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Determination of Yield Line Patterns On the basis of Elastic Fields

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ABSTRACT

The objective of this paper is to develop a simplified method to determine yield line patterns of reinforced concrete floor slabs based on the elastic fields. Unlike other methods mainly focused on the plasticity theory, this paper emphasizes the elastic fields, especially principal moments and maximum shears and shows a link between elasticity field and yield line patterns. General criteria on both positive and negative yield lines are suggested in terms of principal moments and maximum shear forces. The proposed method can predict starting point (or regions) of yielding and the further development of yield lines on whole structures. The yield line patterns determined by the proposed method are shown to be coincident with the classical yield line theory. Furthermore, orthotropy in reinforced concrete slab is investigated and yield line patterns for different type of flat slab with non-isotropic strip are studied broadly.

1. Introduction

The yield line theory is a powerful and efficient tool for the analysis and design of reinforced concrete slabs since the first development by Johansen [1]. Design loads are determined by a postulated collapse mechanism for the slab. For a given slab system, the yield line method always gives an ultimate load which is either correct or too high, so it is called upper bound method. Most correct collapse mechanisms of the slab are well studied and known [2, 3]. However, the designer should investigate whether all possible mechanisms are considered to ensure the design is not too conservative. The complexity and uncertainty of yield line theory is originated from the difficulty in finding the exact collapse mechanism which is composed of yield lines under variant given conditions, such as load types, boundary and geometric conditions. There were several studies about determining the exact pattern of yield lines using optimization [4, 5], yield line elements [6], and mesh adaptivity [7]. Most studies are mainly concentrated on the ultimate condition, i.e. plastic deformation, rather than the intermediate process up to the ultimate condition, i.e. elastic deformation. Little attention has been given to elastic analysis in relation with yield line theory.

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The objective of this paper is to develop a simplified method to determine yield line patterns of slabs based on the elastic fields, especially principal moments and maximum shear forces. The proposed method can effectively predict the starting point, or regions, of yielding and trace the development of yield lines on whole structures. A finite element method was used to obtain the elastic fields of principal moments and shear forces of flat slabs, from which yield line patterns are presented. The results show that the determination of yield line patterns using elastic fields is a very efficient tool for slabs of arbitrary geometry, support and loading conditions, including variation of reinforcement distributions.

2. Determination of Yield Lines from Elastic Fields

In yield line theory, concrete is generally assumed to have the idealized moment curvature relationship, i.e. perfect plastic material. The most basic assumption of yield line analysis is that material either stays rigid or else it goes plastic for simplification purposes. This assumption originated from the actual deformation curve of structures under an ultimate load state in which the amount of plastic deformation is much larger than elastic deformation; therefore we usually neglect the elastic deformation. However, elastic deformation fields have very important information for yield lines. For typical shear force and bending moment diagram of fixed supported beam under uniformly distributed loads, the magnitude of maximum negative moments are greater than that of positive mid span moment, so the first yielding occurs at end points and the positive yielding occur after moment redistribution when the whole beam reaches a failure state.

The criterion for the starting points or regions of positive and negative yielding can be deduced from the shear and moment diagrams as follows:

Assumption 1) Positive yield line occurs at the points of zero shear and maximum positive moments.

Assumption 2) Negative yield line occurs at the points of maximum shear and maximum negative moments.

The shear force and moment diagram for slabs is an elastic surface in which the starting point of the yield line is the point (or region) where the maximum principal moment(s) by external loads exceed the ultimate moments of resistance of slab section.

