한국전산구조공학회
2004년도 봄 학술발표회 논문집

유연한 Timoshenko 빔의 동역학적 유한요소 정식화 및 해석

The Finite Element Formulation and Analysis of the Dynamic Flexible Timoshenko Beam

류지강* 윤성호**
Liu, Zhi-Qiang Yun, Seong-Ho

ABSTRACT

This paper established the dynamic model of a flexible Timoshenko beam with geometrical nonlinearities subject to large overall motions by using the finite element method. The equations of motion are derived by using Hamilton principle based on expressing the kinetic and potential energies of the flexible beam in terms of generalized coordinates. The nonlinear constraint equations are adjoined to the system equations of motion by using Lagrange multipliers.

1. Introduction

The dynamic analysis of flexible body has been developed in these years. It provides more exact and credible results which are needed in design and control of dynamic system. The most popular method for studying this subject is by using finite element method. Flexible beam elements are generally modelled according to the classical Euler-Bernoulli theory. However, for the purpose of higher speed and better system performance, the investigation of flexibility due to the bending and torsion effects of the manipulator is needed. Therefore a more accurate beam element model is developed here by the Timoshenko model of the beam. [(1), (2)]

Two sets of coordinates are defined: one is reference coordinates which describe large rigid-body translation and large angular rotations of the body reference, and the other is elastic coordinates which characterize elastic deformations of the body, i.e. relative translation and angular displacement of infinitesimal volumes at nodal points on the body. [(3)] The location and angular orientation of every infinitesimal volumes in each element can be approximated in terms of its elastic coordinates and the assumed shape functions by the finite element approach. Based on Timoshenko beam theory with the inclusion of the geometrical nonlinearity and the shear deformation, energy equation of each element are derived by the assumed shape functions. And then the elements of each body are assembled using the standard finite element procedures. Algebraic equations prescribing constraints between various bodies are formulated and coupled to the equations of motion by the Lagrange multiplier technique. The resulting equations of motion are usually nonlinear and highly coupled in the inertia terms due to the presence of Coriolis and centrifugal effects as well as inertia due to the rotation of the beam. [(4)]

* 정회원 · 금오공과대학교 기계공학부 석사과정
** 정회원 · 금오공과대학교 기계공학부 부교수

2.Flexible Body Description

2.1. Displacement and deformation analysis

Consider a body in an orthonormal basis $I=(i_1, i_2, i_3)$. Two coordinates of this body will be defined: a before-deformed coordinates, and an after-deformed coordinates, as depicted in fig. 1.

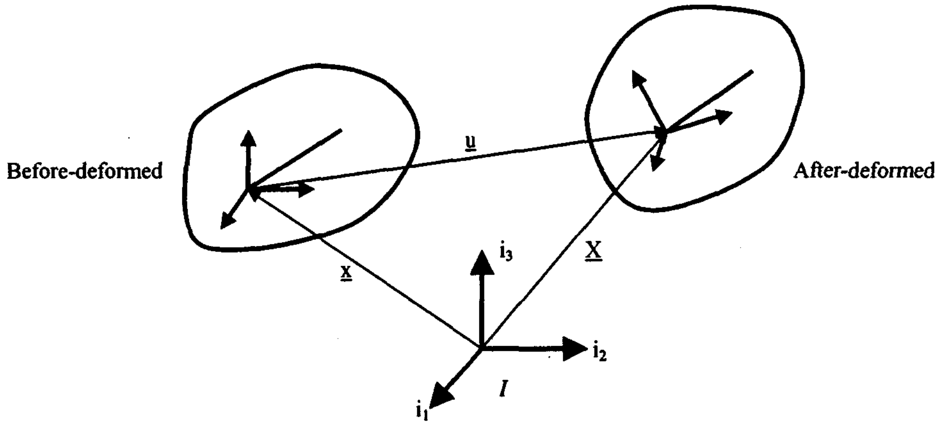

Fig. 1 The before-deformed and after-deformed configurations of a body

Let point o and O be material points in the body, and the position vector of them be $\underline{x}$ and $\underline{X}$ in the before-deformed and the after-deformed coordinates respectively, which are

$$\underline{x}=\underline{x}(x_1,x_2,x_3);\ \underline{X}=\underline{X}(X_1,X_2,X_3). \tag{1}$$

