An Efficient Dynamic Modeling Method for Hybrid Robotic Systems

Goo Bong Chung and Byung-Ju Yi

School of Electrical Engineering and Computer Science, Hanyang University, Ansan, Korea
(Tel : +82-31-400-5218; E-mail: bj@hanyang.ac.kr)

Abstract: In this paper, we deal with the kinematic and dynamic modeling of hybrid robotic systems that are constructed by combination of parallel and serial modules or series of parallel modules. Previously, open-tree structure has been employed for dynamic modeling of hybrid robotic systems. Though this method is generally used, however, it requires expensive computation as the size of the system increases. Therefore, we propose an efficient dynamic modeling methodology for hybrid robotic systems. Initially, the dynamic model for the proximal module is obtained with respect to the independent joint coordinates. Then, in order to represent the operational dynamics of the proximal module, we model virtual joints attached at the top platform of the proximal module. The dynamic motion of the next module exerts dynamic forces to the virtual joints, which in fact is equivalent to the reaction forces exerted on the platform of the lower module by the dynamics of the upper module. Then, the dynamic forces at the virtual joints are distributed to the independent joints of the proximal module. For multiple modules, this scheme can be constructed as a recursive dynamic formulation, which results in reduction of the complexness of the open-tree structure method for modeling of hybrid robotic systems. Simulation for inverse dynamics is performed to validate the proposed modeling algorithm.
Keywords: Dynamic modeling, Hybrid manipulator, Virtual joint, Reaction forces.

1. INTRODUCTION

A hybrid robot denotes a robot system that is constructed by combination of serial and parallel modules or a series of parallel modules. Freeman and Tesar[1], Kang, at al[2], and Cho[3] presented dynamic modeling methodologies for such robotic systmes. Also, Sklar[4] presented a kinematic modeling method for hybrid robotic systems.

In general, the Newton-Euler formulation, the Lagrangian method, and the open-chain method have been employed for dynamic modeling of hybrid robotic systems. Though these methods are well known formulation, however, these methods have shortcoming in that it must calculate dynamics of whole manipulator again even if the structure of robot changes very a little and that requires expensive computation as the size of the system increases.

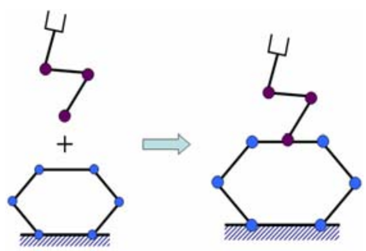

Fig. 1 A hybrid robot composed of well known structures

For example, even though the dynamics of two independent systems, as shown in Fig. 1, are known, the whole dynamics needs to be reconstructed when the two modules are combined as one system.

In this paper, therefore, we propose an efficient dynamic modeling methodology for hybrid robotic systems that are constructed by adding another robot to the existing robot mechanism successively. This approach has advantage in that it is not necessary to calculate the whole dynamics again.

The proposed method is explained as follows. First, the dynamic model for the proximal module is obtained with respect to the independent joint coordinates. Then, we represent the operational dynamics of a lower module as that of equivalent virtual joints attached at the top platform of the lower module. Next, the dynamic model of the upper module including the virtual joints and links is obtained. Eventually, the effective dynamic model of the lower model is represented as the sum of virtual joints' dynamics and its own dynamic model. Here, the physical meaning of the virtual joints is equivalent to the reaction forces exerted on the platform of the lower module by the dynamic motion of the upper module.

The basic idea of the proposed method is described in Fig. 2.

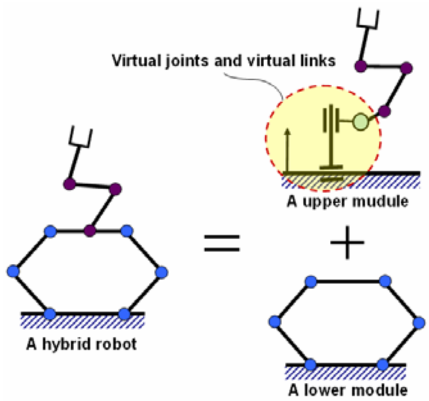

Fig. 2 The concept of the proposed dynamic modeling method for hybrid robotic systems

2. KINEMATICS

As shown in Fig. 1, the hybrid robot structured by combination of a parallel 3 DOF 6-bar linkage and a serial type 3 DOF robot is employed as an illustrative example to explain the proposed dynamic formulation.

The number of minimum joints required to position the system is 6, which are selected by any three joints of the 6-bar linkage and the three joints of the serial robot.

