Fault Detection and Reconstruction for Descriptor Systems with Actuator and Sensor Faults

Tae-Kyeong Yeu, Nobutomo Matsunaga and Shigeyasu Kawaji

Graduate School of Science and Technology, Kumamoto University, Kumamoto 860-8555, JAPAN
(Tel: +81-96-342-3639; E-mail: yeu@actrl.cs.kumamoto-u.ac.jp)
(Tel: +81-96-342-3639; E-mail: matunaga@cs.kumamoto-u.ac.jp)
(Tel: +81-96-342-3631; E-mail: kawaji@cs.kumamoto-u.ac.jp)

Abstract: This paper proposes an application of sliding mode observer to the problem of fault detection and reconstruction for descriptor systems with both actuator and sensor faults. In detecting and reconstructing the faults simultaneously, first, we will consider the fault detection problem for sensor fault. The detection of sensor fault is achieved from the design of the matrix which eliminates the influence of actuator fault. Secondly, the sliding mode observer which adds the general full-order observer for descriptor system to feedforward injection map and feedforward compensation signal is designed, and through which the sensor fault is reconstructed. Finally, with the reconstructed sensor fault, and by eliminating differential term of the sensor fault, the actuator fault is detected and reconstructed.

Keywords: Fault detection and isolation(FDI), Descriptor system, Sliding mode observer

1. INTRODUCTION

The process of detecting and isolating system faults has been of considerable interest during the last two decades[1]-[3]. Research is still under way into development of more effective solution for fault detection and isolation(FDI) in automatic control systems. The purpose of fault detection is to determine occurrence of fault in the plant, whereas fault isolation is to determine location of fault after its detection. The most obvious method for automatic fault detection is the use of hardware redundancy, where measurements from multiple sensors are compared with each other and existence of fault is determined by implementing a voting mechanism. In many cases, however, hardware redundancy may not be possible or desirable, since it imposes penalty in terms of volume, weight and costs etc.. In other cases where fault exists in the actuator, its direct measurement is often not possible. In this case, indirect measurements may be used to infer the component fault status using model of the plant. One method to analytically detect the existence of fault is to look for anomalies in the plant's output relative to model-based estimate of the output. For this FDI problem, the most effective way will be observer-based approach in which the difference between actual and estimated outputs is used as residual vector. That is, function observer for fault detection is to estimate the output of the system from the measurements, and then to construct the residual by properly weighted output estimate error.

In general, a popular and straightforward observer is one by Luenberger[4]. We may select an observer gain such that error state decays suitably fast, but in practical problems assignment of eigenvalues of the error system too far into the left half-plane leads to excessive noise amplification. Whereas, sliding mode observer was proposed by Utkin[5]. The fundamental difference between sliding mode observer and others is that sliding mode observer has discontinuous input terms such that error trajectories move onto a specified attractive hyperplane. Robustness, insensitivity properties, simplicity of design and straightforward implementation are motivations to consider sliding observers as a powerful and deserving observer approaches. A. J. Koshkouei and A. S. I. Zinober[6] proposed a new approach for designing sliding mode observer and proof of stability of the state reconstruction system for linear time-invariant SISO systems using the Lyapunov method. They presented a feedforward injection map and new conditions which ensure the stability of the error reconstruction dynamics and the existence of the sliding mode. C. Edwards et al.[7] provided some new developments in the use of sliding mode observer theory for decoupling the effects of fault signals from the response of the estimated system outputs. Particularly, they have attempted to reconstruct the fault rather than to detect the presence of fault through residual signal. However, these works have not considered for the systems with both sensor fault and actuator faults, since the difficulty of removing the influence of the actuator fault in detecting the sensor fault and the design constraints.

This paper proposes an application of sliding mode observer to the problem of fault detection and reconstruction for descriptor systems with both actuator and sensor faults. In detecting and reconstructing the faults simultaneously, first, we will consider the problem for the sensor fault. The detection of sensor fault is achieved from the design of the matrix which eliminates the influence of actuator fault. Secondly, the sliding mode observer which adds general full-order observer for the descriptor system to feedforward injection map and feedforward compensation signal is designed, and through which the sensor fault is reconstructed. Finally, with the reconstructed sensor fault, and by eliminating differential term of the sensor fault, the actuator fault is detected and reconstructed. Throughout this paper, the notation $||\sharp||$ will be used to represent the Euclidean norm for vectors and spectral norm for matrices and the superscript + denotes the generalized matrix inverse.