Increments in position vector are denoted $d\underline{x}$ and $d\underline{X}$ respectively. A convenient choice for the material coordinates, Lagrangian representation, will be used here. And then $\underline{X}$, $d\underline{X}$ can be expressed as

$$\underline{X}=\underline{X}(x);\ d\underline{X}=\begin{bmatrix} X_{1,1} & X_{1,2} & X_{1,3} \\ X_{2,1} & X_{2,2} & X_{2,3} \\ X_{3,1} & X_{3,2} & X_{3,3} \end{bmatrix} d\underline{x}. \tag{2}$$

where $X_{i,j}=\partial X_i/\partial x_j$ $(i,j=1, 2, 3)$.

The lengths of increments of the lines in the before-deformed and the after-deformed coordinates are

$$ds^2=d\underline{x}^T d\underline{x};\ dS^2=d\underline{X}^T d\underline{X}. \tag{3}$$

According to the description of Green-Lagrange Strain tensor, the change in length of the increment of the position vector is [5]

$$\Delta S=\frac{1}{2}\left(dS^2-ds^2\right)=\frac{1}{2}\left(d\underline{X}^T d\underline{X}-d\underline{x}^T d\underline{x}\right). \tag{4}$$

The displacement relating two coordinates is defined as $\underline{u}\left[u_1\ u_2\ u_3\right]^T$, and easily to find $\underline{u}=\underline{X}-\underline{x}$. Obviously,

$$d\underline{X}=\left(\underline{u}'+\underline{I}\right)d\underline{x}, \tag{5}$$

where $\underline{u}' = \begin{bmatrix} u_{1,1} & u_{1,2} & u_{1,3} \\ u_{2,1} & u_{2,2} & u_{2,3} \\ u_{3,1} & u_{3,2} & u_{3,3} \end{bmatrix}$. Introduce Eq. (5) into Eq. (4), ΔS will be

$$\Delta S = d\underline{x}^T \left(\underline{\varepsilon}_l + \underline{\varepsilon}_{nl} \right) d\underline{x} = d\underline{x}^T \underline{\varepsilon} d\underline{x}, \tag{6}$$

where $\underline{\varepsilon}_l$ and $\underline{\varepsilon}_{nl}$ denote the linear and nonlinear part of $\underline{\varepsilon}$ as $\underline{\varepsilon}_l = \frac{1}{2}\left(\underline{u}'^T + \underline{u}'\right)$ and $\underline{\varepsilon}_{nl} = \frac{1}{2}\left(\underline{u}'^T \underline{u}'\right)$.

After arrangement the relationship between strain and displacement will be found as

$$\underline{\varepsilon} = D\underline{u}. \tag{7}$$

2.2. Motion description of the flexible body

Assume the elastic deformation of point P on an after-deformed body is

$$\underline{u}_f = \underline{N}\underline{q}_f, \tag{8}$$

where $\underline{N}$ is an appropriate shape function, and $\underline{q}_f$ is a set of time dependent elastic coordinates of the body. As shown in Fig. 2, let $\underline{X}$ represent translation tensor and $\underline{R}$ a rotation tensor of the orientation angle θ of the body attached frame $\left(x^*, y^*, z^*\right)$ with respect to the inertial frame I=(i_1, i_2, i_3). Therefore, a set of time-dependent reference coordinates $\underline{q}_r$ associated with the above rigid body motion is introduced to set up the total time-dependent coordinates.

$$\underline{q} = \begin{bmatrix} \underline{q}_r \\ \underline{q}_f \end{bmatrix}; \ \underline{q}_r = \begin{bmatrix} \underline{X} \\ \theta \end{bmatrix}. \tag{9-a), (9-b}$$