2.1 Kinematics of a 6-bar linkage

Kinematic analysis of the 6-bar linkage is processed by using open-chain kinematics. The output of the parallel robot is defined as

$$ {}^s\underline{u} = [x_v \quad y_v \quad \Phi_v]^T \tag{1}$$

and the input vector of each serial sub-chain is defined as

$$ {}^s_1\underline{\phi} = [\theta_1 \quad \theta_2 \quad \theta_3]^T \tag{2}$$

and

$$ {}^s_2\underline{\phi} = [\theta_4 \quad \theta_5 \quad \theta_6]^T \tag{3}$$

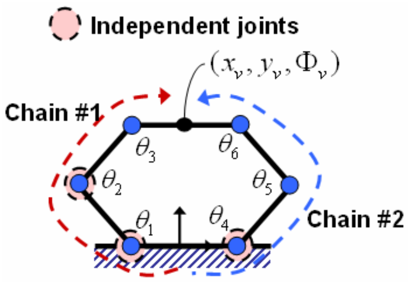

Fig. 3 A 6-bar linkage

Since two serial-chains have the same velocities at the center of platform, we have the constraint equation as follows

$$ {}^s\dot{\underline{u}} = {}^s[{}_1G^u_\phi]\,{}^s_1\dot{\underline{\phi}} = {}^s[{}_2G^u_\phi]\,{}^s_2\dot{\underline{\phi}}\,, \tag{4}$$

where

$$ {}^s\dot{\underline{u}} = [\dot{x}_v \quad \dot{y}_v \quad \dot{\Phi}_v]^T\,, \tag{5}$$

and $[G]$ denotes the Jacobian and the left subscript of G denotes the number of serial sub-chain of this parallel system. The superscript and subscript to the right side of G denote the dependent and independent parameters, respectively.

If we select the independent and dependent parameters as

$$ {}^s\underline{\phi}_a = [\theta_1 \quad \theta_2 \quad \theta_4]^T\,, \tag{6}$$

$$ {}^s\underline{\phi}_p = [\theta_4 \quad \theta_5 \quad \theta_6]^T\,, \tag{7}$$

the velocity vector of the dependent joints is expressed as[5]

$$ {}^s\dot{\underline{\phi}}_p = A^{-1}B\,{}^s\dot{\underline{\phi}}_a \triangleq {}^s[G^p_a]\,{}^s\dot{\underline{\phi}}_a\,, \tag{8}$$

where

$$ A = [{}_1\underline{g}_3 \quad -{}_2\underline{g}_2 \quad -{}_2\underline{g}_3] \tag{9}$$

and

$$ B = [-{}_1\underline{g}_1 \quad -{}_1\underline{g}_2 \quad {}_2\underline{g}_1]\,. \tag{10}$$

And in Eq. (10), ${}_i\underline{g}_j$ is a 3×1 vector that means the j[th] column of the Jacobian of the i[th] serial chain.

Now, the joint velocity of each serial sub-chain can be described in terms of the independent joint variables as

$$ {}^s_1\dot{\underline{\phi}} = {}^s[{}_1G^\phi_a]\,{}^s\dot{\underline{\phi}}_a \tag{11}$$

and

$$ {}^s_2\dot{\underline{\phi}} = {}^s[{}_2G^\phi_a]\,{}^s\dot{\underline{\phi}}_a\,, \tag{12}$$

where

$$ {}^s[{}_1G^\phi_a] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ & {}^s[G^p_a]_{1;} & \end{bmatrix}, \tag{13}$$

$$ {}^s[{}_2G^\phi_a] = \begin{bmatrix} 0 & 0 & 1 \\ & {}^s[G^p_a]_{2;} & \\ & {}^s[G^p_a]_{3;} & \end{bmatrix}, \tag{14}$$

and ${}^s[G^p_a]_{j;}$ denotes the j[th] row of ${}^s[G^p_a]$.

Substituting Eq. (11) into Eq. (4) results in the output velocity vector, given by

$$ {}^s\dot{\underline{u}} = {}^s[G^u_a]\,{}^s\dot{\underline{\phi}}_a\,, \tag{15}$$

where

$$ {}^s[G^u_a] = {}^s[{}_1G^u_\phi]\,{}^s[{}_1G^\phi_a]\,. \tag{16}$$

The output acceleration vector is obtained by differentiating Eq. (4) with respect to time as

$$ {}^s\ddot{\underline{u}} = {}^s[{}_1G^u_\phi]\,{}^s_1\ddot{\underline{\phi}} + {}^s_1\dot{\underline{\phi}}^T\,{}^s[{}_1H^u_{\phi\phi}]\,{}^s_1\dot{\underline{\phi}} \tag{17}$$

and

$$ {}^s\ddot{\underline{u}} = {}^s[{}_2G^u_\phi]\,{}^s_2\ddot{\underline{\phi}} + {}^s_2\dot{\underline{\phi}}^T\,{}^s[{}_2H^u_{\phi\phi}]\,{}^s_2\dot{\underline{\phi}}\,, \tag{18}$$

where

$$ {}^s\ddot{\underline{u}} = [\ddot{x}_v \quad \ddot{y}_v \quad \ddot{\Phi}_v]^T\,. \tag{19}$$

