Determination of Identifiable Parameters and Selection of Optimum Postures for Calibrating Hexa Slide Manipulators

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Abstract: Kinematic calibration enhances absolute accuracy by compensating for the fabrication tolerances and installation errors. Effectiveness of calibration procedures depends greatly on the measurements performed. While the Cartesian postures are measured completely, all of the geometric parameters can be identified to their true values. With partial pose measurements, however, few geometric parameters may not be identifiable and effectiveness of the calibration results may vary significantly within the workspace. QR decomposition of the identification Jacobian matrix can reveal the non-identifiable parameters. Selecting postures for measurement is also an important issue for efficient calibration procedure. Typically, the condition number of the identification Jacobian is minimized to find optimum postures. This paper investigates identifiable parameters and optimum postures for four different calibration procedures – measuring postures completely with inverse kinematic residuals, measuring postures completely with forward kinematics residuals, measuring only the three position components, and restraining the mobility of the end-effector using a constraint link. The study is performed for a six degree-of-freedom fully parallel HexaSlide type parallel manipulator, HSM. Results verify that all parameters are identifiable with complete posture measurements. For the case of position measurements, one and for the case of constraint link, three parameters were found non-identifiable. Optimal postures showed the same trend of orienting themselves on the boundaries of the search space.

Keywords: Hexa Slide Manipulator, Identification Parameters, Kinematic Calibration, Optimal Postures, Parallel Manipulators

1. INTRODUCTION

Recently, increasing attention has been given to the applications of parallel manipulators in different areas. Parallel manipulators are favored for their high accuracy, increased rigidity, and better speed characteristics. Accuracy of the parallel manipulators, however, can be greatly deteriorated because of inaccurate knowledge geometric parameters resulting from fabrication and assembly errors. Kinematic calibration is therefore required to compute the actual values of the geometric parameters and thus enhance the accuracy. Without calibration, the significance and veridicality of results for experimental robotics cannot be gauged. One may expect to spend most of experimental effort in calibration and less in actually running the experiments in control [1].

Kinematic calibration requires redundant sensory information. This information can be acquired by using external sensors or by adding redundant sensors to the system [2-4], or by restraining the motion of the end-effector through some locking device [5-8]. The last two are referred to as self-calibration or autonomous calibration procedures. In another calibration procedures, the end-effector may be needed to traverse precise trajectories while measurement data is collected [9-12].

Classical methods of calibration require measurement of complete or partial postures of the end-effector using some external measuring devices. Numerous devices have been used for calibration of parallel manipulators. Zhuang et al. [3] used electronic Theodolites for the calibration of the Stewart platform along with standard measuring tapes. For a 3 degree-of-freedom (DOF) redundant parallel robot. Ota et al performed calibration of a parallel machine tool, HexaM, using a Double Ball Bar system [13]. Takeda et al. proposed use of low order Fourier series to calibrate parallel manipulators using Double Ball Bar system [14]. Besnard et al. [4] demonstrated that Gough-Stewart platform could be calibrated using two inclinometers. All of the kinematic parameters can be identified when the Cartesian posture is completely measured. However, measuring all components of the Cartesian posture, particularly the orientation, can be difficult and expensive. With partial pose measurements, experimental procedure is simpler but some of the parameters may not be identified.

Zhuang et al. [3] formulated the cost function in terms of the inverse kinematic residuals that results in block diagonal identification Jacobian matrix and the identification procedure can be implemented without solving forward kinematics. Daney et al. [19] presented variable elimination technique to improve the effectiveness of identification procedure when only partial pose information is available. Khalil et al. [15] presented an algorithm to calculate the identifiable parameters for robots with tree structures. Based on QR analyses of the identification Jacobian matrix, Besnard and Khalil [18] analyzed numerical relations between the identifiable and the non-identifiable parameters for different calibration schemes with case study on the Gough-Stewart platform.

Effectiveness of the calibration procedure depends greatly on the measurements performed. While the Cartesian postures are completely measured (3 translations and 3 rotations), the results of calibration are uniform over the workspace and all of the geometric parameters tend to their actual values. With partial pose measurements, however, few geometric parameters may not be identifiable and effectiveness of the calibration results may vary significantly with in the workspace. QR decomposition of the identification Jacobian matrix can reveal the non-identifiable parameters. Selecting postures for measurement is also an important issue for efficient calibration procedure. Typically, the condition number of the identification Jacobian is minimized to find optimum postures. However, other cost functions that can be employed for minimization are summarized by Daney [20].