2. PROBLEM STATEMENT

Consider the following descriptor system with actuator and sensor faults as

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bu(t) + \eta_a f_a(t) \\ y(t) = Cx(t) + \eta_s f_s(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $f_a(t) \in \Re^{q_a}$, $f_s(t) \in \Re^{q_s}$ and $y(t) \in \Re^p$ denote the state, the control input, the actuator fault, the sensor fault and the output respectively. The matrices E, A, B, η_a, η_s and C are constant and real-valued with appropriate dimensions. E, rank $E = r$, is singular matrix and η_a and η_s are assumed to be full column rank. Throughout this paper we assume that the system (1) is regular, and is controllable and observable in sense of the Rosenbrock. Also, $f_a(t)$ and $f_s(t)$ are bounded, i.e., there exist a non-negative real number m_a and m_s such that $|f_a(t)| \leq m_a$ and $|f_s(t)| \leq m_s$ respectively.

Now, the problem is to determine a proper matrix in order to detect the faults of the system (1). Further, through the sliding mode observer with feedforward injection map and compensation signal, we attempt the reconstruction of the faults.

3. RECONSTRUCTION OF SENSOR FAULT

In this section, we consider the reconstruction of measurement fault $f_s(t)$ for the system (1) as

$$\begin{cases} E\dot{x}_s(t) = Ax_s(t) + Bu(t) + \eta_a f_a(t) \\ y_s(t) = Cx_s(t) + \eta_s f_s(t) \end{cases} \tag{2}$$

First, to reconstruct the sensor fault without the influence of actuator fault $f_a(t)$, the actuator term needs to be eliminated in system (2). For this, the matrix R_a which is satisfied with $R_a\eta_a = 0$ may be determined as

$$R_a = (I_n - Q_aC)\left\{I - E^\sharp\eta_a(CE^\sharp\eta_a)^+C\right\}E^\sharp \tag{3}$$

where the matrices $E^\sharp \in \Re^{n\times n}$ and $C^\sharp \in \Re^{n\times p}$ are satisfied with

$$\begin{bmatrix} E^\sharp & C^\sharp \end{bmatrix}\begin{bmatrix} E \\ C \end{bmatrix} = I_n \tag{4}$$

and Q_a is an arbitrary matrix which makes $(I_n - Q_aC)$ nonsingular. As multiplying the system (2) by the matrix R_a, the following system which actuator term is eliminated is obtained

$$\begin{cases} R_aE\dot{x}_s(t) = R_aAx_s(t) + R_aBu(t) \\ y_s(t) = Cx_s(t) + \eta_s f_s(t) \end{cases} \tag{5}$$

where we assume conditions as follows

$$(i) \quad \text{rank}\begin{bmatrix} R_aE \\ C \end{bmatrix} = n$$

$$(ii) \quad \text{rank}\begin{bmatrix} R_a(sE - A) & 0 \\ C & \eta_s \end{bmatrix} = n + q_s$$

When sensor fault is known, i.e., $f_s(t) = \hat{f}_s(t)$, the state observer for the system (5) may be described as

$$\begin{cases} \dot{z}_s(t) = N_sz_s(t) + L_sy_s(t) + J_su(t) - L_s\eta_s\hat{f}_s(t) \\ \hat{x}_s(t) = V_sy_s(t) - z_s(t) \end{cases} \tag{6}$$

where $\hat{x}_s \in \Re^n$ and $z_s \in \Re^n$ are the estimated state vector and the transformed state vector respectively. And N_s, L_s, J_s and V_s are unknown matrices of appropriate dimensions. Then, we have the following lemma which describes the relation between descriptor system and observer.

Lemma 1

The system (6) is an state observer for the descriptor system (5) if

$$\text{Re}\ \lambda(N_s) < 0 \tag{7}$$

and there exist matrices $R_s \in \Re^{n\times n}$ and $V_s \in \Re^{n\times p}$ such that

$$N_s = R_sR_aE + K_sC \tag{8}$$

$$L_s = K_s - N_sV_s \tag{9}$$

$$J_s = -R_sR_aB \tag{10}$$

$$V_s\eta_s = 0 \tag{11}$$

$$R_sR_aE + V_sC = I_n \tag{12}$$

where $\text{Re}\lambda[\cdot]$ denotes the real part of eigenvalues.

