecture does when $n_R = n_T$. Computer simulation is given to verify performances of our proposed codes.**

I. INTRODUCTION

The deployment of multiple transmit and receive antennas, leading to the so-called multiple-input multiple-output (MIMO) systems, is a potential way for future wireless communication systems to cope with the limitation in power and bandwidth. Foschini *et al.* [1] and Telatar [2] have predicted that significant spectral efficiencies can be obtained with multiple antennas in rich scattering environments providing channel variations can be accurately tracked. It is this prediction that leads to the development of various MIMO techniques such as space-time coding [3]-[5] and V-BLAST architecture [6].

Space-time block code was first introduced by Alamouti [3] for a simple transmit diversity scheme with two branches. Then, the problem was generalized to an arbitrary number of transmit antennas by Tarokh *et al.* [4]. The key feature of STBCs is that it achieves a full diversity gain with a simple maximum-likelihood decoding algorithm. Perhaps, the biggest disadvantage of STBCs is that their transmission rates, and hence their spectral efficiencies, are very low no matter how many transmit and receive antennas are used. Consequently, they are not appropriate for high data rate applications. The very recent STCs proposed in [5] have been declared to have full rate and full diversity. Unfortunately, they have much higher complexity in the decoding process than STBCs, hence limiting their application to MIMO systems with large number of transmit and receive antennas.

In contrast to STBCs, V-BLAST architecture [6] can provide enormous spectral efficiencies, which are in proportion to the number of transmit antennas. Nonetheless, its detection based on interference suppression and interference cancellation techniques such as zero forcing (ZF) [5] and QR decomposition [7] has reduced its diversity order. Particularly, when the number of receive antenna, n_R, is equal to the number of transmit antenna, n_T, interference suppression process causes the diversity order of the first detected symbol to decrease to one, leading to a V-BLAST system with very low bit error rate (BER) performance. The problem can be overcome by using more receive antennas than transmit antennas, *i.e.*, $n_R > n_T$. This, however, may lead to practical limitations of V-BLAST. For example, when V-BLAST is to be applied to a system where a spectral efficiency of 20 bits/s/Hz and a diversity order of 5 are required for both forward link and reverse link, one may use $n_T = 5$, $n_R = 9$ and 16QAM modulation scheme for the forward link. What about the reverse link? Definitely, one cannot achieve the goal for this link. If spectral efficiency requirement is satisfied, then diversity order requirement is not, and vice versa. It is undoubted from practical standpoint that the numbers of transmit and receive antennas should be equal in order for the forward and reverse links in the system to have equal bit rate as well as BER performance.

In this paper, we first introduce the "nulling diversity gains" concept for a general space-time code through QR decomposition of the equivalent channel matrix corresponding to that code. Based on this concept, we devise two new space-time codes for MIMO applications, called ST1 and ST2, which respectively have transmission rates of less than one and two symbols, *i.e.*, $(n_T - 1)$ and $(n_T - 2)$, per channel use in comparison with V-BLAST architecture for the same number of transmit antennas. For the sake of convenience, MIMO systems have n_T transmit antennas and n_R receive antennas that utilize our proposed codes are denoted by ST1 n_T Tx n_R Rx and ST2 ST1 n_T Tx n_R Rx systems. The big advantage of our proposed space-time codes over V-

BLAST is that when $n_R = n_T$ they offers a minimum nulling diversity order of three and five, respectively. As a result, the ST1 n_T Tx n_R Rx and ST2 n_T Tx n_R Rx systems can employ a very simple detection algorithm based on QR decomposition without having to consider optimum ordering. Moreover, for equal or even higher spectral efficiencies, it still has much higher BER performances than its V-BLAST counterpart does as shown by simulation results.

II. THE INVESTIGATION OF NULLING DIVERSITY GAINS VIA QR DECOMPOSITION

Let us consider a MIMO system consisting of n_T transmit and n_R receive antennas. In this system, the input information sequence is encoded by a space-time encoder to give a transmission matrix $\mathbf{S}$ of n_T rows L columns with a total number of N symbols. . It is clear that the transmission rate of this space-time code is $R = N/L$ symbols per channel use. Assume that the relationship between transmitted and received signal vectors can be expressed in the following linear model:

$$\underline{\mathbf{r}} = \mathbf{H}\underline{\mathbf{s}} + \underline{\mathbf{w}} \tag{1}$$

where $\underline{\mathbf{r}} = [r_1 \quad r_2 \quad \ldots \quad r_M]^T$ is the $M \times 1$ equivalent received signal vector, $\underline{\mathbf{s}} = [s_1 \quad s_2 \quad \ldots \quad s_N]^T$ is the $N \times 1$ vector consisting of N transmitted symbol within the matrix $\mathbf{S}$, T denotes the transpose of a matrix, $\mathbf{H}$ is the $M \times N$ equivalent channel matrix whose entries are assumed to be i.i.d. zero-mean complex Gaussian random variables with unity variance or possibly zero so that they satisfy the following normalized condition:

$$\sum_{m=1}^{N} E\left[\left|h_{m,n}\right|^2\right] = n_T, \quad n = 1, 2, \ldots, M, \tag{2}$$