This paper investigates non-identifiable parameters and optimum postures for four different calibration procedures of parallel manipulators. The study is performed for a 6 degree-of-freedom (DOF) fully parallel Hexa Slide manipulator, HSM. The four calibration procedures are (i) Postures measured completely and inverse kinematics residuals (ii) Postures measured completely and forward kinematics residuals (iii) Only positions measured and (iv) Mobility of the end-effector restrained by using a constraint link. For optimum postures, the problem was formulated as a constrained optimization problem by specifying limits on the search space where the search space is defined in the Cartesian coordinates.

This paper is organized as follows: Hexa Slide Manipulator is introduced in Section 2. Section 3 discusses identifiable parameters for three different calibration procedures. Optimum postures for calibration of HSM are presented in section 4. Section 5 concludes the study.

2. DESCRIPTION OF THE MECHANISM

The Hexa Slide mechanism, HSM, on which the study is performed, is a 6-degree-of-freedom fully parallel manipulator of PRRS type as shown in figures. Figure 1 shows the identification parameters and figure 2 elaborates the base frame definition. A_{i0} and A_{i1}, in figure 1, denote the start and the end points of the i^{th} (i=1,2,…,6) rail axis. A_i denotes the center of i^{th} universal joint and it lies on the line segment $A_{i0}A_{i1}$. All of the rail axes are identical and the nominal link length ℓ for each leg is equal. The articular variable, λ_i, is the distance between the points A_{i0} and A_i. The point B_i denotes the center of spherical joint at the platform.

Posture of the mobile platform is represented with a position vector of the mobile frame center in the base frame and with three Euler angles as

$$X = \begin{bmatrix} x & y & z & \theta & \psi & \phi \end{bmatrix} \tag{1}$$

The Euler angles are defined as: ψ rotation about the global X-axis, θ rotation about the global Y-axis and ϕ rotation about the rotated local z-axis. Orientation is thus given by: $R = R_{Y,\theta} R_{X,\psi} R_{z,\phi}$

$$R = \begin{bmatrix} C\theta C\phi + S\theta S\phi S\psi & -C\theta S\phi + S\theta S\psi C\phi & S\theta C\psi \\ C\psi S\phi & C\psi C\phi & -S\psi \\ -S\theta C\phi + C\theta S\psi S\phi & S\theta S\phi + C\theta S\psi C\phi & C\theta C\psi \end{bmatrix} \tag{2}$$

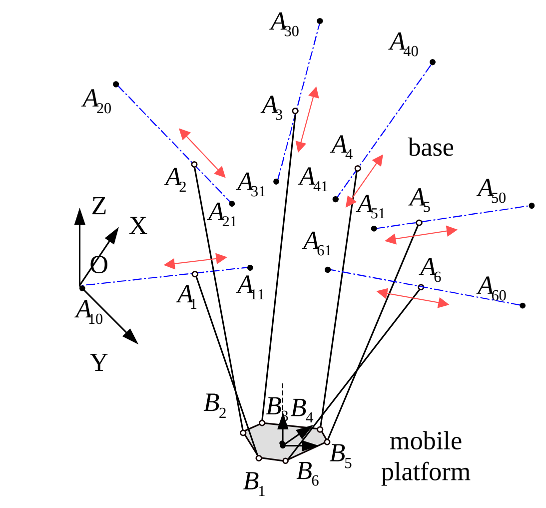


Fig. 1 Schematic of HSM

2.1 The Inverse Kinematics

The problem of inverse kinematics is to compute the articular variables for a given position and orientation of the mobile platform. For the HSM, the problem of inverse kinematics is simple and unique and is solved individually for each kinematic chain. Considering a single link chain, the inverse kinematics relation can be expressed as

$$\lambda = \mathbf{a}^T \mathbf{A}_0\mathbf{B} - \sqrt{\ell^2 - \left\|\mathbf{A}_0\mathbf{B}\right\|^2 + \left(\mathbf{a}^T \mathbf{A}_0\mathbf{B}\right)^2} \tag{3}$$

where **a** is the unit vector along the direction of the rails.

