## A POSITION TRACKING ALGORITHM WITH RADAR MEASUREMENT

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#### **ABSTRACT**

This paper describes the remote tracking algorithm using measurements (azimuth, elevation, and slant range) of the radar ground station. Kalman filter model for noise reduction of the measured information is first derived by linearizing with respect to angle, angular rate, range, and range rate. And then a tracking algorithm is introduced to calculate the position of the vehicle during in-flight. The simulation results show that the algorithm is practical and effective enough tracking position of the vehicle in considerably less error.

Keywords: Kalman filter, noise reduction, radar, tracking system, transponder

## 1. INTRODUCTION

Position and trajectory data of in-flight vehicle are important information to determine flight safety of vehicle. In general tracking system, radar and transponder are used to acquire position information. Vehicle position and trajectory can be determined by RF communication between radar ground station and in-flight rocket, and antenna position data.

The radar ground station within range of the vehicle can interrogate the vehicle transponder continuously using a synchronized rapid sequential time-sharing technique. The round-trip transit time of the pulsed signal is measured and used by the radar computer to determine the range of the vehicle. The elevation and azimuth angles of the returning signal are transferred to the radar computers and recorders directly from elevation and azimuth shaft encoders to determine vehicle location.

In this paper, we present a algorithm for tracking position of the vehicle on the radar ground station. This algorithm also includes a noise reduction part through Kalman filter. This paper is organized as follows. In section II, Kalman filter model for noise reduction is derived. Section III introduces the position tracking procedure using the filtered measurements. In section IV, simulation results are depicted including the simulation conditions.

#### 2. FILTER DESIGN FOR NOISE REDUCTION

For the best performance, noise reduction of the measured data is the most important factor in the position tracking system. Many filters are considered for this problem through papers (Lawton et al. 1998, Yao et al. 2002). The filters are an alpha-beta filter, an adaptive filter, a Kalman

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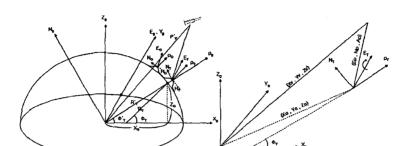


Figure 1. Transformation from Tracker Site to Geocentric Coordinate.

filter, and an extended Kalman filter etc. Because of the optimality for data filtering and the simplicity for implementation, we use the Kalman filter for noise reduction of the radar tracking. The implementation of a Kalman filer, however, requires the statistical model. The dynamic equations of azimuth angle (AZ), elevation angle (EL), and slant range (SR) are derived with a piecewise constant acceleration model and the Kalman filter model is defined as

$$X(k+1) = FX(k) + W(k)$$
  
$$Z(k) = HX(k) + V(k)$$

where state vector  $X(k) = [AZ(k) \ EL(k) \ SR(k) \ AZ(k) \ EL(k) \ SR(k) \ AZ(k) \ EL(k) \ SR(k)]^T$ , and Z(k) is the measurement at time k.  $\delta t$  is the sample interval of measurements. W(k) is the process noise and V(k) is the measurement noise. F and H are like this

$$F = \begin{bmatrix} 1 & 0 & 0 & \delta t & 0 & 0 & \frac{\delta t^2}{2} & 0 & 0 \\ 0 & 1 & 0 & 0 & \delta t & 0 & 0 & \frac{\delta t^2}{2} & 0 \\ 0 & 0 & 1 & 0 & 0 & \delta t & 0 & 0 & \frac{\delta t^2}{2} \\ 0 & 0 & 1 & 0 & 0 & \delta t & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & \delta t & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \delta t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \delta t \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

## 3. POSITION TRACKING ALGORITHM

The geocentric coordinate system  $(N_g, E_g, \rho_g)$  was selected with the vertical axis through the ground station with the East axis through the equator, and with the North axis on the longitudinal plane through the ground station. The tracker site coordinate system data  $(N_T, E_T, \rho_T)$  of the ground station is transformed to geocentric coordinate data by rotating the tracker site data through an angle equal to the tracker site latitude and then translating the tracker site data to the Earth's center. The geocentric coordinate system data may then be transformed to any location on the earth by translation to the earth's surface and then two rotations equal to the offset latitude and the longitude difference between the tracker site and the launcher site  $(N_O, E_O, \rho_O)$  (Oklahoma State Univ. 1981). The relation of these coordinates is shown in Figure 1.

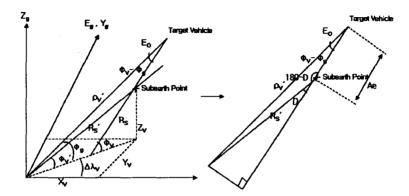


Figure 2. Transformation from Geocentric Coordinate to Geodetic coordinate.

From Figure 1, the offsets from the tracker site to geocentric coordinates are

$$X_o = (R_T' + H_T)\cos\phi_T, \ Z_o = (R_T' + H_T)\sin\phi_T'$$
  
 $R_T' = R_e - 21.39\sin^2\phi_T, \text{ and } \phi_T' = \phi_T - e^2\sin\phi_T\cos\phi_T/(1 - e^2\sin^2\phi_T)$ 

where  $\phi_T$ ,  $\lambda_T$ ,  $H_T$  means the latitude, longitude and altitude of tracker site.  $R_e$  and e means equatorial radius and eccentricity of the earth.