By manipulation of these two loop constraint equations, a relation between the dependent joint acceleration vector and the independent joint acceleration vector is expressed as below

$$ {}^s\ddot{\underline{\phi}}_p = {}^s[G^p_a]\,{}^s\ddot{\underline{\phi}}_a + {}^s\dot{\underline{\phi}}_a^T\,{}^s[H^p_{aa}]\,{}^s\dot{\underline{\phi}}_a\,, \tag{20}$$

where, ${}^s[H^p_{aa}]$ is a $3\times3\times3$ matrix obtained by

$$ {}^s[H^p_{aa}] = A^{-1}\circ({}^s[{}_2G^\phi_a]^T\,{}^s[{}_2H^u_{\phi\phi}]\,{}^s[{}_2G^\phi_a] - {}^s[{}_1G^\phi_a]^T\,{}^s[{}_1H^u_{\phi\phi}]\,{}^s[{}_1G^\phi_a])\,. \tag{21}$$

Therefore, the joint acceleration vector of each chain is described as follows

$$ {}^s_1\ddot{\underline{\phi}} = {}^s[{}_1G^\phi_a]\,{}^s\ddot{\underline{\phi}}_a + {}^s\dot{\underline{\phi}}_a^T\,{}^s[{}_1H^\phi_{aa}]\,{}^s\dot{\underline{\phi}}_a \tag{22}$$

and

$$ {}^s_2\ddot{\underline{\phi}} = {}^s[{}_2G^\phi_a]\,{}^s\ddot{\underline{\phi}}_a + {}^s\dot{\underline{\phi}}_a^T\,{}^s[{}_2H^\phi_{aa}]\,{}^s\dot{\underline{\phi}}_a\,, \tag{23}$$

where ${}^s[{}_iH^\phi_{aa}]$ is a three dimensional array given by

$$ {}^s[{}_1H^\phi_{aa}] = \begin{bmatrix} [0]_{3\times3} & & \\ & [0]_{3\times3} & \\ & & [H^p_{aa}]_{1;;} \end{bmatrix} \tag{24}$$

and

$$ {}^{s}[{}_{2}H^{\phi}_{aa}] = \begin{bmatrix} [0]_{3\times 3} & & \\ & [H^{p}_{aa}]_{2;;} & \\ & & [H^{p}_{aa}]_{3;;} \end{bmatrix}. \quad (25) $$

Consequently, we obtain the output acceleration represented in terms of the independent joint variables as follows [5]

$$ {}^{s}\underline{\ddot{u}} = {}^{s}[G^{u}_{a}]\,{}^{s}\underline{\ddot{\phi}}_{a} + {}^{s}\underline{\dot{\phi}}_{a}^{T}\,{}^{s}[H^{u}_{aa}]\,{}^{s}\underline{\dot{\phi}}_{a}\,, \quad (26) $$

where

$$ {}^{s}[H^{u}_{aa}] = {}^{s}[{}_{1}G^{u}_{a}] \circ {}^{s}[{}_{1}H^{\phi}_{aa}] + {}^{s}[{}_{1}G^{\phi}_{a}]^{T}\,{}^{s}[{}_{1}H^{u}_{\phi\phi}]\,{}^{s}[{}_{1}G^{\phi}_{a}]. \quad (27) $$

2.2 Kinematics of the robot with virtual joints and links

Assume that the motion of the lower module is expressed as three virtual joints and links as shown in Fig. 4.

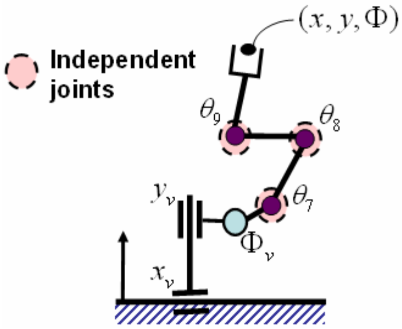

Fig. 4 The upper module including virtual joints and links

Then, the input vector of the upper module with additional virtual joints and virtual links is defined as

$$ {}^{v}\underline{\phi} = \left[x_v \quad y_v \quad \Phi_v \quad \theta_7 \quad \theta_8 \quad \theta_9\right]^{T}. \quad (28) $$