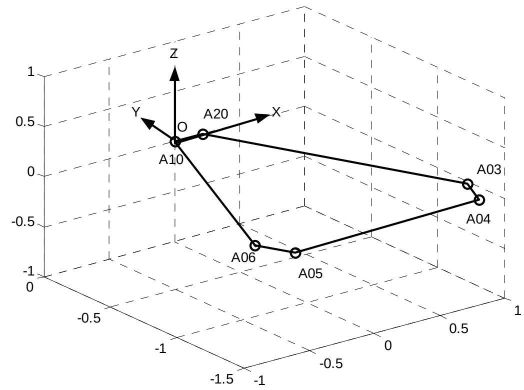


Fig. 2 Global Reference Coordinate System

2.2 The Forward Kinematics

In forward kinematics, the position and orientation of the mobile platform are computed for given values of articular variables. Unlike the inverse kinematics, the problem is difficult and may yield many solutions. Forward kinematics of HSM is solved numerically according to the following algorithm [24]:

- Suppose $\mathbf{X}_g$, an initial posture (6x1 vector)
- Calculate $\mathbf{Q}_g$=IK($\mathbf{X}_g$)
- Update posture as: $\mathbf{X}_g = \mathbf{X}_g + \mathbf{J}_{inv}^{-1}(\mathbf{Q}_g - \mathbf{Q}_d)$
- Calculate $\mathbf{Q}_g$=IK($\mathbf{X}_g$)
- If $\|\mathbf{Q}_d - \mathbf{Q}_g\|$ > tolerance, goto step (iii)
- else $\mathbf{X}_g$ is the forward kinematics solution

where Q_d is the vector of measured articular variables, Q_g is the articular variable vector calculated from IK in step (ii), J_{inv} is the inverse Jacobian and X_g is the solution posture. Note that the inverse manipulator Jacobian used in the above computations needs to be transformed into the inverse Jacobian of Euler angles where the transformation depends on the choice of Euler angles used.

2.3 Frames and Identification Parameters

The number of identification parameters depends on the way the reference frames are assigned. By assigning the reference frames properly, the complexity of the calibration problem can be reduced significantly. Fassi et al. studied the manipulator under consideration for minimum, complete, and parametrically continuos model for kinematic calibration and found that 54 parameters are required when measurements are performed externally. Number of parameters, however, can be reduced by proper frame assignments [4].

For this study, origin of the base frame, OXYZ, is located at the A_{10}. X-axis of the base frame is defined along the line segment A_{10}-A_{20}. Z-axis of the base frame is directed opposite to the gravity acceleration and the OXYZ system forms a RHS. With this frame assignment, the following 5 parameters will be zero and therefore will not be considered as identification parameters.

$$A_{10x} = A_{10y} = A_{10z} = A_{20y} = A_{20z} = 0$$

Origin of the mobile frame, P, is located at the center of the mobile platform. The PX'Y'Z' also forms a RHS and is parallel to OXYZ when rotation angles are zero.

Considering a kinematic chain of the HSM, following are the identification parameters in general:

S Joints' location:	**B**	3 parameters/chain
Slider Axis Start Point:	$\mathbf{A}_0$	3 parameters/chain
Slider direction vector:	**a**	2 parameters/chain
Link Length:	ℓ	1 parameters/chain

Note that the unit vectors of the sliders' are specified by two components; say, the x and the y component. This makes 9 parameters for each link chain and 54 parameters in total for the mechanism. Note that all parameters are measured in the units of length. Note also that the **B** points are defined with respect to the PX'Y'Z' frame while the $\mathbf{A_0}$ points are defined with respect to the OXYZ frame.

From equation (3), 6 parameters will always be zero. Therefore, the total number identification parameters are reduced to 49.

3. IDENTIFIABLE PARAMETERS FOR FOUR CALIBRATION PROCEDURES

To perform the calibration procedure, we should solve the following linearized equation.

$$\Delta Y(X,u) = J(X,u)\bullet\Delta u \tag{4}$$

where ΔY is the vector of error residuals, X is the vector of Cartesian posture, u is the vector of the identification parameters, and J is the Identification Jacobian matrix.

In general, it is not always possible to calibrate kinematic parameters and it is important to study if all of the parameters are identifiable with particular calibration procedure. Besnard and Khalil [22] proposed use of QR decomposition of the identification jacobian matrix to find the non-identifiable parameters for different calibration schemes. Below, we present briefly the QR decomposition.