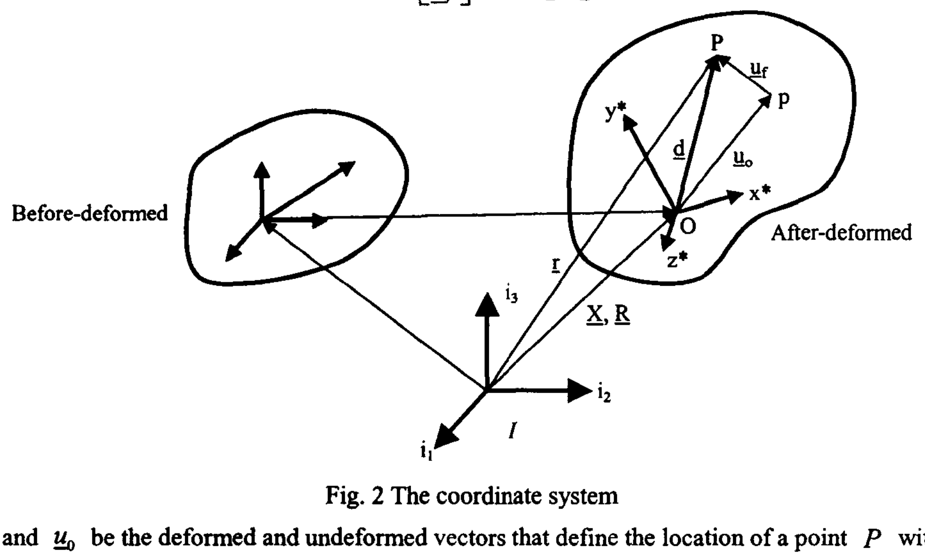

Fig. 2 The coordinate system

Let $\underline{d}$ and $\underline{u}_0$ be the deformed and undeformed vectors that define the location of a point P with respect to the body $\left(x^*, y^*, z^*\right)$ coordinate system. Then $\underline{u}_f$ is elastic deformation at point P, such that,

$$\underline{d} = \underline{u}_0 + \underline{u}_f = \underline{u}_o + \underline{N}\underline{q}_f. \tag{10}$$

The global location of point P with respect to the inertial frame is

$$\underline{r} = \underline{X} + \underline{R}\underline{d} = \underline{X} + \underline{R}\underline{u}_o + \underline{R}\underline{N}\underline{q}_f. \tag{11}$$

2.3. Energy equations

The velocity vector comes from Eq. (11) by taking the differentiation respect to time,

$$\dot{\underline{r}} = \dot{\underline{X}} + \dot{\underline{R}}\underline{d} + \underline{R}\dot{\underline{d}} = \dot{\underline{X}} + \dot{\underline{R}}\underline{d} + \underline{R}\underline{N}\dot{\underline{q}}_f, \tag{12}$$

and the second term can be written in the form of $\underline{R}_\theta \underline{d}^i \dot{\theta}$ by the chain rule $\dot{\underline{R}} = \underline{R}_\theta \dot{\theta}$. Therefore,

$$\dot{\underline{r}} = \dot{\underline{X}} + \underline{H}\dot{\theta} + \underline{R}\underline{N}\dot{\underline{q}}_f, \tag{13}$$

where $\underline{H} = \underline{R}_\theta \underline{d}$. In partitioned form, the Eq. (12) is $\dot{\underline{r}} = \begin{bmatrix} \underline{I} & \underline{H} & \underline{R}\underline{N} \end{bmatrix} \begin{bmatrix} \dot{\underline{X}}^T & \dot{\theta}^T & \dot{\underline{q}}_f^T \end{bmatrix}^T$.