When the output vector is defined as

$$ {}^{v}\underline{u} = [x \quad y \quad \Phi]^{T}, \quad (29) $$

the velocity vector of the end-effecter can be expressed directly in terms of the joint velocity vector as

$$ {}^{v}\underline{\dot{u}} = {}^{v}[G^{u}_{\phi}]\,{}^{v}\underline{\dot{\phi}} \quad (30) $$

and the acceleration relation between the output and the input vector is obtained by differentiating Eq. (30) with respect to time as

$$ {}^{v}\underline{\ddot{u}} = {}^{v}[G^{u}_{\phi}]\,{}^{v}\underline{\ddot{\phi}} + {}^{v}\underline{\dot{\phi}}^{T}\,{}^{v}[H^{u}_{\phi\phi}]\,{}^{v}\underline{\dot{\phi}}\,. \quad (31) $$

Finally, by substituting Eq. (15) and Eq. (26) into Eq. (30) and Eq. (31), we can obtain the velocity and the acceleration of the hybrid robot described in terms of independent joints.

2.3 Kinematics of the hybrid robot

The mobility of the hybrid robot given in Fig. 5 is 6. Thus, six independent inputs are required to drive the system. The system can be visualized as two serial chains by cutting a joint or a link.

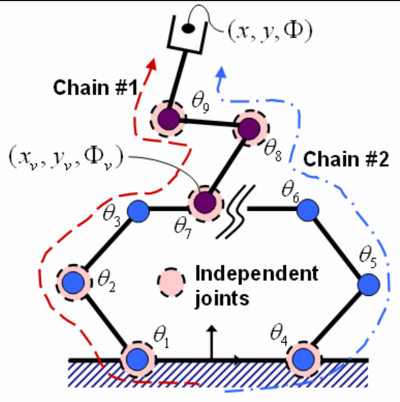

Fig. 5 A hybrid robot

The input vector of each serial sub-chain of the hybrid robot is defined as

$$ {}^{h}_{1}\underline{\phi} = [\theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \theta_6 \;\; \theta_7 \;\; \theta_8]^{T} \quad (32) $$

and

$$ {}^{h}_{2}\underline{\phi} = [\theta_4 \;\; \theta_5 \;\; \theta_6]^{T}. \quad (33) $$

The output of the hybrid robot is defined as

$$ {}^{h}\underline{u} = [x \quad y \quad \Phi]^{T}. \quad (34) $$

When the independent joint variables are chosen as

$$ {}^{h}\underline{\phi}_{a} = \left[\theta_1 \quad \theta_2 \quad \theta_4 \quad \theta_7 \quad \theta_8 \quad \theta_9\right]^{T}, \quad (35) $$

the output velocity and acceleration of the robot are described, respectively, as

$$ {}^{h}\underline{\dot{u}} = {}^{h}[G^{u}_{a}]\,{}^{h}\underline{\dot{\phi}}_{a} \quad (36) $$

and

$$ {}^{h}\underline{\ddot{u}} = {}^{h}[G^{u}_{a}]\,{}^{h}\underline{\ddot{\phi}}_{a} + {}^{h}\underline{\dot{\phi}}_{a}^{T}\,{}^{h}[H^{u}_{aa}]\,{}^{h}\underline{\dot{\phi}}_{a}\,, \quad (37) $$

where

$$ {}^{h}[G^{u}_{a}] = {}^{v}[G^{u}_{\phi}] \begin{bmatrix} {}^{s}[G^{u}_{a}] & [0]_{3\times 3} \\ [0]_{3\times 3} & I_{3\times 3} \end{bmatrix}. \quad (38) $$

Similarly, ${}^{s}[H^{u}_{aa}]$ can be also obtained.

In Eq. (38), $[0]_{3\times3}$, and $I_{3\times3}$ are a 3×3 null matrix, a 3×3 identity matrix, respectively. And ${}^{v}[G^{u}_{\phi}]$ is given by Eq. (30), and ${}^{s}[G^{u}_{a}]$ is given by Eq. (16).

3. DYNAMICS

3. 1 Dynamics of the hybrid robot by the open-tree structure method

In this section, we deal with the dynamic formulation of the hybrid robot by the open-tree structure method that is well known method for general robot dynamics. By cutting a link as shown in Fig. 5, we have two open chain structures.

Using the Lagrange's form of d'Alembert principle proposed by Freeman and Tesar[1], the dynamic model for the ith chain is represented as

$$ {}^{h}_{i}\underline{\tau}_{\phi} = {}^{h}[{}_{i}I^{*}_{\phi\phi}]\,{}^{h}_{i}\underline{\ddot{\phi}} + {}^{h}_{i}\underline{\dot{\phi}}^{T}\,{}^{h}[{}_{i}P^{*}_{\phi\phi\phi}]\,{}^{h}_{i}\underline{\dot{\phi}}\,, \quad (39) $$

where ${}^{h}_{i}\underline{\tau}_{\phi}$ represents the torque vector applied to the joints of the i[th] serial sub-chain. $[{}_iI^*_{\phi\phi}]$ and $[{}_iP^*_{\phi\phi\phi}]$ are the joint-referenced effective inertia matrix and the effective inertia power array relating to centrifugal and Coriolis forces, respectively.