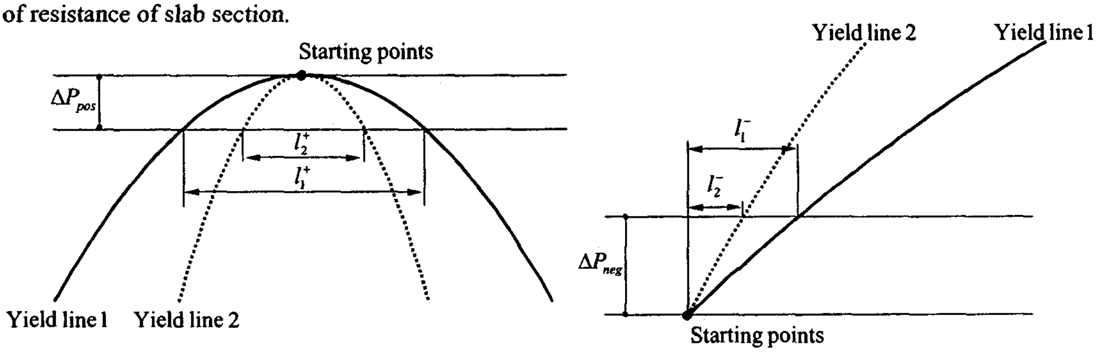

Figure 1 Postulated yield line patterns along principal moments

To discuss the development of yield lines, two different types of yield line are assumed along principal moment contours. Based on the assumption 1 and 2, the starting region of a positive yield line is determined as shown in Figure 1 (a). For the next yield line under slightly increased external loads (ΔP_{pos} in Figure 1) due to moment redistribution, yield line 1 can develop a greater yielding region than yield line 2. Graphically, the length of the yielding region for yield line 1 (l_1^+ in Figure 1 (a)) is larger than that (l_2^+) of yield line 2. It means that yield line 2 requires additional external loads in comparison with yield line 1 for the same yielding regions. As a result, the collapse load of yield line 1 is less than that of yield line 2, i.e. yield line 1 is a more exact yield line pattern. The same behavior applies to negative yield lines shown in Figure 1 (b). We can deduce from these an assumption for the development of yield lines as follows:

Assumption 3) Yield lines develop in a direction that gives the smallest collapse load; the length of maximum principal moments under additional external loads is largest.

3. Yield Line Patterns for Various Slabs

On the basis of previous assumptions derived from the results of elastic analysis, several yield line patterns of slabs under unit uniformly distributed loads are investigated with different boundary conditions, including column supports. After conducting elastic finite element analysis, both principal moment and maximum shear coefficients are illustrated as dimensionless values using contour plots.

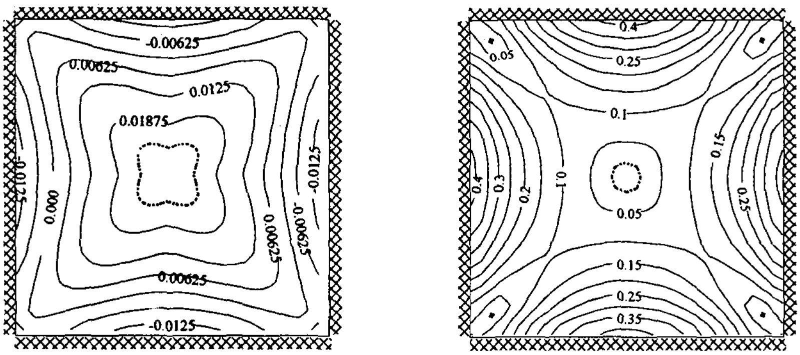

Figure 2 Moment and shear contours for slab with all fixed edges

CASE I: Fixed slabs

The elastic principal moment and shear force coefficient contours for an isotropic square slab with all clamped edges are shown in Figure 2. The first positive yielding starts from the centre of the slab (according to assumption 1) and gradually develops in a direction which results in the length of maximum principal moment contour being largest (according to assumption 3). For the purpose of comparison, 95% of maximum principal moments and 5% of minimum shear force contours are plotted as dotted lines in Figure 2. The final pattern of positive yield lines therefore develops in diagonal directions. With respect to maximum shear forces, the yield line develops in a direction in which the shear force is close to zero. The negative yielding occurs at centre of supports simultaneously and develops along the support lines (according to assumption 3).

CASE II: Two edges clamped and two edges free

Figure 3 shows principal moment and shear coefficients for a slab with two edges clamped under uniformly distributed load. Yield line patterns can be derived from the principal moment and shear force coefficient contours, especially from the shear contours where the near zero lines constitute the positive yield line.