Let m be the number of measurement data and r be the number of kinematic calibration parameters. Size of the Identification Jacobian matrix will then be m×r, where m>>r. QR decomposition of the matrix can be expressed as

$$\Delta Y = \begin{bmatrix} q_{11} & \cdots & q_{1m} \\ \vdots & \ddots & \vdots \\ q_{m1} & \cdots & q_{mm} \end{bmatrix} \bullet \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1r} \\ 0 & r_{21} & \cdots & r_{1r} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & r_{rr} \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} \Delta u \tag{5}$$

For the non-identifiable parameters, the corresponding diagonal entity of the matrix R is zero. While performing numerical computations, the values may be small but still not zero. Thus, τ is defined as numerical zero and if $|r_{ii}| \le \tau$, it is taken zero. The tolerance is defined as $\tau = r\times\varepsilon\times\max|r_{ii}|$, where ε is the machine accuracy [16].

3.1 Case-I: Complete Posture measurements with Inverse kinematics residuals

Expressing the error residual in terms of the articular variables results in block diagonal identification Jacobian matrix. Also, the derivatives required for establishing the identification Jacobian can be expressed in closed form thereby avoiding the numerical inaccuracies. The problem of identification, thus, can be solved individually for each kinematic chain as

$$\Delta\lambda_i = \frac{\partial f_i}{\partial u_1}\Delta u_1 + \frac{\partial f_i}{\partial u_2}\Delta u_2 + \ldots + \frac{\partial f_i}{\partial u_{j-1}}\Delta u_{j-1} + \frac{\partial f_i}{\partial u_j}\Delta u_j \tag{6}$$

where j represents the number of identification parameters for each kinematic chain. Note that at least j postures are required to solve the identification problem in this case. When postures are measured completely, 6 values are measured for each posture – the three positions and the three rotations. If *k* measurements are performed, then minimum number of postures, j, can be expressed as

$$j \ge k/6 \tag{7}$$

Equation 4 can be expressed in detail for this case as

$$\begin{bmatrix} \Delta\lambda_1^1 \\ \vdots \\ \Delta\lambda_6^1 \\ \Delta\lambda_1^2 \\ \vdots \\ \Delta\lambda_i^{k-1} \\ \vdots \\ \Delta\lambda_6^k \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1^1}{\partial u_1} & \frac{\partial f_1^1}{\partial u_2} & \cdots & \frac{\partial f_1^1}{\partial u_{j-1}} & \frac{\partial f_1^1}{\partial u_j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_6^1}{\partial u_1} & \frac{\partial f_6^1}{\partial u_2} & \cdots & \frac{\partial f_6^1}{\partial u_{j-1}} & \frac{\partial f_6^1}{\partial u_j} \\ \frac{\partial f_1^2}{\partial u_1} & \frac{\partial f_1^2}{\partial u_2} & \cdots & \frac{\partial f_1^2}{\partial u_{j-1}} & \frac{\partial f_1^2}{\partial u_j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_i^{k-1}}{\partial u_1} & \frac{\partial f_i^{k-1}}{\partial u_2} & \cdots & \frac{\partial f_i^{k-1}}{\partial u_{j-1}} & \frac{\partial f_i^{k-1}}{\partial u_j} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_6^k}{\partial u_1} & \frac{\partial f_6^k}{\partial u_2} & \cdots & \frac{\partial f_6^k}{\partial u_{j-1}} & \frac{\partial f_6^k}{\partial u_j} \end{bmatrix} = \begin{bmatrix} \Delta u_1 \\ \Delta u_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \Delta u_{j-1} \\ \Delta u_j \end{bmatrix} \tag{8}$$

Computations show that all of the 49 kinematic parameters are identifiable in this case. Further, the condition number of its identification jacobian can gauge the effectiveness of any calibration procedure. Condition number of the identification jacobian is minimum in this case as will be shown later. Also, the identification jacobian is homogenous – all entities bear the same units.

3.2 Case-II: Complete Posture measurements with Forward kinematics residuals

In this case, the error residual is expressed in terms of the components of the Cartesian posture. Thus, for each measurement, 3 rows of the identification jacobian are computed based on the position components and 3 based on the rotations. The matrix, thus, is not homogenous and may need scaling. The problem of identification can be expressed as

$$\Delta X_i = \frac{\partial g_i}{\partial u_1}\Delta u_1 + \frac{\partial g_i}{\partial u_2}\Delta u_2 + \ldots + \frac{\partial g_i}{\partial u_{j-1}}\Delta u_{j-1} + \frac{\partial g_i}{\partial u_j}\Delta u_j \tag{9}$$

In this case, the identification Jacobian matrix is computed numerically through perturbation of the identification parameters. Numerical value of perturbation was taken as 10^{-7}.