If the body has n elements, the kinetic energy of the body with the elemental volume V^e and the elemental density ρ^e is

$$KE = \sum_{e=1}^{n} \frac{1}{2} \int_{V^e} \rho^e \dot{\underline{r}}^T \dot{\underline{r}} dV^e. \tag{14}$$

The substitution of Eq. (13) into Eq. (14) and the denotation $\dot{\underline{q}} = \begin{bmatrix} \dot{\underline{X}}^T & \dot{\theta}^T & \dot{\underline{q}}_f^T \end{bmatrix}^T$ result into the kinetic energy expression as

$$KE = \frac{1}{2} \dot{\underline{q}}^T M(\underline{q}) \dot{\underline{q}}, \tag{15}$$

where

$$M(\underline{q}) = \sum_{e=1}^{n} \frac{1}{2} \int_{V^e} \rho^e \begin{bmatrix} I & \underline{H} & \underline{R}\underline{N} \\ \underline{H}^T & \underline{H}^T \underline{H} & \underline{H}^T \underline{R}\underline{N} \\ (\underline{R}\underline{N})^T & (\underline{R}\underline{N})^T \underline{H} & \underline{N}^T \underline{N} \end{bmatrix} dV^e = \sum_{e=1}^{n} \begin{bmatrix} m_{RR}(\underline{q}) & m_{R\theta}(\underline{q}) & m_{Rf}(\underline{q}) \\ m_{\theta R}(\underline{q}) & m_{\theta\theta}(\underline{q}) & m_{\theta f}(\underline{q}) \\ m_{fR}(\underline{q}) & m_{f\theta}(\underline{q}) & m_{ff}(\underline{q}) \end{bmatrix}. \tag{16}$$

Furthermore, by substitution of Eq. (9-a), the kinetic energy expression in Eq. (15) becomes

$$KE = \frac{1}{2} \begin{bmatrix} \dot{\underline{q}}_r \\ \dot{\underline{q}}_f \end{bmatrix}^T \begin{bmatrix} m_{rr}(\underline{q}) & m_{rf}(\underline{q}) \\ m_{fr}(\underline{q}) & m_{ff}(\underline{q}) \end{bmatrix} \begin{bmatrix} \dot{\underline{q}}_r \\ \dot{\underline{q}}_f \end{bmatrix}. \tag{17}$$

The strain energy expression is

$$SE = \sum_{e=1}^{n} \frac{1}{2} \int_{V^e} \underline{\varepsilon}^T E \underline{\varepsilon} dV^e. \tag{18}$$

The substitution of Eq. (7) and (8) into Eq. (18) yeilds

$$SE = \frac{1}{2} \underline{q}_f^T (\underline{K}_l + \underline{K}_{nl}) \underline{q}_f = \frac{1}{2} \underline{q}_f^T \underline{K} \underline{q}_f, \tag{19}$$

where the total stiffness matrix $\underline{K}$ is the sum of linear stiffness $\underline{K}_l$ and nonlinear stiffness $\underline{K}_{nl}$ accounted for the large displacements.

2.4 The system equation of motion

According to Hamilton principle, [6], [7]

$$\delta \int_{t_i}^{t_f} (SE - KE) dt = \int_{t_i}^{t_f} \delta W_{ext} dt - \left[\int_{V^e} \hat{\underline{p}}^T \delta \underline{q} dV^e \right]_{t_i}^{t_f}, \tag{20}$$

where $\left[\int_{V^e} \hat{\underline{p}}^T \delta \underline{q} dV^e \right]_{t_i}^{t_f}$ is the virtual work done by the externally applied momenta and it vanish at the initial and final times. The virtual work done by the externally applied loads $\hat{\underline{t}}$ over the elemental surface dS^e and body force $\underline{b}$ is

$$\delta W_{ext} = \int_{t_i}^{t_f} \left\{ \int_V \underline{b}^T \delta \underline{q} dV + \int_S \underline{\hat{t}}^T \delta \underline{q} dS^e \right\} dt = -\int_{t_i}^{t_f} \delta \underline{\Phi}\left(\underline{q}\right) dt = \int_{t_i}^{t_f} \underline{Q}^T \delta \underline{q} dt , \tag{21}$$

where $\underline{\Phi}\left(\underline{q}\right)$ is the total potential of the applied loads and $\underline{Q}$ the generalized external force.