(Proof)

First, the matrices $E_s{}^\sharp \in \Re^{n\times n}, \text{rank}E_s{}^\sharp = n$ and $C_s{}^\sharp \in \Re^{n\times p}, \text{rank}C_s{}^\sharp = p$ are defined as

$$\begin{bmatrix} E_s{}^\sharp & C_s{}^\sharp \end{bmatrix}\begin{bmatrix} R_aE \\ C \end{bmatrix} = I_n \tag{13}$$

And the state error is defined as

$$e_s(t) = R_sR_aEx_s(t) + z_s(t) \tag{14}$$

Then $\dot{e}_s(t)$ is governed by

$$\begin{aligned} \dot{e}_s(t) &= N_se_s(t) + (-N_sR_sR_aE + L_sC + R_sR_aA)x_s(t) \\ &+ (J_s + R_sR_aB)u(t) + L_s\eta_sf_s(t) - L_s\eta_s\hat{f}_s(t) \\ &- V_s\eta_s\dot{f}_s(t) \end{aligned} \tag{15}$$

which by invoking (7) - (11) reduces to

$$\dot{e}_s(t) = N_s e_s(t) \tag{16}$$

It follows from (7) that $e_s(t) \to 0$ as $t \to \infty$. Then, $\lim_{t\to\infty}[x_s(t) - \hat{x}_s(t)] = 0$.

□

For the design of state observer, we will solve (8) - (12) in the following. First, from (12),

$$V_s C = I_n - R_s R_a E \tag{17}$$

Substitution of (13) into (17) yields

$$V_s C \;=\; V_s(E_s{}^\sharp)^+(I_n - E_s{}^\sharp R_a E) = I_n - R_s R_a E \tag{18}$$

from which the matrix V_s is obtained as

$$V_s = (I_n - G R_a E) C_s{}^\sharp \tag{19}$$

where

$$G = R_s - V_s(C_s{}^\sharp)^+ E_s{}^\sharp$$

Multiplying of the matrix C into (19) yields

$$V_s C = (I_n - G R_a E)(I_n - E_s{}^\sharp R_a E) \tag{20}$$

Since rank $E = r$, we find

$$R_s = E_s{}^\sharp + G(I_n - R_a E E_s{}^\sharp) \tag{21}$$

Next, from (11) and (19), we get

$$C_s{}^\sharp \eta_s = G R_a E C_s{}^\sharp \eta_s \tag{22}$$

whose solution exists if rank $R_a E C_s{}^\sharp \eta_s =$ rank $\eta_s = q_s$. Note that $p \geq r - q_a > q_s$, i.e., the number of output and rank E - rank η_a must be great than that of fault. The general solution of (22) can be written as

$$\begin{aligned} G \;=\; & C_s{}^\sharp \eta_s (R_a E C_s{}^\sharp \eta_s)^+ + Q_s\{I_n - R_a E C_s{}^\sharp \\ & \eta_o (R_a E C_s{}^\sharp \eta_s)^+\} \end{aligned} \tag{23}$$

where Q_s is an arbitrary matrix. By substituting G in (23) into (19), we get

$$V_s = (I_n - Q_s R_a E)\{I_n - C_s{}^\sharp \eta_s (R_a E C_s{}^\sharp \eta_s)^+ R_a E\} C_s{}^\sharp \tag{24}$$

and can see form (24) that there exists the matrix Q_s which makes $(I_n - Q_s R_a E)$ nonsingular and the rank $V_s = p - q_s$. The remaining problem is to find the matrix K_s which stabilizes the matrix N_s.

The general form of sliding mode observer for the system (5) can be obtained as

$$\left\{ \begin{array}{l} \dot{z}_s(t) = N_s z_s(t) + L_s y_s(t) + J_s u(t) - \lambda_s v_s(t) \\ \hat{x}_s(t) = V_s y_s(t) - z_s(t) \end{array} \right. \tag{25}$$

where $v_s(t)$ is external feedforward compensation signal and $\lambda_s \in \Re^{n\times p}$ is the feedforward injection map such that $C\lambda_s \neq 0$. Without loss of the generality we assume that $C\lambda_s > 0$. And we assume that (N_s, λ_s) is a completely controllable.