Again, in this case, all of the 49 parameters were found identifiable. Condition number of the identification Jacobian matrix, however, is higher as compared to the first case.

3.3 Case – III: Measuring only the 3 position components

When only partial pose information is available, we cannot use the inverse kinematics residuals and computations are based on the forward kinematic model. Equation 9 holds valid for this case with the difference that the computed error vector consists of only position components. For each measurement, only 3 rows of the identification Jacobian are computed numerically.

The number of identifiable parameters was found to be 48 in this case. This means one of the identification parameters cannot be identified. Also, the condition number was found much higher than the first two cases. The identification Jacobian, however, is homogenous.

3.4 Case – IV: Using constraint link

When constraint link is employed, all of the measured postures are equidistant (equal to the length of the constraint link) from certain point. This fact is exploited to perform the identification. The problem of the identification can thus be expressed as

$$\Delta l_i = \frac{\partial g_i}{\partial u_1}\Delta u_1 + \frac{\partial g_i}{\partial u_2}\Delta u_2 + ... + \frac{\partial g_i}{\partial u_{j-1}}\Delta u_{j-1} + \frac{\partial g_i}{\partial u_j}\Delta u_j \qquad (10)$$

For each measurement, a single row of the homogenous identification Jacobian matrix is computed numerically.

In this case, we need to define extra parameters, as the exact position of the ends of the constraint link may not be known precisely. Figure 3 shows the schematic of the calibration procedure for the case under consideration. It can be seen that parameters for two offsets need to be added. Adding 6 more parameters to makes total of 55 identification parameters.

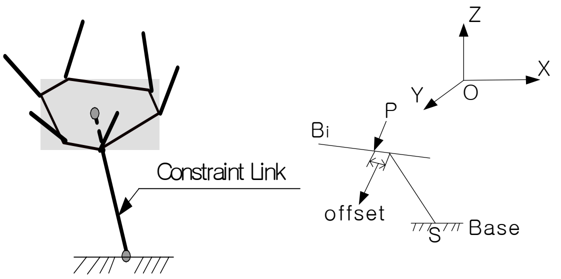


Fig. 3 Schematic diagram of calibration method using a constraint link

QR decomposition reveals that the number of identifiable parameters is 46, thus making 9 parameters non-identifiable. Condition number of the identification Jacobian is highest of all the cases.

3.5 Comparison of the Calibration procedures

Table 1 summarizes the results for the calibration procedures. If the postures are measured completely, all of the parameters can be identified. Also, the inverse kinematics residuals give better results due to lower value of the condition number. With only position measurements, all parameters cannot be identified. Note that the error residual may still reduce significantly. The calibrated parameters, however, may give varying accuracy in different region of the workspace. The case of the constraint link shows as many as 9 non-identifiable parameters and does not seem a practical solution to calibration.

For cases III and IV, additional information is required to make possible identification of all of the parameters. Use of inclinometers and/or rotary encoder sensors can be studied to identify all calibration parameters.

Table 1 Comparison of Calibration Procedures

	Case I	Case II	Case III	Case IV
Identifiable Parameters	49	49	48	46
Condition Number	1.4e2	1.5e3	2.4e3	1.4e5

4. OPTIMUM POSTURES FOR CALIBRATION

Measuring postures involve time and may be expensive sometimes. Performing measurements at optimum postures can assure better performance of calibration procedures. It can thus reduce experimental cost and effort.

Typically, the condition number of the identification Jacobian is minimized for searching the optimum postures and in this study it is employed as the cost function. However, many different criteria have been used for this optimization problem, including [20].

- $Det\left(J^T J\right)$ where Det represents determinant
- $\frac{\sqrt[L]{\sigma_L \cdots \sigma_1}}{\sqrt{m}}$ where m is the number of measurement postures, L is the number of singular values and σ represents the singular values.
- $\frac{\sigma_L}{\sigma_1}$ - is the inverse of the condition number and is maximized for optimization. The maximum value of this cost function is 1.
- σ_L
- $\frac{\sigma_L^2}{\sigma_1}$

In this study, the problem of optimization is formulated as constrained minimization where the search is specified in Cartesian coordinates and is limited by specifying constraints. Note that ideally the search space of the optimum postures should be the entire workspace of the manipulator. However, boundaries of the actual workspace are mostly complex functions of the spatial coordinates and its difficult to model them exactly in the Cartesian space. Therefore, a simple rectangular region was chosen for search in approximate middle of the workspace. If the search space is specified in the joint space, more working volume can be exploited. However, postures generated within such space may need to be checked for being valid. Table 2 shows the values of the specified constant constraints. Note that the "fmincon" function of the MATLAB optimization toolbox was used to solve the problem with condition number of the identification Jacobian taken as the cost function.