The Eq. (20) is rewritten as

$$\delta \int_{t_i}^{t_f} \left(SE - KE + \underline{\Phi}\right) dt = 0 \tag{22}$$

$$\delta \int_{t_i}^{t_f} \left(\frac{1}{2} \underline{\dot{q}}^T M\left(\underline{q}\right) \underline{\dot{q}} - \frac{1}{2} \underline{q_f}^T \underline{K} \underline{q_f} + \underline{q}^T \underline{Q} + \underline{\Phi}^T \left(\underline{q}\right) \underline{\lambda} \right) dt = 0, \tag{23}$$

where the last term is caused by the constrained forces which is described by constraint conditions $\underline{\Phi}\left(\underline{q}\right) = \underline{0}$ and Lagrange multiplier vector $\underline{\lambda}$.

Performing the variation operation of Eq. (23),

$$\delta \int_{t_i}^{t_f} \left\{ \frac{\partial}{\partial \underline{q}} \left(\frac{1}{2} \underline{\dot{q}}^T M\left(\underline{q}\right) \underline{\dot{q}} \right) \delta \underline{q} + M\left(\underline{q}\right) \underline{\dot{q}} \delta \underline{\dot{q}} - \underline{K} \underline{q_f} \delta \underline{q_f} + \underline{Q} \delta \underline{q} + \frac{\partial \underline{\Phi}^T}{\partial \underline{q}} \lambda \delta \underline{q} \right\} dt = 0 \tag{24}$$

After some substitutions,

$$\int_{t_i}^{t_f} \left(M\left(\underline{q}\right) \underline{\ddot{q}} + \underline{K} \underline{q_f} \right) \delta \underline{q} dt = \int_{t_i}^{t_f} \left\{ \underline{Q} - \dot{M}\left(\underline{q}\right) \underline{\dot{q}} + \frac{\partial}{\partial \underline{q}} \left(\frac{1}{2} \underline{\dot{q}}^T M\left(\underline{q}\right) \underline{\dot{q}} \right) + \underline{\Phi}_q^T \left(\underline{q}\right) \underline{\lambda} \right\} \delta \underline{q} dt , \tag{25}$$

where for arbitrary $\delta \underline{q}$ states that

$$M\left(\underline{q}\right) \underline{\ddot{q}} + \underline{K} \underline{q_f} = \underline{Q} - \dot{M}\left(\underline{q}\right) \underline{\dot{q}} + \frac{\partial}{\partial \underline{q}} \left(\frac{1}{2} \underline{\dot{q}}^T M\left(\underline{q}\right) \underline{\dot{q}} \right) + \underline{\Phi}_q^T \left(\underline{q}\right) \underline{\lambda} . \tag{26}$$

The second and third term, denoted by the force vector $\underline{F}$, contain the Coriolis components and the Gyroscopic force since they include the quadratic form of the velocity. Finally, the equation of motion can be rewritten as

$$\begin{bmatrix} m_{rr} & m_{rf} \\ m_{fr} & m_{ff} \end{bmatrix} \begin{Bmatrix} \underline{\ddot{q}_r} \\ \underline{\ddot{q}_f} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \underline{K} \end{bmatrix} \begin{Bmatrix} \underline{q_r} \\ \underline{q_f} \end{Bmatrix} = \begin{Bmatrix} \underline{Q_r} \\ \underline{Q_f} \end{Bmatrix} + \begin{Bmatrix} \underline{F_r} \\ \underline{F_f} \end{Bmatrix} + \begin{Bmatrix} \underline{\Phi}_{qr}^T \\ \underline{\Phi}_{qf}^T \end{Bmatrix} \underline{\lambda} . \tag{27}$$