Now, we determine the feedforward injection map and compensation signal which guarantee the stability of the error reconstruction dynamics and the condition of the existence through the same method proposed by T. K. Yeu and S. Kawaji[11]. Subtracting (25) from (5), the error dynamics is

$$\dot{e}_s(t) = N_s e_s(t) + L_s \eta_s f_s(t) - \lambda_s v_s(t) \tag{26}$$

and the output reconstruction error $e_{y_s}(t)$ is as follows

$$\dot{e}_{y_s}(t) = C N_s e_s(t) + C L_s \eta_s f_s(t) - C\lambda_s v_s(t) \tag{27}$$

Hence, the equation of the ideal sliding mode are obtained from conditions such as $e_{y_s}(t) = 0$ and $\dot{e}_{y_s}(t) = 0$. Therefore, the equivalent feedforward signal is given by

$$v_{eq_s}(t) = \frac{1}{C\lambda_s}(C N_s e_s + C L_s \eta_s f_s(t)) \tag{28}$$

Consider the discontinuous feedforward input as

$$v_s(t) = W_s \mathrm{sgn}(e_{y_s}(t)), \quad e_{y_s}(t) \neq 0 \tag{29}$$

where W_s is a positive real number or diagonal matrix, whose elements are $w_{sj} \geq \frac{|C_j L_s \eta_s|}{C_j \lambda_{sj}} m_s$, $j = 1, \cdots, p$. Hence, C_j and λ_{sj} are the j-th column element and row element of matrices C and λ_s respectively. We now establish conditions so that $\lim_{t\to\infty} e_s(t) = 0$.

Let P_s be the u.p.d.s. the solution of Lyapunov equation

$$N_s P_s + P_s{}^T N_s = -Q_s \tag{30}$$

where Q_s is an arbitrary p.d.s. matrix. For reaching the stability of the reconstruction error system, the feedforward injection map λ is chosen as

$$\lambda_s = P_s{}^{-1} C^T \tag{31}$$

and the following is considered

$$L_s \eta_s = \lambda_s \rho_s \tag{32}$$

so that the error dynamics obtained by substituting (28) into $v_s(t)$ in (26) be independent of the sensor fault, where ρ_s is a constant matrix. A Lyapunov function candidate for (26) is

$$V(e_s(t)) = e_s{}^T(t) P_s e_s(t) \tag{33}$$

where P_s is defined in (30). Then, $\dot{V}(e_s(t)) < 0$ and consequently $\lim_{t\to\infty} e_s(t) = 0$.

Next, under the above conditions, from (27) we have

$$C\lambda_s v_s(t) = C L_s \eta_s f_s(t) \tag{34}$$

hence the discontinuous component in (29) is replaced by the continuous approximation such as

$$v_{\delta_s}(t) = W_s \frac{e_{y_s}(t)}{||e_{y_s}(t)|| + \delta_s} \tag{35}$$

where δ_s is a small positive scalar. It follows from (34) that

$$f_s(t) \approx (CL_s\eta_s)^+(C\lambda_s)v_{\delta_s}(t) \quad (36)$$

Thus, the sensor fault is reconstructed as well as is detected from (36).

4. RECONSTRUCTION OF ACTUATOR FAULT

In the section 3, the sensor fault $f_s(t)$ was already reconstructed from the feedforward signals. Using the obtained fault, in this section, the actuator fault will be reconstructed.

Consider the following the descriptor system

$$\begin{cases} E\dot{x}_a(t) = Ax_a(t) + Bu(t) + \eta_a f_a(t) \\ y_a(t) = Cx_a(t) + \eta_s f_s(t) \end{cases} \quad (37)$$

To reconstruct the actuator fault, $f_a(t)$, the differential term of sensor fault must be eliminated. This problem is solved from the following lemma.

Lemma 2

As Lemma 1, for the system (37) the matrix ${C_a}^\sharp \in \Re^{n\times p}$ is defined as

$${C_a}^\sharp = (I_n - QE)\left\{I_n - C^\sharp\eta_s(EC^\sharp\eta_s)^+E\right\}C^\sharp$$

which is satisfied with the following conditions

$$(i) \quad \begin{bmatrix} {E_a}^\sharp & {C_a}^\sharp \end{bmatrix}\begin{bmatrix} E \\ C \end{bmatrix} = I_n$$

$$(ii) \quad {C_a}^\sharp\eta_s = 0$$

where Q is an arbitrary matrix. From the matrix ${C_a}^\sharp$, the differential term of sensor fault is eliminated.