Table 2 Search space for optimum postures

X(m)	Y(m)	Z(m)	ψ (°)	θ (°)	ϕ (°)
-0.3 - 0.3	-1.1 - -0.5	-1.3 - -1.0	$\pm$ 15	$\pm$ 15	$\pm$ 30

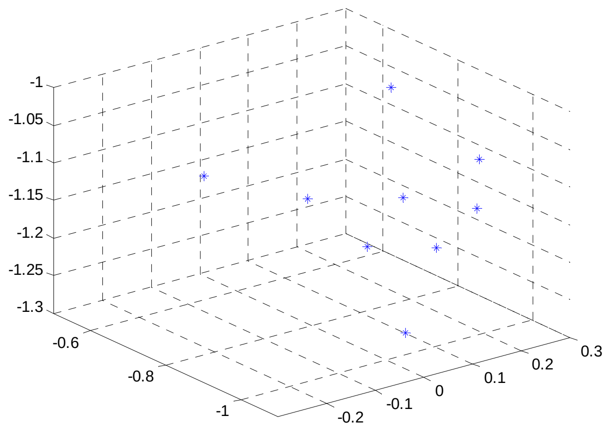
4.1 Case I

Starting with randomly generated postures, the optimum postures were found after few iterations of the optimization function. Table 3 shows the values of the condition number before and after the minimization.

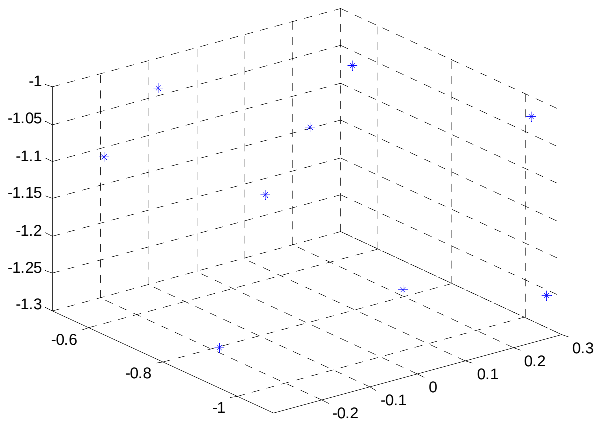
Table 3 Comparison of Condition number (Case I)

Number of Postures	Condition number
9 postures - Random	2.888e+3
9 postures - Optimum	140.2

Figure 4 shows the orientation of postures with in the search space before and after the optimization. It can easily be observed that the trend of postures is to orient themselves near the boundaries of the workspace.



(a) Postures before optimization



(b) Postures after optimization

Fig. 4 Optimum postures - case I

4.2 Cases II, III, and IV

The trend of postures to orient themselves on the boundaries is same for all the cases. So, in this subsection we compare and discuss only the condition number for cases II, III, and IV. Table 4 – Table 6 compare the condition numbers of the identification Jacobians before and after the optimization.

Table 4 Comparison of Condition number (Case II)

Number of data	Condition number
9 postures - Random	2.153e+4
9 postures - Optimum	1537.3

Table 5 Comparison of Condition number (Case III)

Number of data	Condition number
16 postures - Random	3.3916e+4
16 postures - Optimum	2430.2

Table 6 Comparison of Condition number (Case IV)

Number of data	Condition number
46 postures - Random	6.0120e+5
46 postures - Optimum	1.4523e+5

Note that the reduction of the cost function, condition number of the identification Jacobian, is much less for case III and case IV as compared to case I and case II. Generally, the higher the condition number, the less reliable will be the calibration results. High values of condition number stress the need of augmenting more information for Cases III and IV.

5. CONCLUSIONS

Computer simulations results show that for case I and case II, all of the 49 geometric parameters 49 are identifiable. For case III, only 48 geometric parameters can be identified. For case IV, the number of identifiable parameters is just 46 out of the total 55 parameters in this case.

Optimization for the measurement postures results in significant reduction of the condition number for case I and case II. For case III and case IV the reduction is not very significant. A same trend was observed during optimization for all of the cases – the measurement postures move towards the boundary of the search space.

The last two cases require more information for effective calibration and some other sensors should be used to augment the measurement information.

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