$\underline{\mathbf{s}}$ is the $N \times 1$ transmitted signal vector whose elements are drawn from a space-time code, and $\underline{\mathbf{w}}$ is the $M \times 1$ vector of additive white Gaussian noises (AWGN) at the receive antennas with noise variance per receive antenna being σ^2, $M = n_R L$. Moreover, we assume that the channel is quasi-static, *i.e.*, $\mathbf{H}$ is constant in one block, and changes from this block to the next.

Using quadrature residue (QR) factorization, we can express the channel matrix as:

$$\mathbf{H} = \mathbf{QR} \tag{3}$$

where $\mathbf{Q}$ is an $M \times N$ unitary matrix, *i.e.*, $\mathbf{Q}^H\mathbf{Q} = \mathbf{I}_N$, and $\mathbf{R}$ is an $N \times N$ upper triangular matrix defined by:

$$\mathbf{R} = \begin{bmatrix} R_{1,1} & R_{1,2} & \ldots & R_{1,N} \\ 0 & R_{2,2} & \ldots & R_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & R_{N,N} \end{bmatrix} \tag{4}$$

Pre-multiplying both sides of Equation (1) by $\mathbf{Q}^H$, we get:

$$\underline{\mathbf{v}} = \mathbf{R}\underline{\mathbf{s}} + \underline{\mathbf{n}} \tag{5}$$

where $\underline{\mathbf{v}} = \mathbf{Q}^H\underline{\mathbf{r}}$ and $\underline{\mathbf{n}} = \mathbf{Q}^H\underline{\mathbf{w}}$ are respectively $N \times 1$ received signal vector and $N \times 1$ noise vector induced by QR decomposition. It is straightforward to show that the elements of $\underline{\mathbf{n}}$ are still Gaussian distributed with zero-mean and variance σ^2 per receive antenna. Equation (5) allows us to easily detect transmitted symbols in $\underline{\mathbf{s}}$. The first symbol to be detected is based on the decision statistic from the last equation of (5):

$$v_N = R_{N,N}s_N + n_N \tag{6}$$

From Equation (6), the signal-to-noise ratio (SNR) after QR decomposition of the channel matrix $\mathbf{H}$, or we may call the post-detection SNR, for symbol s_N is given by:

$$\rho_{post}^N = (R_{N,N})^2 E\left[|s_N|^2\right]/\sigma^2 = G_D^N \rho_{pre}^N \tag{7}$$

where the expectation in the numerator is taken over the signal constellation set, $G_D^{n_T} = (R_{n_T,n_T})^2$, and $\rho_{pre}^N = E\left[|s_N|^2\right]/\sigma^2$ is the pre-detection SNR for symbol s_N.

The post-detection SNR of s_N in Equation (6) can be interpreted as the corresponding pre-detection SNR multiplied by a real factor G_D^N. Since G_D^N may improve or reduce the pre-detection SNR depending on whether it is greater or less than 1 and results from the decomposition of the channel matrix $\mathbf{H}$ we call it the "nulling diversity gain" for the symbol s_N. Analogously, the square of the i^{th} diagonal element, $R_{i,i}$, of the matrix $\mathbf{R}$ is the diversity gain corresponding to the transmitted symbol s_i and is denoted by $G_D^i = (R_{i,i})^2$.

III. THE PROPOSED SPACE-TIME CODE FOR HIGH SPECTRAL EFFICIENCIES AND HIGH PERFORMANCE

From QR decomposition perspective, the nulling diversity order of the first detected symbol of a V-BLAST*K*Tx*K*Rx system, $K = n_T = n_R$, is always equal to one. Therefore, as long as interference suppression and interference cancellation techniques are employed for detection, the performance of V-BLAST*K*Tx*K*Rx system is very poor. In this paper, we propose two space-time codes for MIMO applications that have their corresponding nulling diversity orders of three and five for the first detected symbols. Although these codes have lower transmission rates than V-BLAST architecture, there high nulling diversity gains enable the systems to employ higher-level modulation schemes. As a result, for comparable or even higher spectral efficiencies, the TS1*K*Tx*K*Rx and TS2*K*Tx*K*Rx systems adopting our proposed codes always outperform their V-BLAST counterpart.