The tracker site rectangular coordinate data are derived form elevation angle, azimuth angel and slant range measured on the tracker site as follows

$$A_P = SR_P \sin EL_P$$
,  $N_P = GR_P \cos AZ_P$ ,  $E_P = GR_P \cos AZ_P$ 

The transformation from tracker site to geocentric coordinates is

$$\left[ \begin{array}{c} X_V \\ Y_V \\ Z_V \end{array} \right] = \left[ \begin{array}{ccc} 0 & -\sin\phi_T & \cos\phi_T \\ 1 & 0 & 0 \\ 0 & \cos\phi_T & \sin\phi_T \end{array} \right] \left[ \begin{array}{c} E_P \\ N_P \\ A_P \end{array} \right] + \left[ \begin{array}{c} X_o \\ Y_o \\ Z_o \end{array} \right],$$

and the transformation matrix of euler angles is calculated from rotation angle  $(\phi_T)$  of  $E_P$  axis like Figure 1.

The latitude and longitude of the subearth point of the vehicle and the mean sea level altitude of the vehicle are useful parameters. The longitude of the vehicle subearth point is from Figure 2.

$$\lambda_V = \lambda_T - \Delta \lambda_V$$

where  $\Delta \lambda_V = \tan^{-1}{(Y_V/X_V)}$  means the longitude difference between tracker site and subearth point of the vehicle.

The latitude of the vehicle cannot be determined directly but must be calculated in an iterative loop. The iteration loop proceeds as follows

$$\begin{array}{ll} Step \ 1: \phi_V = \phi_g & Step \ 2: D = 0 \\ Step \ 3: R_s' = R_e - 21.39 \sin^2 \phi_V & Step \ 4: D = e^2 \sin \phi_V \cos \phi_V / (1 - e^2 \sin^2 \phi_V) \\ Step \ 5: \phi_V = \phi_g + R_s' D / \rho_V' & Step \ 6: \text{goto Step } 3 \end{array}$$

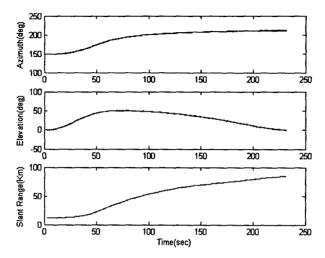


Figure 3. Measured information from radar ground station.

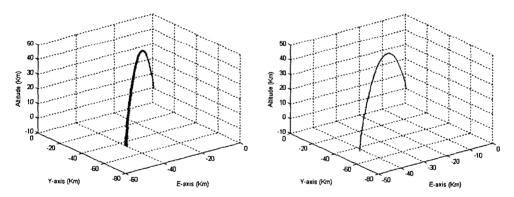


Figure 4. Position of the vehicle using the tracking algorithm.

The vehicle mean sea level altitude is calculated as follows from Figure 2

$$A_e = \rho_V' \cos(\phi_V - \phi_g) - R_s' \cos D$$

The final values to be calculated are the north, east, and ground range distances from the vehicle subearth point to the origin of launcher site. These values are calculated as follows

$$E_{eo} = (\lambda_o - \lambda_V) R'_o \cos \phi'_o$$

$$N_{eo} = \{110.575 + 1.11 \sin^2(\phi_o + \phi_V/2)\} (\phi_V - \phi_o)$$

$$GR_{eo} = \sqrt{E_{eo}^2 + N_{eo}^2}$$

## 4. SIMULATION RESULTS

It is assumed that the target trajectory used for the numerical results is KSR-III trajectory created with WGS-84 earth model. And the origin of the tracker site is located on latitude (36.6890°),

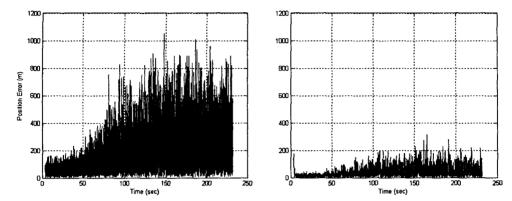


Figure 5. Effects of the noise reduction filter.

longitude (126.3095°) and altitude (17.8 m), and the origin of the launcher site is located on latitude (36.6052°), longitude (126.2374°) and altitude (4.8 m). The angular error on the radar is 3 mrad, and the range error is 25 m. The measurement rate is one measurement every 0.02sec. Figure 3 shows the measurement of the radar.

Figure 4a represents the tracking result without noise reduction. The plots in Figure 5a illustrate that position errors are induced about 800 m by the measurement error. Figure 4b and 5b show the tracking result with noise reduction by the Kalman filter. The position error settles down about 200 m.

#### 5. CONCLUSIONS

This paper deals with a remote tracking algorithm using measurements of the radar ground station. Kalman filter model for noise reduction of the measured information has been established through linearization with respect to angle, angular rate, range, and range rate. And then a tracking algorithm has been introduced to calculate the position of the vehicle during in-flight. By interpreting computer simulation results, the implemented algorithm is practical and effective enough tracking position of the vehicle in considerably less error. This algorithm is useful for real-time tracking or trajectory analysis.

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