The dynamic model for the six independent joints is obtained by virtual work principle between the Lagrangian coordinates and the minimum independent joints as[5]

$$^h\underline{\tau}_a = {}^h[I^*_{aa}]\,{}^h\underline{\ddot{\phi}}_a + {}^h\underline{\dot{\phi}}_a^T\,{}^h[P^*_{aaa}]\,{}^h\underline{\dot{\phi}}_a , \quad (40)$$

where $^h\underline{\tau}_a \in R^6$, and

$$^h[I^*_{aa}] = \sum_{i=1}^{2} {}^h[{}_iG^\phi_a]^T\,{}^h[{}_iI^*_{\phi\phi}]\,{}^h[{}_iG^\phi_a] \quad (41)$$

and

$$^h[P^*_{aaa}] = \sum_{i=1}^{2}\{({}^h[{}_iG^\phi_a]^T\,{}^h[{}_iI^*_{\phi\phi}])\circ{}^h[{}_iH^\phi_{aa}] + {}^h[{}_iG^\phi_a]^T({}^h[{}_iG^\phi_a]^T\circ{}^h[{}_iP^*_{\phi\phi\phi}])\,{}^h[{}_iG^\phi_a]\} . \quad (42)$$

In Eq. (41) and (42), $^h[{}_iG^\phi_a]$ and $^h[{}_iH^\phi_{aa}]$ denote the Jacobian and Hessian relating the joints of the i[th] serial sub-chain to the independents joints of the hybrid robot.

3. 2 Dynamics of the 6-bar linkage

The dynamic model of the 6-bar linkage, shown in Fig. 3, can be also obtained using the open-tree structure model formed by cutting the center of the platform.

The dynamic model for the three independent joints is obtained by virtual work principle between the Lagrangian coordinates and the minimum independent joints as[5]

$$^s\underline{\tau}_a = {}^s[I^*_{aa}]\,{}^s\underline{\ddot{\phi}}_a + {}^s\underline{\dot{\phi}}_a^T\,{}^s[P^*_{aaa}]\,{}^s\underline{\dot{\phi}}_a , \quad (43)$$

where $^s\underline{\tau}_a \in R^3$, and

$$^s[I^*_{aa}] = \sum_{i=1}^{2} {}^s[{}_iG^\phi_a]^T\,{}^s[{}_iI^*_{\phi\phi}]\,{}^s[{}_iG^\phi_a] \quad (44)$$

and

$$^s[P^*_{aaa}] = \sum_{i=1}^{2}\{({}^s[{}_iG^\phi_a]^T\,{}^s[{}_iI^*_{\phi\phi}])\circ{}^s[{}_iH^\phi_{aa}] + {}^s[{}_iG^\phi_a]^T({}^s[{}_iG^\phi_a]^T\circ{}^s[{}_iP^*_{\phi\phi\phi}])\,{}^s[{}_iG^\phi_a]\} . \quad (45)$$

By the relationship between the output vector and the torque vector at the independent joints

$$^s\underline{\tau}_u = {}^s[G^a_u]^T\,{}^s\underline{\tau}_a \quad (46)$$

the dynamic model in the operational space can be also obtained. In Eq. (46), $^s\underline{\tau}_u$ is the operational torque vector and $^s[G^a_u]$ is the inverse matrix of the forward Jacobian $^s[G^u_a]$.

3.3 Efficient dynamic model of a hybrid robot

In the proposed method, it is assumed that the whole kinematic and dynamic models both the six-bar linkage and the 3DOF serial module are already given.

Now, the dynamic model of the upper module including the virtual joints will be described. The dynamic equation of the upper module is described as

$$^v\underline{\tau}_\phi = {}^v[I^*_{\phi\phi}]\,{}^v\underline{\ddot{\phi}} + {}^v\underline{\dot{\phi}}^T[P^*_{\phi\phi\phi}]\,{}^v\underline{\dot{\phi}} \triangleq \begin{bmatrix} {}^v\underline{\tau}_v \\ {}^v\underline{\tau}_a \end{bmatrix}, \quad (47)$$

where $^v\underline{\tau}_v$ denotes the 3×1 dynamic forces at the virtual joints to support the dynamics of the upper module and $^v\underline{\tau}_a$ denotes the dynamics of the upper module itself. Then, the whole dynamic model with respect to the independent joints of the hybrid robot can be represented as

$$^h\underline{\tau}_a = \begin{bmatrix} {}^s[G^u_a]^T\;{}^v\underline{\tau}_v + {}^s\underline{\tau}_a \\ {}^v\underline{\tau}_a \end{bmatrix}. \quad (48)$$

It implies that the dynamic equation of a hybrid robot can be easily derived by union of the dynamic model of the augmented virtual joints and those of the two given modules.

