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Calculation of Characteristics for Electromagnetic Waves Scattering in Discrete Non-uniform Media

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Abstract

Signals of the short wave part of centimetre, millimetre and optic wave length ranges are being broadly used in the communication, location and remote sensing systems with space channels. In this case the presence of discrete non-uniform mediums like orbital debris, space dust and other discrete formations in the propagation channel may have substantial influence upon the characteristics of wave processes, and thus upon the data system quality. Mathematical models for studying the discrete non-uniform mediums effect on the characteristics of electromagnetic wave propagation are analyzed in this paper.

1. INTRODUCTION

It is known [1] that the task of studying the electromagnetic wave propagation and scattering in the random discrete non-uniform media is in general statement solved by two steps: first of all it is necessary to analyse the characteristics of scattering and absorption for a single particle and then to study the characteristics of the wave process at a presence of a large number of particles, distributed arbitrary in space. The complexity of solving this task even in particular cases leads to a necessity to use the numerical methods for obtaining the final quantitative result.

That is why an aim was set to develop the multi-purpose mathematical models, procedures and software complex realizing them, which would allow to calculate the characteristics of propagation in the near space for both optic waves and radio waves at a presence of formations consisting of the random discrete non-uniform media and having arbitrary density and arbitrary dimensions, permittivity and permeability for separate particles.

2. ELECTROMAGNETIC WAVE SCATTERING AND ABSORPTION BY A SINGLE PARTICLE

The back scattering cross-section σ_b , total scattering cross-section σ_s , absorption cross-section σ_a and attenuation cross-section $\sigma_t = \sigma_s + \sigma_a$ are usually used for describing the scattering and absorbing properties of a single particle. These characteristics may be calculated using the mathematical methods, known from the diffraction theory [1-4]: Rayleigh approximation, Born (Rayleigh-Debye) approximation, Wentzel-Kramers-Brillouin approximation and Mie theory.

The Mie theory, which gives a general rigorous analytic solution for a uniform spherical particle by factoring the fields into elementary spherical functions, is the universal one. Application of Mie theory for a particle with complex relative dielectric permeability $\dot{\varepsilon} = \dot{\varepsilon}' - i\dot{\varepsilon}''$ and complex relative magnetic permittivity $\dot{\mu} = \dot{\mu}' - i\dot{\mu}''$ allows to obtain characteristics in the form of the following infinite sums:

$$\sigma_b = \frac{\pi R^2}{\alpha^2}\left|\sum_{n=1}^{\infty}(2n+1)(-1)^n\left(\dot{a}_n - \dot{b}_n\right)\right|^2,$$

(1)

$$\sigma_s = \frac{2\pi R^2}{\alpha^2}\sum_{n=1}^{\infty}(2n+1)\left(\left|\dot{a}_n\right|^2 + \left|\dot{b}_n\right|^2\right),$$

(2)

$$\sigma_t = \frac{2\pi R^2}{\alpha^2}\sum_{n=1}^{\infty}(2n+1)\mathrm{Re}\left(\dot{a}_n + \dot{b}_n\right)$$

(3)

where R – particle radius, $\alpha = 2\pi R/\lambda$, λ – wave length in vacuum,

$$\dot{a}_n = \frac{\dot{\mu}\psi_n(\alpha)\psi_n'(\beta) - \sqrt{\dot{\mu}\dot{\varepsilon}}\psi_n(\beta)\psi_n'(\alpha)}{\dot{\mu}\xi_n(\alpha)\psi_n'(\beta) - \sqrt{\dot{\mu}\dot{\varepsilon}}\psi_n(\beta)\xi_n'(\alpha)},$$

(4)

$$\dot{b}_n = \frac{\sqrt{\dot{\mu}\dot{\varepsilon}}\psi_n(\alpha)\psi_n'(\beta) - \dot{\mu}\psi_n(\beta)\psi_n'(\alpha)}{\sqrt{\dot{\mu}\dot{\varepsilon}}\xi_n(\alpha)\psi_n'(\beta) - \dot{\mu}\psi_n(\beta)\xi_n'(\alpha)},$$

(5)

$$\psi_n(x) = \sqrt{\pi x/2}\,J_{n+1/2}(x),$$

$$\xi_n(x) = \sqrt{\pi x/2}\,H_{n+1/2}^{(1)}(x), \qquad (6)$$

$$\beta = \sqrt{\dot{\mu}\dot{\varepsilon}}\,\alpha,$$

$J_{n+1/2}(x)$, $H_{n+1/2}^{(1)}(x)$ – Bessel and Hankel functions of the first kind of half-integer indexes.