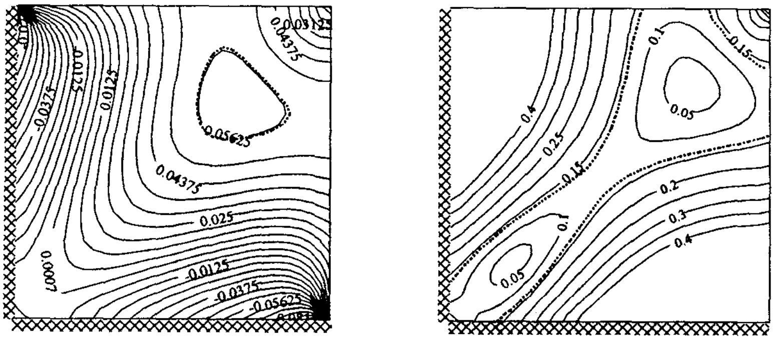

Figure 3 Moment and shear contours for slab with two fixed edges

CASE III: One edge clamped with columns on two opposite edges

Figure 4 shows maximum principal moment and maximum shear force coefficient contours for a slab one edge clamped with columns on two opposite edges. As with the previous example, a positive yield line occurs along the centre line between column and clamped edge and a negative yield line develops around the column. On the basis of moment and shear contours, if the positive yield line occurs along the centre line, then the slab divides into two parts, one is a cantilever slab (including its clamped edge) and the other is a flat slab (including column). Therefore, at ultimate failure, a negative yield line is required both in the cantilever and flat slab part that are divided after a positive yield line develops along the centre line.

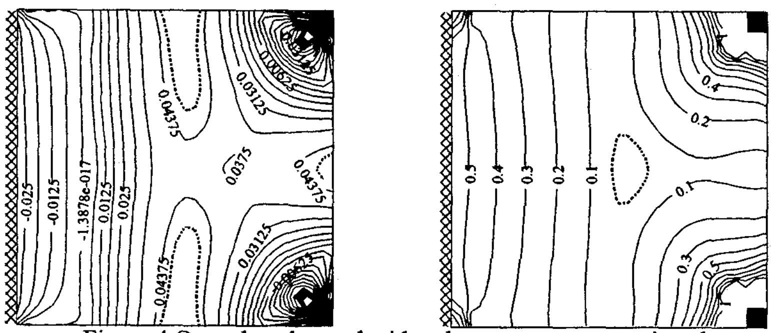

Figure 4 One edge clamped with columns on two opposite edges

CASE IV: Four column supported slabs

Figure 5 shows the principal moment and maximum shear force coefficient contours for a slab supported by four columns. A positive yield line develops from the centre point and progresses in the shape of a cross. Negative yield line develops around each of the columns.

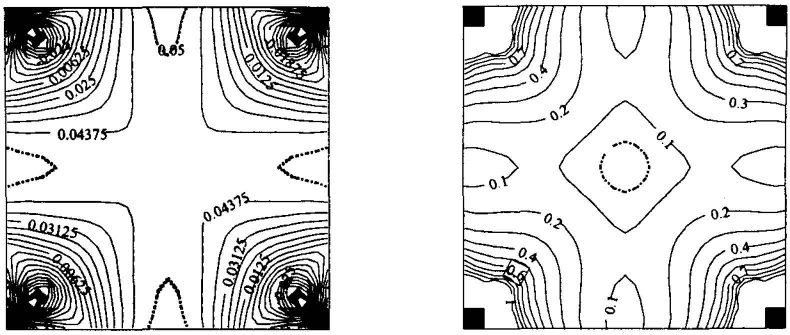

Figure 5 Four column supported slab

The variation of principal moment and shear force coefficients according to different boundary conditions is summarized in Table 1. For slabs with column supports, the maximum shear and negative principal moment increased rapidly due to the column effect. Therefore, the detailed behavior of column supports should be investigated using a special model. Both the column and corner effects are not covered in this paper.