The transmission matrices for ST1 and ST2 codes are respectively defined by:

$$\mathbf{S}_{ST1} = \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \\ s_{3n_T-8} & s_{3n_T-7} & s_{3n_T-6} \\ s_{3n_T-5} & s_{3n_T-4} & s_{3n_T-3} \\ s_{3n_T-4} & s_{3n_T-3} & s_{3n_T-5} \end{bmatrix} \quad (8)$$

$$\mathbf{S}_{ST2} = \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ \mathrm{M} & \mathrm{M} & \mathrm{M} \\ s_{3n_T-11} & s_{3n_T-10} & s_{3n_T-9} \\ s_{3n_T-8} & s_{3n_T-7} & s_{3n_T-6} \\ s_{3n_T-10} & s_{3n_T-9} & s_{3n_T-11} \\ s_{3n_T-7} & s_{3n_T-6} & s_{3n_T-8} \end{bmatrix} \quad (9)$$

The vertical and horizontal axes of the transmission matrix respectively represent the spatial and temporal domain of the proposed code. It is clearly seen that these codes have transmission rates of $R = (n_T - 1)$ and $R = (n_T - 2)$ symbols per channel use, respectively.

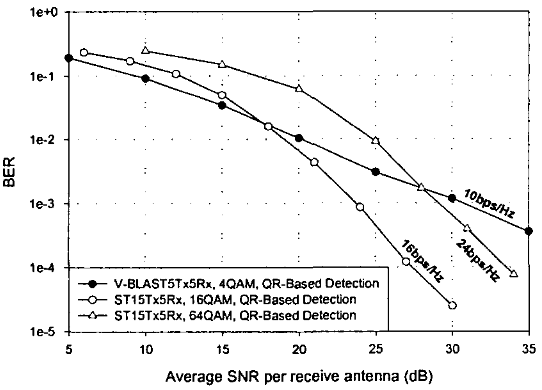

Fig. 1: Comparison of BER performances between ST15Tx5Rx system and V-BLAST5Tx5Rx architecture

For illustration of performance of our proposed codes let us consider the codes for $n_T = 5$ transmit antennas given by:

$$\mathbf{S}_{ST1} = \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ s_7 & s_8 & s_9 \\ s_{10} & s_{11} & s_{12} \\ s_{11} & s_{12} & s_{10} \end{bmatrix} \quad (10)$$

$$\mathbf{S}_{ST2} = \begin{bmatrix} s_1 & s_2 & s_3 \\ s_4 & s_5 & s_6 \\ s_7 & s_8 & s_9 \\ s_5 & s_6 & s_4 \\ s_8 & s_9 & s_7 \end{bmatrix} \quad (11)$$

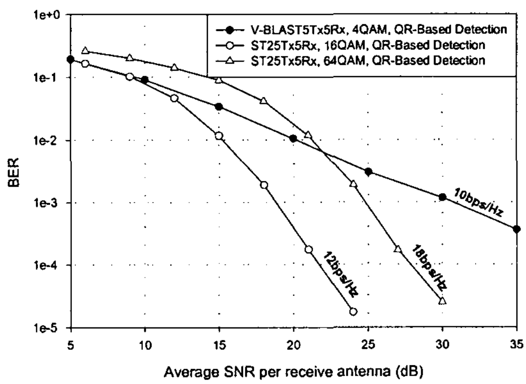

Fig. 2: BER performances of ST15Tx5Rx system in comparison with that of V-BLAST5Tx5Rx architecture

Fig. 1 and Fig. 2 respectively compare BER performances of ST15Tx5Rx and ST25Tx5Rx systems with V-BLAST5Tx5Rx. In the simulation, V-BLAST5Tx5Rx system adopts 4QAM, resulting in a spectral efficiency of 10 bits/s/Hz, while ST1T5Tx5Rx and ST2T5Tx5Rx systems utilize 16QAM and 64QAM, respectively yielding spectral efficiencies of 16 bits/s/Hz and 24 bits/s/Hz for the former and 12 bits/s/Hz and 18 bits/s/Hz for the later. All three systems employ QR decomposition for detection. It can be seen from Fig. 1 and Fig. 2 that even for higher spectral efficiencies, our proposed ST15Tx5Rx and ST25Tx5Rx systems still significantly outperform V-BLAST5Tx5Rx architecture.

IV. CONCLUSIONS

In this paper, we propose two novel space-time codes, called ST1 and ST2 space-time code2, for MIMO systems. Beside its high spectral efficiencies, when equal number of transmit and receive antennas are deployed, this code offers much higher BER performance as compared to V-BLAST. Moreover, it allows a very simple detection technique based on QR decomposition to be used. Consequently, it is a potential candidate for constructing future high-bit-rate high-performance wireless communication systems.

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