The proposed scheme is useful in that it is able to use the dynamic models of the given modules consisting of the hybrid robot. To examine its meaning, Eq. (47) is rewritten as

$$\begin{bmatrix} {}^v\underline{\tau}_v \\ {}^v\underline{\tau}_a \end{bmatrix} \triangleq \begin{bmatrix} [I^*_{\phi\phi}]_{vv} & [I^*_{\phi\phi}]_{va} \\ [I^*_{\phi\phi}]_{av} & [I^*_{\phi\phi}]_{aa} \end{bmatrix}\begin{bmatrix} {}^v\underline{\ddot{\phi}}_v \\ {}^v\underline{\ddot{\phi}}_a \end{bmatrix} + \begin{bmatrix} {}^v\underline{C}_v \\ {}^v\underline{C}_a \end{bmatrix}, \quad (49)$$

where $^v\underline{\ddot{\phi}}_v$ and $^v\underline{\ddot{\phi}}_a$ denote the accelerations of virtual joints and the independent joints of the upper module, respectively. $^v\underline{C}_v$ and $^v\underline{C}_a$ are the terms corresponding to the nonlinear velocity term. $[I^*_{\phi\phi}]_{vv}$ and $[I^*_{\phi\phi}]_{aa}$ are the inertia matrix of virtual joints and that of the upper module, respectively.

Manipulating Eq. (43), (48), and (49) gives the dynamic model with respect to the six independent joints:

$$^h\underline{\tau}_a = {}^h[I^*_{aa}]\,{}^h\underline{\ddot{\phi}}_a + {}^h\underline{C} , \quad (50)$$

where

$$^h[I^*_{aa}] = \begin{bmatrix} {}^s[I^*_{aa}] + {}^s[G^u_a]^T[I^*_{\phi\phi}]_{vv}\,{}^s[G^u_a] & {}^s[G^u_a]^T[I^*_{\phi\phi}]_{va} \\ [I^*_{\phi\phi}]_{av}\,{}^s[G^u_a] & [I^*_{\phi\phi}]_{aa} \end{bmatrix}, \quad (51)$$

$$^h\underline{\ddot{\phi}}_a = \begin{bmatrix} {}^s\underline{\ddot{\phi}}_a \\ {}^v\underline{\ddot{\phi}}_a \end{bmatrix}, \quad (52)$$

and $^h\underline{C}$ is the vector regarding nonlinear velocities.

It is observed from Eq. (51) that the module dynamics given by $^s[I^*_{aa}]$ and $[I^*_{\phi\phi}]_{aa}$ are still used in this dynamic model, and that only the terms $[I^*_{\phi\phi}]_{vv}$ and $[I^*_{\phi\phi}]_{va}$ denoting the dynamic model of the virtual joints need to be calculated additionally.

Conclusively, instead of calculating the dynamics of the open tree structure, given by Eq. (40), the proposed method is found to be computationally efficient. Furthermore, it is very appealing in that the dynamic models of the given modules are still used instead of developing totally new dynamic model.

4. SIMULATION

In this section, simulation for inverse dynamics is performed to validate the proposed modeling algorithm. The goal of this section is to show that torque values of the independent joints of the hybrid robot obtained by the proposed method are the same as those of the dynamic model employing the open-tree structure method.

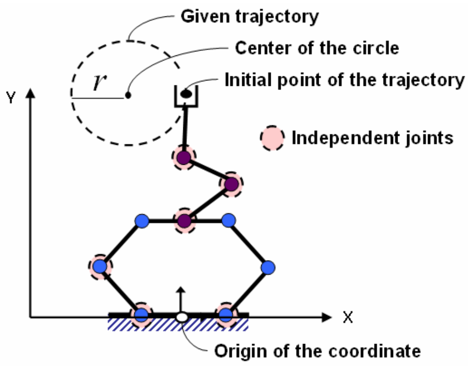

Fig. 6 Trajectory and initial configuration of the robot

A given operational trajectory shown in the Fig. 6 is a circle whose the radius(r) is 5 cm. The center point (x_c, y_c) and an initial point of the end-effecter are obtained by initial joint angles given in Eq. (53) and (54). The rotational angle(Φ) between the distal link and the x-axis is fixed as 90°. And the trajectory is followed by the robot in four seconds. The initial and last velocities of the end-effecter are zero. The above mentioned trajectory can be represented as follows

$$ {}^{h}_{1}\underline{\phi} = [120^\circ\ \text{-}60^\circ\ \text{-}60^\circ\ 60^\circ\ 60^\circ\ \text{-}30^\circ\ ]^T , \quad (53)$$

