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시간 지연을 이용한 연속시간 TS 퍼지 시스템의 카오스화

김택룡*, 주영훈**, 박진배*
* 연세대학교 전기전자공학과 ** 군산대학교 정보전자공학부

Chaotifying a Continuous-Time TS Fuzzy System with Time-Delay

Taek-Ryong Kim*, Young-Hoon Joo**, Jin-Bae Park*
*Dept. of Electrical & Electronic Eng., Yonsei Univ. **School of Electronic & Information Eng., Kunsan Univ.

Abstract - In this paper, a systematic design approach based on the parallel distributed compensation technique is proposed for chaotifying a general continuous-time Takagi-Sugeno (TS) fuzzy system. The fuzzy parallel distributed compensation controller (FPDCC) is composed of the feedback gain and time-delay feedback. The verification of chaos in the controlled continuous-time TS fuzzy system is done by the following procedures. First, we establish an asymptotically approximate relationship between a time-delay continuous-time TS fuzzy system and a discrete-time TS fuzzy system. Then, Marotto theorem is applied. Therefore, the generated chaos is in the sense of Li and Yorke. The boundedness in the controlled continuous-time TS fuzzy system is also proven via its associated discrete-time TS fuzzy system.

1. Introduction

The "chaotification" or "anticontrol of chaos" means making a nonchaotic system chaotic or keeping existing chaos of a chaotic system. We propose the chaotification method for a stable continuous-time TS fuzzy system. Specifically, the TS fuzzy system is composed of the fuzzy rules, which characterize local relations of the system in the state space. The local relations are sometimes called the subsystem of the TS fuzzy system. We use the theorem proposed in [1], which establish an asymptotically approximate relationship between delayed differential equation and difference equation, to establish an asymptotically approximate relationship between time delay continuous-time TS fuzzy system and discrete-time TS fuzzy system. From this, we can verify the chaos in the controlled continuous-time TS fuzzy system by applying Marotto theorem to its associated discrete-time TS fuzzy system.

The fuzzy parallel distributed compensation controller (FPDCC) is designed with the feedback gain and time-delay feedback. The feedback gain is obtained from linear matrix inequality (LMI) method. For the time-delay feedback, we use a simple sinusoid functions which was used in [2], [3] and [4]. In fact, the folding and stretching effect of this function and time-delay play a major role in making the controlled TS fuzzy system chaotic.

This paper is organized as follows. Section 2 briefly reviews the continuous-time TS fuzzy system and an asymptotically approximate relationship between the delayed differential equation and the difference equation established in [1]. The FPDCC design for chaotifying a stable continuous-time TS fuzzy system, an asymptotically approximate relationship between the time-delay continuous-time TS fuzzy system and discrete-time TS fuzzy system and the verification of chaos in the controlled TS fuzzy system is presented in Sections 3, 4 and 5 respectively. Some conclusions are finally given in Section 6.

2. Preliminary

2.1 The continuous-time TS fuzzy model

A single input TS fuzzy system is described as follows:

Plant Rule i:

IF $z_1(t)$ is M_{i1} $\cdots$ and $z_n(t)$ is M_{in},

THEN $\dot{x}(t) = A_i x(t) + B_i u(t)$ (1)

The final output of the fuzzy system is inferred by

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i \{A_i x(t) + B_i u(t)\} \quad (2)$$

in which μ_i can be regarded as the firing strength of the IF-THEN rules.

Throughout this paper, we denote x^j is the j th element of the vector x, T^{ij} is the ij th element of the matrix T, and T^{*j} is the j th column of the matrix T.

2.2 Approximate relationship between the delayed-differential equation and the difference equation

Let us consider an n th order single-input single-output stable linear time-invariant (LTI) system described by phase variable form.

$$\dot{x}(t) = Ex(t) + Dv(t), \qquad v(t) = h(x(t-\tau)^1) \quad (3)$$

where

$$E = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix}, \qquad D = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and $v(t)$ is the time-delay feedback input, $x(t)^1$ is a system output, and $h(\cdot)$ is a continuous scalar function with bounded magnitude.