Relations (1) – (3) require substantial time for computation at numerical summation. That is why for small ($\alpha \ll 1$) and large ($\alpha \gg 1$) particles it is advisable to use asymptotic approximations.

In the first case ($\alpha \ll 1$, $\left|\sqrt{\dot{\mu}\dot{\varepsilon}}\right| \cdot \alpha \ll 1$) it is sufficient to use two first members of the Mie series, which finally results in the following relations:

$$\sigma_b = 4\pi R^2\alpha^4\left|\frac{\dot{\varepsilon}-1}{\dot{\varepsilon}+2} - \frac{\dot{\mu}-1}{\dot{\mu}+2}\right|^2,$$

(7)

$$\sigma_s = \frac{8}{3}\pi R^2\alpha^4\left(\left|\frac{\dot{\varepsilon}-1}{\dot{\varepsilon}+2}\right|^2 + \left|\frac{\dot{\mu}-1}{\dot{\mu}+2}\right|^2\right).$$

(8)

$$\sigma_t = 12\pi R^2\alpha\left(\frac{\varepsilon''}{(\varepsilon'+2)^2 + (\varepsilon'')^2} + \frac{\mu''}{(\mu'+2)^2 + (\mu'')^2}\right), \quad (9)$$

which at $\dot{\mu} = 1$ correspond to the classical formula of Rayleigh approximation [1].

In the second case ($\alpha \gg 1$, $\left|\sqrt{\dot{\mu}\dot{\varepsilon}}\right| \cdot \alpha \gg 1$) the computations may be simplified by using the asymptotic approximations for functions (6):

$$\psi_n(x) = \cos(x + (n+1)\pi/2),$$

$$\xi_n(x) = (-1)^{n+1}e^{ix},$$

which gives the following expressions for the coefficients (4) and (5), used in (1) – (3):

$$\dot{a}_n = i^{n+1}e^{-i\alpha}\frac{\dot{\mu}\cos x\,tgy - \sqrt{\dot{\mu}\dot{\varepsilon}}\sin x}{\dot{\mu}\,tgy + i\sqrt{\dot{\mu}\dot{\varepsilon}}},$$

(10)

$$\dot{b}_n = i^{n+1}e^{-i\alpha}\frac{\sqrt{\dot{\mu}\dot{\varepsilon}}\cos x\,tgy - \dot{\mu}\sin x}{\sqrt{\dot{\mu}\dot{\varepsilon}}\,tgy + i\dot{\mu}},$$

(11)

where

$$x = \alpha - (n+1)\pi/2, \quad y = \beta - (n+1)\pi/2. \quad (12)$$

Thus, the relations (1) - (12) comprise a mathematical model for calculating the scattering and absorption characteristics for a spherical particle with arbitrary radius and electro physical characteristics and generalize the classical results [1-3] for the case $\mu\neq 1$. The software for calculating the characteristics of electromagnetic wave interaction with single particle is developed on the basis of the model presented.

Scattering and absorption characteristics for a single spherical particle as functions of its relative dimension α , obtained using the developed software, are presented in Figures 1, 2 and 3 as an example for

$\varepsilon' = 10, \varepsilon'' = 0.9, \mu' = 1 \text{ and } 10, \mu'' = 0$.

Apparently from the schedules resulted on the Figure 1, for the spherical particle having dielectric with losses properties, dependence of coefficients of scattering and absorption on parameter α has classical character. At occurrence in a particle of magnetic properties renders essential influence on characteristics of scattering and absorption (Figure 2).

The most essential changes occur to coefficient of back scattering. Apparently from the Figure 3 practically for all option values an alpha the level of the back scattering coefficient at μ'=10 on 25 … 30 dB is less, than in case of not magnetic material.