$$ {}^{h}_{2}\underline{\phi} = [60^\circ\ 60^\circ\ 60^\circ\ ]^T , \quad (54)$$

$$ x = x_c + r\cos\alpha , \quad (55)$$

$$ y = y_c + r\sin\alpha , \quad (56)$$

$$ \alpha = \frac{6\pi}{{t_f}^2}t^2 - \frac{4\pi}{{t_f}^3}t^3, \qquad (t_f = 4) \quad (57)$$

and

$$ \Phi = 90^\circ . \quad (58)$$

When all lengths and masses of links are given 0.4 m and 0.1 kg, respectively, torque values obtained by the proposed method and the open-tree structure method are represented in Fig. 7 and Fig. 8, respectively.

As shown in Fig. 7 and 8, torque values obtained by the two methods are identical. The computational effort of the proposed method is much less than that of the open-chain method since it is not necessary to calculate the whole dynamics again, which was the case of the usual open-chain dynamic modeling methodology. Namely, it validates that the proposed method is an efficient dynamic modeling method for hybrid robotic systems.

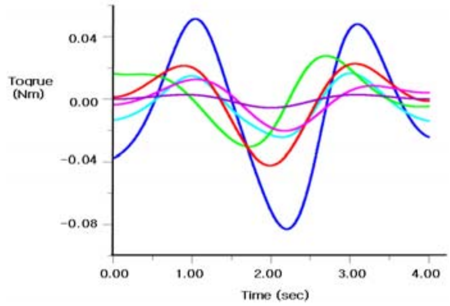

Fig. 7 Torque values obtained by the proposed method

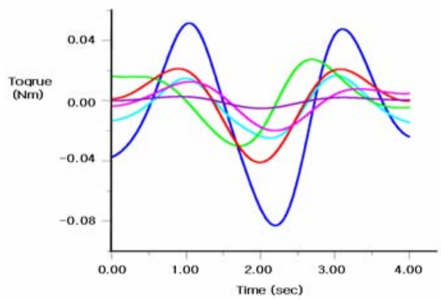

Fig. 8 Torque values obtained by the open-chain method

5. GENERAL FRAMEWORK FOR MODULAR-TYPE HYBRID ROBOTIC SYSTEMS

For multiple modules, the proposed scheme can be expanded as a recursive dynamic formulation, which results in reduction of the complexness of the open-tree structure method for the modeling of hybrid robotic systems. As shown in Fig. 9, the first-order kinematics and dynamics of the hybrid robotic systems that consist of 'n' modules are obtained by adding upper module's dynamics to the base module's dynamics one by one.

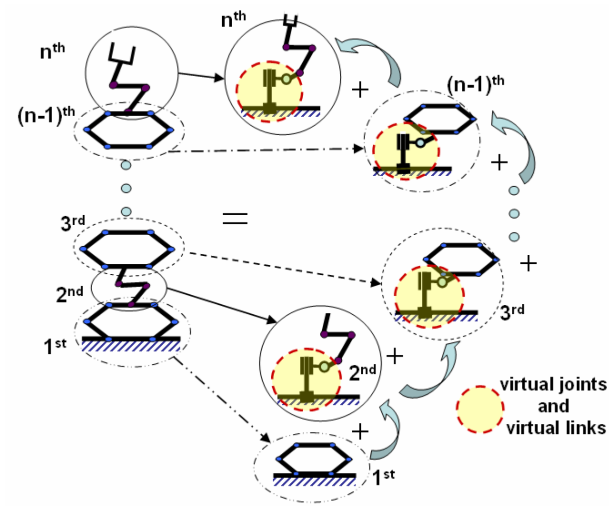

Fig. 9 An equivalent dynamic model of a hybrid robot constructed by 'n' modules

5. 1 Kinematics

The first-order kinematics of the (i-1)th module can be described, from Eq. (36), as

$$ {}_{i-1}\underline{\dot{u}} = {}^{h}_{i-1}[G^u_a]\, {}^{h}_{i-1}\underline{\dot{\phi}}_a , \quad (59)$$

where ${}_{i-1}\dot{\underline{u}}$, ${}^{h}_{i-1}\dot{\underline{\phi}}_{a}$, and ${}^{h}_{i-1}[G^{u}_{a}]$ are the output velocity vector of the i-1th module, the total independent joint velocity vector of the whole module up to i-1th module, and the Jacobian that relates the output vector to the total independent joints up to i-1th module. Also, the output velocity vector of the ith module can be expressed in terms of the ith virtual joints and the independent joints of the ith module as