Since the system (3) is assumed to be a stable system, the system matrix E is a Hurwitz stable matrix. This LTI system can be computed iteratively on each τ-time interval $(m\tau,(m+1)\tau]$ for $m=0,1,\cdots$. Denote $x(t)\equiv x(m\tau+\hat{t})\equiv x(m,\hat{t})$ for $t=m\tau+\hat{t}, \hat{t}\in(0,\tau]$.
It follows that

$$x(m,\hat{t})=e^{E\hat{t}}x(m-1,\tau)+\int_0^{\hat{t}} e^{E(\hat{t}-t')}Dh(x(m-1,t')^1)dt' \quad (4)$$

Lemma 1 [1] For a sufficiently large τ and a large $\hat{t}\in(t_0,\tau]$, (4) can be approximated to the following.

$$x(m,\hat{t})_1\approx a_0^{-1}h(x(m-1,\hat{t})^1), \qquad x(m,\hat{t})^d\approx 0 \quad (5)$$
$$\text{for } m=0,1,\cdots \text{ and } d=2,\cdots,n$$

Lemma 1 establish an asymptotically approximate relationship between the delayed differential Eq.(3) and the difference Eq.(5). This enables us to establish an asymptotically approximate relationship between the time-delay continuous-time TS fuzzy system and the discrete-time TS fuzzy system.

3. The FPDCC Design for Chaotifiying Continuous-Time TS Fuzzy System

For a stable continuous-time TS fuzzy system, the FPDCC is designed to make it chaotic. Each control rule of FPDCC is constructed from the corresponding rule of the TS fuzzy system.

Control Rule i:
IF $z_1(t)$ is M_{i1} $\cdots$ and $z_n(t)$ is M_{in}
THEN $u(t)=-Kx(t)+\nu_i(t)$ (6)

where $\nu_i(t)$ is the time-delay feedback,

$$\nu_i(t)=\sigma\sin\left(\frac{\beta\pi}{\sigma}(T_i^{-1}x(t-\tau))^1\right) \quad (7)$$

Note that $\nu_i(t)$ is bounded inputs. That is $|\nu_i(t)|<\sigma$. T_i is a transformation matrices, which is determined later. The overall fuzzy controller is inferred by

$$u(t)=\sum_{i=1}^{r}\mu_i\{-Kx(t)+v_i(t)\} \quad (8)$$

Substituting (8) into (2) we obtain the closed-loop system

$$\dot{x}=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\{G_ix(t)+B_i\nu_j(t)\} \quad (9)$$

where $G_i=A_i-B_iK$

The equation (8) can be seen the final output of the following continuous-time TS fuzzy system, which has r^2 subsystems.

Rule ij:
IF $z_1(t)$ is M_{i1} and M_{j1} $\cdots$ and $z_n(t)$ is M_{in} and M_{jn}
THEN $\dot{x}(t)=G_ix(t)+B_i\nu_j(t)$ (10)

where G_i is the system matrix of the subsystem of (10).
The feedback gain matrix K is designed to make that the system matrices of each subsystem of (10), G_i, are Hurwitz stable matrices. To this purpose we use LMI method.

Theorem 1 [7] The feedback gain matrix K is designed from the following LMI to make that G_i are Hurwitz stable matrices.

$$QA_i^T-M^TB_i^T+A_iQ-B_iM<0, \qquad i=1,\cdots,r$$
$$Q>0$$

where $Q=P^{-1}$, $M=KQ$,
where P and Q are positive symmetric matrices. K is solved by $K=MP$.

4. An Asymptotically Approximate Relationship between the Time-Delay Continuous-Time TS Fuzzy System and the Discrete-Time TS Fuzzy System

Here, we establish an asymptotically approximate relationship between the time-delay continuous-time TS fuzzy system and the discrete-time TS fuzzy system as the first step to verify chaos in controlled TS fuzzy system by Marotto theorem. We consider the subsystem of the closed-loop TS fuzzy system (10) in the local area corresponding to each rule. Specifically the subsystem of (10) in the *ij-th* local area is the following.

$$\dot{x}(t)=G_ix(t)+B_i\nu_j(t) \quad (11)$$

Define a new state vector $\hat{x}_i$ by

$$x=T_i\hat{x}_i$$

Note that we use the same transformation matrix for same i index.

Then,

$$\dot{\hat{x}}_i(t)=T_i^{-1}G_iT_i\hat{x}_i(t)+T_i^{-1}B_i\nu_j(t)$$
$$=E_i\hat{x}_i(t)+D_i\nu_j(t) \quad (12)$$

where

$$E_i=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_{i0} & -a_{i1} & -a_{i2} & \cdots & -a_{i(n-1)} \end{bmatrix}, D_i=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

Theorem 2 There is an asymptotically approximate relationship between the continuous-time TS fuzzy system with time-delay (11) and the following discrete-time TS fuzzy system if the time-delay τ is a sufficiently large and all element of $(T_i)^{-1}$ are nonzero.