3. Two Dimensional Timoshenko Beam Element

As shown in Fig. 3, the relationship between the physical coordinate x and the natural ξ with its Jacobian J in the longitudinal direction of a 2D Timoshenko beam element are

$$\xi = \frac{x}{l}; \; J = \frac{dx}{d\xi} = l . \tag{28}$$

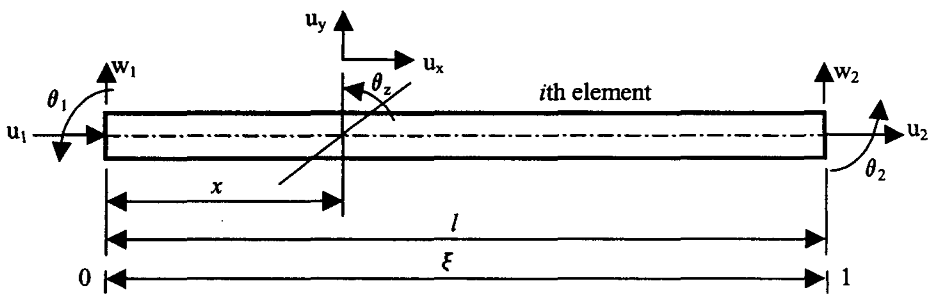

Fig. 3 The element configuration of a 2D Timoshenko beam element

The deformation of any point in the neutral axis of the ith element is

$$\underline{u}_f^{\,e} = \begin{bmatrix} u_x & u_y & \theta_z \end{bmatrix}^T = \underline{N}\underline{q}_e^{\,e}, \tag{29}$$

where the notation $(\bullet)^e$ is the quantity related to the ith element and $q_e^{\,e}$ is the element coordinate which defined as $\underline{q}_e = [u_1 \quad w_1 \quad \theta_1 \quad u_2 \quad w_2 \quad \theta_2]^T$. The shape function $\underline{N}$ is given as

$$\underline{N}^i = \begin{bmatrix} \underline{N}_1 \\ \underline{N}_2 \\ \underline{N}_3 \end{bmatrix} = \begin{bmatrix} 1-\xi & 0 & 0 & \xi & 0 & 0 \\ 0 & 1-\xi & 0 & 0 & \xi & 0 \\ 0 & 0 & 1-\xi & 0 & 0 & \xi \end{bmatrix}. \tag{30}$$

As depicted in Fig. 4, in the planar case the relationship between the reference configuration $[x \quad y \quad \theta]^T$ and final configuration $[x^* \quad y^* \quad \theta^*]^T$ is

$$\begin{Bmatrix} x \\ y \\ \theta \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x^* \\ y^* \\ \theta^* \end{Bmatrix} = \underline{R} \begin{Bmatrix} x^* \\ y^* \\ \theta^* \end{Bmatrix}; \quad \underline{R}_\theta = \begin{bmatrix} -\sin\theta & -\cos\theta & 0 \\ \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{(31-a), (31-b)}$$

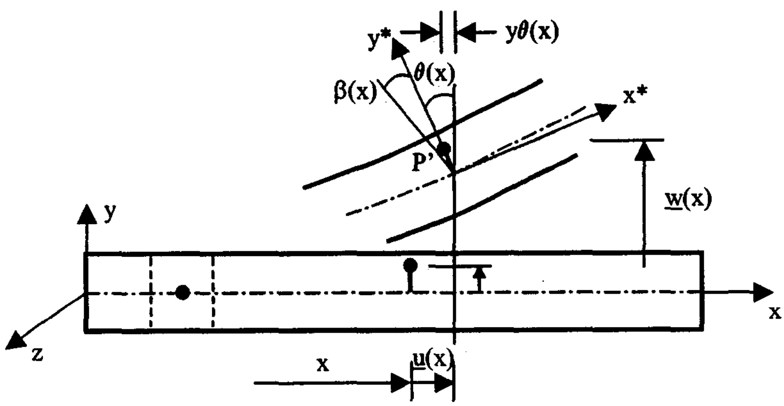

Fig. 4 The relationship between reference and final configuration

From Eq. (13) and (31-b),

$$\underline{H} = \underline{R}_\theta \underline{d} = \underline{R}_\theta \left(\underline{u}_o + \underline{N}\underline{q}_f \right), \tag{32}$$

where $\underline{u}_o = \begin{bmatrix} u_{ox} & u_{oy} \end{bmatrix}^T$ defines the undeformed position of the element and assume $\underline{q}_o = \begin{bmatrix} x_i & x_j \end{bmatrix}^T$, then

$$\underline{u}_o = \begin{bmatrix} 1-\xi & \xi \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ x_j \end{bmatrix} = \underline{N}_o \underline{q}_o. \tag{33}$$