$$ {}_{i}\dot{\underline{u}} = {}_{i}[G^{u}_{va}]\begin{bmatrix} {}_{i}\dot{\underline{\phi}}_{v} \\ {}_{i}\dot{\underline{\phi}}_{a} \end{bmatrix}, \tag{60} $$

where ${}_{i}\dot{\underline{\phi}}_{v}$, ${}_{i}\dot{\underline{\phi}}_{a}$, and ${}_{i}[G^{u}_{va}]$ are the velocity vectors of the ith virtual joints and the independent joints of the ith module, and the Jacobian of the ith module including virtual joints, respectively. Since the virtual joints' velocity of the ith module is identical to the output velocity of the (i-1)th module, Eq. (60) can be written as

$$ {}_{i}\dot{\underline{u}} = {}_{i}[G^{u}_{va}]\begin{bmatrix} {}_{i-1}\dot{\underline{u}} \\ {}_{i}\dot{\underline{\phi}}_{a} \end{bmatrix}, $$

$$ = \begin{bmatrix} {}_{i}[G^{u}_{v}] & {}_{i}[G^{u}_{a}] \end{bmatrix}\begin{bmatrix} {}_{i-1}\dot{\underline{u}} \\ {}_{i}\dot{\underline{\phi}}_{a} \end{bmatrix}. \tag{61} $$

Substituting Eq. (59) into Eq. (61) gives

$$ {}_{i}\dot{\underline{u}} = \begin{bmatrix} {}_{i}[G^{u}_{v}]\,{}^{h}_{i-1}[G^{u}_{a}] & \cdot \\ \cdot & {}_{i}[G^{u}_{a}] \end{bmatrix} {}^{h}_{i}\dot{\underline{\phi}}_{a}, \tag{62} $$

where

$$ {}^{h}_{i}\dot{\underline{\phi}}_{a} = \begin{bmatrix} {}^{h}_{i-1}\dot{\underline{\phi}}_{a} \\ {}_{i}\dot{\underline{\phi}}_{a} \end{bmatrix}. \tag{63} $$

Therefore, the Jacobian ${}^{h}_{i}[G^{u}_{a}]$ of the whole modules ranging from the first module to the ith module is obtained from Eq. (59) and (62) as

$$ {}^{h}_{i}[G^{u}_{a}] = \begin{bmatrix} {}_{i}[G^{u}_{v}]\,{}^{h}_{i-1}[G^{u}_{a}] & \cdot \\ \cdot & {}_{i}[G^{u}_{a}] \end{bmatrix}. \tag{64} $$

5. 2 Dynamics

The dynamic equation ${}^{h}_{i}\underline{\tau}_{a}$ for a hybrid robotic system that is composed of 'i' modules is obtained by reiteration of the process proposed in section 3.3.

The dynamics of the first module is given by

$$ {}^{h}_{1}\underline{\tau}_{a} = {}_{1}\underline{\tau}_{a}, \tag{65} $$

where ${}_{1}\underline{\tau}_{a}$ denotes the motion torque with respect to the independent joints of the first module itself.

The dynamics of the second module is given by

$$ {}^{h}_{2}\underline{\tau}_{a} = \begin{bmatrix} {}^{h}_{1}[G^{u}_{a}]^{T}\;{}_{2}\underline{\tau}_{v} + {}^{h}_{1}\underline{\tau}_{a} \\ {}_{2}\underline{\tau}_{a} \end{bmatrix}, \tag{66} $$

where ${}_{2}\underline{\tau}_{v}$ and ${}_{2}\underline{\tau}_{a}$ denotes the motion torques at the virtual joints and the independent joints of the second module itself, respectively.

Conclusively, the dynamics ${}^{h}_{i}\underline{\tau}_{a}$ of the hybrid robotic system that consists of 'i' modules is obtained as

$$ {}^{h}_{i}\underline{\tau}_{a} = \begin{bmatrix} {}^{h}_{i-1}[G^{u}_{a}]^{T}\;{}_{i}\underline{\tau}_{v} + {}^{h}_{i-1}\underline{\tau}_{a} \\ {}_{i}\underline{\tau}_{a} \end{bmatrix}, \tag{67} $$

where ${}_{i}\underline{\tau}_{v}$ denotes the dynamics at the virtual joints to support the dynamic motion of the ith module, ${}_{i}\underline{\tau}_{a}$ denotes the dynamics corresponding to the independent joints of the ith module itself, and ${}^{h}_{i-1}\underline{\tau}_{a}$ denotes the dynamics of the (i-1) modules, respectively.

6. CONCLUSIONS

In this paper, we proposed a new efficient dynamic modeling methodology for hybrid robotic system and validated the proposed method by comparing torque values of the proposed method with those of the existing method. Our future work is to apply the proposed method to general type of hybrid robotic mechanisms and design modular type hybrid robotic systems.

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