Table 1 Comparison of coefficients according to boundary conditions

CASES	Moment 1)		Shear 2)	
	Max.	Min.	Max.	Min.
All clamped	0.0230	-0.0154	0.4140	0.0090
Three edges clamped one free	0.0436	-0.0282	0.9884	0.0498
Two edges clamped two free	0.0590	-0.0904	2.7534	0.0033
Two edges clamped with one column	0.0457	-0.1267	7.6739	0.0000
One edge clamped with two columns	0.0478	-0.1335	8.2131	0.0000
One edge clamped with one column	0.0446	-0.1384	7.4679	0.0000
Four columns supported	0.0525	-0.1500	9.3900	0.0000

4. Applications

Slabs with different coefficient of orthotropy

According to the classical yield line 'affinity theorems', an orthotropic square slab under uniformly distributed loads can be considered as rectangular isotropic slab where the length of l_y becomes $l_y/\sqrt{\mu}$, where μ is known as the coefficient of orthotropy and defined as the ratio of ultimate moments of resistance per unit width in the x and y direction. To investigate the varying yield line patterns of orthotropic slabs, four different orthotropic slabs with the value of μ from 1.00 to 0.25 are analyzed for both clamped and column supported slabs. As the coefficient of orthotropy decreases the yield line patterns goes to that of rectangular slabs, which is consistent with the affinity theorems.

Positive and negative moment variations according to different orthotropy are summarized in Table 2 for clamped slabs. Due to the orthotropy in the y direction, the slab behaves as a rectangular one with the longer length in the y direction, such that the short span moment (Mx) increases as the coefficient of orthotropy decreases as shown in Table 2.

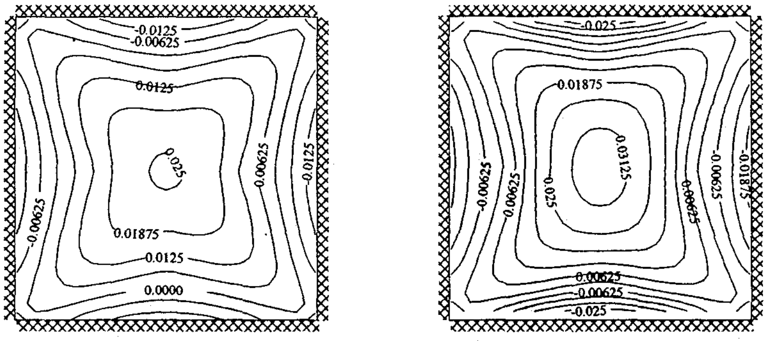

Figure 6 Principal moment variation according to orthotropy coefficients

Table 2 Variation of moments according to orthotropy: Clamped slabs

	Positive moments			Negative moments		
μ	Mx(+)	My(+)	Mp(+)	Mx(-)	My(-)	Mp(-)
1.00	0.02299	0.02299	0.02299	-0.05129	-0.05129	-0.01538
0.75	0.02566	0.02016	0.02566	-0.05565	-0.04557	-0.01823
0.50	0.02904	0.01687	0.02904	-0.06107	-0.03815	-0.02289
0.25	0.03406	0.01343	0.03406	-0.06836	-0.02771	-0.02771

Therefore, the higher ordinate region on principal moment contour develops along the direction of y. Also, the positive principal moment (Mp(+)) is strongly dependent on the maximum value of Mx or My.

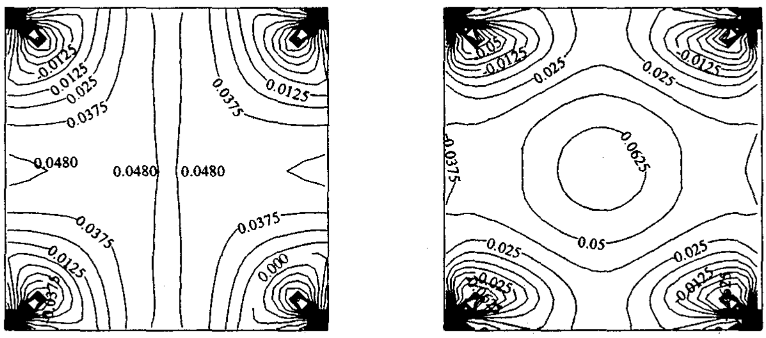

Figure 7 Principal moment variations: Column supported slabs

For the clamped slabs there is a gradual increase in principal moment (Mp) with decreasing μ, but for the column supported slab there is a sudden increase where $\mu < 0.5$.