In the condition of Fig. 4, the total strain energy of the beam is

$$SE = \frac{1}{2} \int_{V^e} \left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_{xy} \gamma_{xy} \right) dV^e = \frac{1}{2} \int_{V^e} \left(E\varepsilon_x^{\,2} + kG\gamma_{xy}^2 \right) dV^e \tag{34}$$

And in the figure also be shown that

$$\underline{u}_x(x,y) = u(x) - y\theta(x); \tag{35}$$

$$\underline{u}_y(x,y) = w(x,y) = w(x), \tag{36}$$

where $\underline{u}_x$ and $\underline{u}_y$ are due to axial load and shearing. $\theta(x)$ is the rotation due to the pure bending.

According to the Eq. (7), the relationship between strain and stress,

$$\varepsilon_x = \left(\frac{\partial u}{\partial x} - y\frac{\partial \theta}{\partial x}\right) + \frac{1}{2}\left\{\left(\frac{\partial u}{\partial x} - y\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x} - \frac{\partial y}{\partial x}\right)^2\right\}; \tag{37}$$

$$\gamma_{xy} = \left(-\theta + \frac{\partial w}{\partial x}\right) + \left(\frac{\partial u}{\partial x} - y\frac{\partial \theta}{\partial x}\right)(-\theta). \tag{38}$$

where ε_x consists of ε_a and ε_b which caused by axial load and shearing, respectively. The strain energy now is

$$SE = \frac{1}{2}\int_{V^e} E\varepsilon_a^2 dV^e + \frac{1}{2}\int_{V^e} E\varepsilon_b^2 dV^e + \frac{1}{2}\int_{V^e} EG\gamma_{xy}^2 dV^e . \tag{39}$$

Here only take the linear case into account, therefore

$$\begin{aligned} SE &= \frac{1}{2}EA\int_0^l \left(\frac{\partial u}{\partial x}\right)^2 dx + \frac{1}{2}EI\int_0^l \left(\frac{\partial \theta}{\partial x}\right)^2 dx + \frac{1}{2}kGA\int_0^l \left(\frac{\partial w}{\partial x} - \theta\right)^2 dx \\ &= \frac{1}{2}\underline{q}_f^{\,T}\left(\underline{K}_{Axial} + \underline{K}_{Bending} + \underline{K}_{Shear}\right)\underline{q}_f \end{aligned} \tag{40}$$

After some arrangements,

$$\begin{aligned} \underline{K}_{Axial} &= \frac{1}{2}EA\int_0^1 \frac{1}{J^2}\underline{N}_{1,\xi}^{\,T}\underline{N}_{1,\xi} J d\xi; \\ \underline{K}_{Bending} &= EI\int_0^1 \frac{1}{J^2}\underline{N}_{3,\xi}^{\,T}\underline{N}_{3,\xi} J d\xi; \\ \underline{K}_{Shear} &= kGA\int_0^1 \left(\frac{1}{J}\underline{N}_{2,\xi} - \underline{N}_3\right)^T \left(\frac{1}{J}\underline{N}_{2,\xi} - \underline{N}_3\right) J d\xi \end{aligned} \tag{41}$$

Combining these three terms, the stiffness matrix can be got as follow.

$$\underline{K} = \frac{EI}{l}\begin{bmatrix} \frac{A}{I} & 0 & 0 & -\frac{A}{I} & 0 & 0 \\ 0 & \frac{kGA}{EI} & \frac{l}{2}\frac{kGA}{EI} & 0 & -\frac{kGA}{EI} & \frac{l}{2}\frac{kGA}{EI} \\ 0 & \frac{l}{2}\frac{kGA}{EI} & \frac{l^2}{3}\frac{kGA}{EI}+1 & 0 & -\frac{l}{2}\frac{kGA}{EI} & \frac{l^2}{6}\frac{kGA}{EI}-1 \\ -\frac{A}{I} & 0 & 0 & \frac{A}{I} & 0 & 0 \\ 0 & -\frac{kGA}{EI} & -\frac{l}{2}\frac{kGA}{EI} & 0 & \frac{kGA}{EI} & -\frac{l}{2}\frac{kGA}{EI} \\ 0 & \frac{l}{2}\frac{kGA}{EI} & \frac{l^2}{6}\frac{kGA}{EI}-1 & 0 & -\frac{l}{2}\frac{kGA}{EI} & \frac{l^2}{3}\frac{kGA}{EI}+1 \end{bmatrix} \tag{42}$$