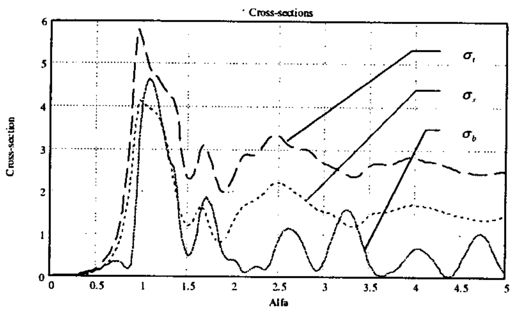

Figure 1

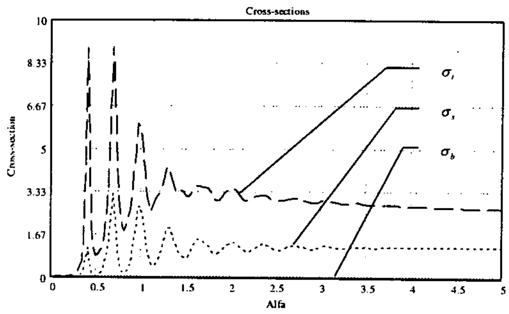

Figure 2

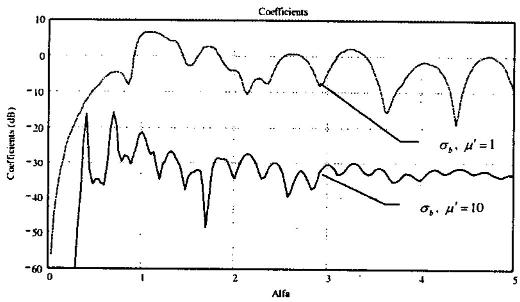

Figure 3

3. ELECTROMAGNETIC WAVE PROPAGATION AND SCATTERING IN THE LAYER

A rigorous statistic wave description of signal propagation through the random discrete media is given by a theory of multiple scattering, one of the variants of which is developed by V.Twersky [1].

But the integral equation of the theory of multiple scattering does not have a general analytic solution and numerical procedures a rather bulky, even for the modern computing devices. That is why the first approximation of the theory of multiple scattering [1] is used as a mathematical instrument in the present paper. A model for the formation of spherical particle in the form of linear layer of thickness L, located in the far zone relative to the emission source, was chosen for studies. Under

Figure 4

these conditions the intensity of direct signal, having passed through the layer of constant density ρ is defined by the expression:

$$I_f = e^{-\gamma} + 2\pi \int_0^{\pi/2} \frac{|f(\theta)|^2}{\sigma_t} \frac{e^{-\gamma} - e^{-\gamma/\cos\theta}}{1-\cos\gamma} \sin\theta d\theta, \quad (13)$$

where $\gamma = \rho\sigma_t L$ - attenuation coefficient, θ - angle of scattering measured from the normal line to the layer boundary, $f(\theta)$ - complex amplitude of a signal scattered by a single particle.

In general case the integration in (13) is made numerically, and asymptotic approximations are used for small and large particles.

The signal intensity for the back scattering from the layer under the studied conditions is defined by the following formula:

$$I_b = (1-e^{-2\gamma})\sigma_b/(8\pi\sigma_t). \quad (16)$$

The "visibility" coefficient is a relation of powers for the back scattered signal, received by the location system in the case of absence and presence of the random discrete non-uniform layer:

$$K_v = I_f^2 + I_b S/\sigma_0, \quad (14)$$

where S is the layer surface area and σ_0 is the back scattering cross-section for the object being located.

Figure 4 for $\lambda = 3\ mm$, $L = 1\ m$ shows an example for the dependences of the energy characteristics of signals vs R, calculated by the software realizing the model (13) - (14) ($\varepsilon' = 10$, $\varepsilon'' = 0.9$, $\mu' = 1$, $\mu'' = 0$).

4. CONCLUSION

Calculation of power characteristics of the radio signal extending in discrete non-uniform environments is great importance at designing radar and communication systems. The developed mathematical models, procedures and software may be used for studying the influence of discrete non-uniform mediums on the quality indexes for the location and communication systems and for designing the systems for the near space remote sensing.

The results submitted in the article allow to draw a conclusion, that use of discrete environments with magnetic properties renders essential influence on target characteristics and can be used at the decision a many practical problems.

References

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