Orthotropic slabs with different angle of material

To investigate the effect of material angle to yield line patterns, two different type of orthotropic slab with material angle of 22.5 and 45 were analyzed. Figure 8 shows variations of principal moment contours according to the coefficient of orthotropy with different material angle. For both slab, a positive yield line develops along weak diagonal direction. However, as the coefficient of orthotropy decreases from 0.75 to 0.25, yield line pattern of slab with material angle 22.5 follows to that of rectangular slab except strong diagonal direction as shown in Figure 8.

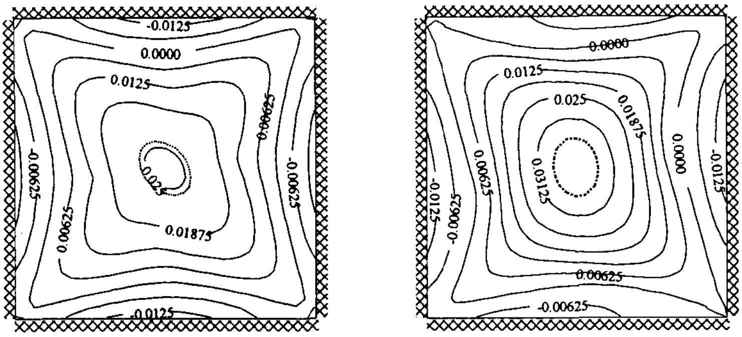

Figure 8 Principal moment variations: material angle of 22.5

Slabs with non-isotropic strips

Certain types of flat slabs are composed of reinforced concrete units joined along their edges by narrow strips of infill material having a much lower (typically μ=0.25) equivalent Young's modulus. An example is the precast system developed by BRE using Densit infill [12] or the system developed by Altimimi using FRC infill [13]. These flat slabs are idealized in this study. Slab A is a single bay, whilst slab B is modeled as a quarter of two by two bay flat slab using its symmetry in both x and y direction (The intersection region with horizontal and vertical strip is assumed as isotropic property with μ=1).

From the principal moment contour for slab A, the positive yield line develops perpendicular to the strip. It is difficult to determine the pattern of yield line on the basis of principal shear only because the potential positive yield line, which has almost zero principal shear value, develops as a cross shape.

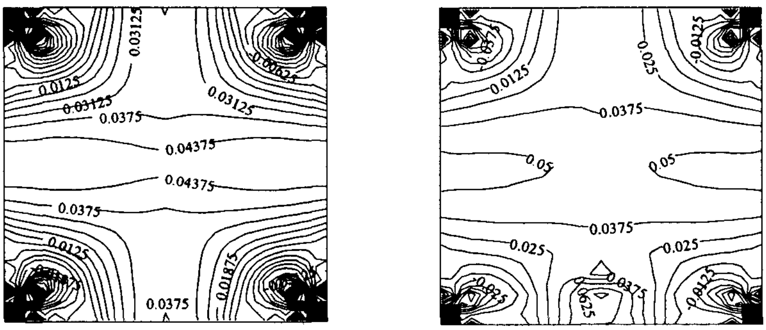

Figure 9 Elastic fields of slab A and B

In the yield line pattern for slab B, the positive yield line again develops perpendicular to the strips, so the final yield line pattern is a T shape. The principal shear for slab B is only slightly different from Slab A. So far, the orthotropy of slabs are represented as fully cracked section, that is, the ratio of orthotropy coefficient is exactly same as that of equivalent Young's modulus.

5. Conclusions

In this paper, a simplified method to determine yield line patterns of reinforced concrete floor slabs based on the elastic fields is presented. Yield line patterns for various slabs with different boundary conditions and orthotropic slabs are investigated. The yield line patterns determined by the proposed method are shown to be coincident with the classical yield line theory. Moreover, orthotropic slab and slab with non-isotropic strip are analyzed. The results are summarized as the following: (1) Elastic fields give an important information about the starting point and the further development of yield lines; (2) the positive yield line occurs at the points or regions of the maximum principal moment and the minimum shear forces; (3) the negative yield line occurs at the points or regions of the maximum negative principal moment and the maximum shear forces; (4) the coefficient of orthotropy has a great effect onto structural behavior if the value is lesser than 0.25 in case of flat slab; (5) the relationship between the coefficients of orthotropy and equivalent Young's modulus is derived on the basis of cracking progress; (6) the proposed method can effectively predict yield line patterns for flat slab with non-isotropic strips.

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