Introducing Eq. (32), (39) into Eq. (27) yields the equation of motion in the global configuration.

It should be noticed here, in the generalized force $\underline{Q} = \left[\underline{Q}_r^T \quad \underline{Q}_f^T\right]^T$, the time field of $\underline{Q}_r$ is from inertial time t_i to final time t_f and of $\underline{Q}_f$ is from t_i to ∞. And

$$\left\{\underline{Q}_f\right\}_{local} = \int_0^l f\underline{N}_1^T \underline{q}_f dx + h\left[\underline{N}_1^T \underline{q}_f\right]_{\xi=\xi_h} + \int_0^l p\underline{N}_2^T \underline{q}_f dx + Q\left[\underline{N}_2^T \underline{q}_f\right]_{\xi=\xi_Q} + M\left[\underline{N}_3^T \underline{q}_f\right]_{\xi=\xi_M} \quad (43)$$

where f is the longitudinal force per unit length, h the longitudinal point force, p the transverse force per unit length, Q the transverse point force, M the point moment. And, ξ_h, ξ_Q, ξ_M are the acted position of p, Q and M, respectively.

In the process of conversion of the coordinates, $\underline{Q}_f$ in the global configuration will be

$$\left\{\underline{Q}_f\right\}_{globle} = \underline{\mathbf{R}}\left\{\underline{Q}_f\right\}_{local}, \quad (44)$$

where $\underline{\mathbf{R}}$ here extends to be $\underline{\mathbf{R}} = \begin{bmatrix} \underline{R} & \underline{0} \\ \underline{0} & \underline{R} \end{bmatrix}_{6\times 6}$. $\underline{Q}_r$ will not change because it is not related to the nodal point, but is applied at the mass center of the element.

4. Conclusion

This paper derived the finite element equation of motion for general flexible Timoshenko beam. It provided an alternative way to formulate the Coriolis and centrifugal effects, as the other inertia effects due to the motion. In the computation of stiffness matrix, the nonlinearities are omitted for simplifying the problem and will be added in future for more in-depth study. In addition, a set of numerical algorism will be developed to evaluate the efficiency of the proposed formulation. Further, the analysis for 3D beam will be extended.

Reference

1. Stephen P. Timoshenko, J. N. Goodier, *Theory of Elasticity 3rd edition*, McGraw Hill College Div; December 1970, pp.15~86

2. M. Schanz and H. Antes, "A Boundary Integral Formulation for the Dynamic Behavior of a Timoshenko Beam," *Electronic Journal of Boundary Elements*, Vol. BETEQ 2001, No. 3, pp.348~359

3. Simo J.C., Vu-Quoc, L. "On the dynamics of flexible beams under large overall motions", *Journal of applied mechanics*, Vol. 53, 1986, pp.849~863

4. J.L. Meriam, L.G. Kraige, *Engineering Mechanics Dynamics. Forth edition*, John Wiley & Sons, Inc. 1998, p.395, p.581

5. Olivier A. Bauchau, *Flexible Multibody Dynamics*, April 2002, Chp.11

6. Golo, G., V. Talasila, A.J. van der Schaft, "A Hamiltonian formulation of the Timoshenko beam model," *Proc. of Mechatronics 2002*, University of Twente, June 2002, pp.24~26

7. A. Macchelli and C.Melchiorri, *Distributed Port Hamiltonian Formulation of the Timoshenko Beam: Modeling and Control*, University of Bologna-DEIS, Italy, 2003