Rule ii $(i=j)$:
IF $z_1(k)$ is M_{i1} and M_{i1} $\cdots$ and $z_n(k)$ is M_{in}, and M_{in},

$$\text{THEN}\quad x(k+1)\approx\begin{bmatrix}\frac{\sigma(T_i)^{11}}{\alpha_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)^{11}}x_1(k)\right)\\ \frac{\sigma(T_i)^{21}}{\alpha_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)^{21}}x_2(k)\right)\\ \vdots\\ \frac{\sigma(T_i)^{n1}}{\alpha_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)^{n1}}x_n(k)\right)\end{bmatrix}.$$

Rule ij $(i\neq j)$:
IF $z_1(k)$ is M_{i1} and M_{j1} $\cdots$ and $z_n(k)$ is M_{in} and M_{jn},

$$\text{THEN}\quad x(k+1)\approx\begin{bmatrix}\frac{\sigma(T_i)^{11}}{\alpha_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{11}}x_1(k)\right)\\ \frac{\sigma(T_i)^{21}}{\alpha_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{21}}x_2(k)\right)\\ \vdots\\ \frac{\sigma(T_i)^{n1}}{\alpha_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{n1}}x_n(k)\right)\end{bmatrix}$$

The final output of this discrete-time TS fuzzy system is inferred by

$$x(k+1)\approx\sum_{i=1}^{r}\mu_i^2\begin{bmatrix}\frac{\sigma(T_i)^{11}}{\alpha_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)^{11}}x_1(k)\right)\\ \frac{\sigma(T_i)^{21}}{\alpha_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)^{21}}x_2(k)\right)\\ \vdots\\ \frac{\sigma(T_i)^{n1}}{\alpha_{i0}}\sin\left(\frac{\beta\pi}{\sigma(T_i)^{n1}}x_n(k)\right)\end{bmatrix}$$
$$+\sum_{i=1}^{r}\sum_{j=1,j\neq i}^{r}\mu_i\mu_j\begin{bmatrix}\frac{\sigma(T_i)^{11}}{\alpha_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{11}}x_1(k)\right)\\ \frac{\sigma(T_i)^{21}}{\alpha_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{21}}x_2(k)\right)\\ \vdots\\ \frac{\sigma(T_i)^{n1}}{\alpha_{i0}}\sin\left(\frac{\beta\pi(T_j^{-1}T_i)^{11}}{\sigma(T_i)^{n1}}x_n(k)\right)\end{bmatrix}$$
$$\equiv F(x) \qquad (13)$$

Proof: The proof is omitted due to lack of space. ■

Theorem 3 (boundedness) The final output of the discrete-time TS fuzzy system (13) is bounded by the following constant.

$$x(k)\leq\begin{bmatrix}\max_{1\leq i\leq n}\left\{\left|\frac{(T_i)^{11}}{\alpha_{i0}}\right|\right\}\sigma\\ \max_{1\leq i\leq n}\left\{\left|\frac{(T_i)^{21}}{\alpha_{i0}}\right|\right\}\sigma\\ \vdots\\ \max_{1\leq i\leq n}\left\{\left|\frac{(T_i)^{n1}}{\alpha_{i0}}\right|\right\}\sigma\end{bmatrix}<\infty,\qquad k=1,2,\cdots$$

Proof: The proof is omitted due to lack of space. ■

5. Verification of Chaos in the Controlled TS Fuzzy System

We show the time-delay continuous-time TS fuzzy system (9) or (10) is chaotic as proving that its associate discrete-time TS fuzzy system (13) is chaotic by Marotto theorem. Therefore the generated chaos is in the sense of Li and Yorke.

Theorem 4 Suppose that $\mu_i(k)\mu_j(k)$, $i,j=1,2,\cdots,r$ are continuously differentiable in the neighborhood of the fixed point, $x^*=0$, of the controlled system (13). Then there exists a positive constant $\bar{\beta}$ such that if $\beta>\bar{\beta}$, then the controlled discrete-time TS fuzzy system (13) is chaotic in the sense of Li and Yorke.
Proof: The proof is omitted due to lack of space. ■

6. Conclusions

A systematic approach for anticontrol of chaos in a general continuous-time TS fuzzy system was developed in this paper. The FPDCC is very simple and effective. To verify the chaos in closed-loop TS fuzzy system, an asymptotically approximate relationship between the time-delay continuous-time TS fuzzy system and the discrete-time TS fuzzy system was derived. Then Marotto theorem was applied. Therefore, the generated chaos is in the sense of Li and Yorke.

Many systems can be represented by TS fuzzy system. Therefore this methodology of making a general continuous-time TS fuzzy system chaotic provides the opportunity to investigate further interaction between fuzzy theory and chaos theory. That has great potential to future engineering.

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