(Proof)

The proof is omitted. □

As section 3, consider a suitable state estimation for descriptor system (37) so that the reconstruction error dynamics is asymptotically stable. If the state error equation is defined as

$$e_{a_s}(t) = E^\sharp Ex_a(t) + z_a(t) \quad (38)$$

The general form of the sliding mode observer for system (37) can be described as

$$\begin{cases} \dot{z}_a(t) = N_a z_a(t) + L_a y_a(t) + J_a u(t) - L_a\eta_s\hat{f}_s(t) \\ \qquad - E^\sharp\lambda_a v_a(t) \\ \hat{x}_a(t) = C^\sharp y_a(t) - z_a(t) \end{cases} \quad (39)$$

where $\hat{x}_a \in \Re^n$ and $z_a \in \Re^n$ are estimated state vector and the transformed state vector respectively. And

$$\begin{aligned} N_a &= E^\sharp E + K_a C, \quad L_a = K_a - N_a C^\sharp \\ J_a &= -E^\sharp B \end{aligned}$$

Note that the matrix K_a which stabilizes the matrix N_a is feedback control gain. Hence, $v_a(t)$ is an external feedforward compensation signal and $\lambda_a \in \Re^{n\times p}$ is the feedforward injection map such that $C\lambda_a \neq 0$.

Now, we determine the feedforward injection map and compensation signal. Subtracting (39) from (37), the error dynamics $\dot{e}_a(t)$ is

$$\dot{e}_a(t) = N_a e_a(t) + E^\sharp\eta_a f_a(t) - \lambda_a v_a(t) \quad (40)$$

and the output error dynamics, $\dot{e}_{y_a}(t)$ is as follows

$$\dot{e}_{y_a}(t) = CN_a e_a(t) + CE^\sharp\eta_a f_a(t) - C\lambda_a v_a(t) \quad (41)$$

The 'virtual' equivalent feedforward signal may be given by

$$v_{eq_a} = \frac{1}{C\lambda_a}(CN_a e_a + CE^\sharp\eta_a f_a) \quad (42)$$

We desire to find λ_a and $v_a(t)$ such that the stability property of the system is preserved. We can interpret λ_a and $v_a(t)$ as the control input distribution map and the input of the reconstruction error system, respectively.

Consider the discontinuous feedforward input as

$$v_a(t) = W_a \text{sgn}(e_{y_a}(t)), \quad e_{y_a}(t) \neq 0 \quad (43)$$

where W_a is a positive real number or diagonal matrix, whose elements are $w_{aj} \geq \frac{|C_j E^\sharp\eta_a|}{C_j\lambda_{aj}} \cdot m_a$. We now establish conditions so that $\lim_{t\to\infty} e_a(t) = 0$.

Let P_a be the u.p.d.s. solution of Lyapunov equation

$$N_a P_a + {P_a}^T N_a = -Q_a \quad (44)$$

where Q_a is an arbitrary p.d.s. matrix. Choose

$$\lambda_a = {P_a}^{-1}C^T, \quad (45)$$

and suppose the following holds

$$E^\sharp\eta_a = \lambda_a\rho_a \quad (46)$$

where ρ_a is a constant matrix. The choice (45) and the condition (46) are now lead to stability of the reconstruction error system. A Lyapunov function candidate for (40) is

$$V(e_a(t)) = {e_a}^T(t)P_a e_a(t) \quad (47)$$

where P_a is defined in (44). Then, $\dot{V}(t) < 0$ and consequently $\lim_{t\to\infty} e_a(t) = 0$. Under the above conditions, from (41) we have

$$C\lambda_a v_a(t) = CE^\sharp\eta_a f_a(t) \quad (48)$$

Here, an alternative approach[7] will be employed, then the discontinuous component in (43) is replaced by the continuous approximation such as

$$v_{\delta_a}(t) = W_a\frac{e_{y_a}(t)}{||e_{y_a}(t)|| + \delta_a} \quad (49)$$

where δ_a is a small positive scalar. It can be shown that the equivalent output injection can be approximated to any degree of accuracy by (49) for a small enough choice of δ_a. It follows from (48) that

$$f_a(t) \approx (CE^\sharp\eta_a)^+(C\lambda_a)v_{\delta_a}(t) \quad (50)$$

From (50), the actuator fault, $f_a(t)$ is reconstructed.

5. SIMULATION RESULTS

Consider a descriptor system with both measurement fault and input fault such as

$$E = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}, \quad \eta_a = \begin{bmatrix} 0.0 \\ 1.0 \\ 1.0 \\ 0.0 \end{bmatrix},$$

$$A = \begin{bmatrix} -2.2 & 0.0 & 0.0 & 0.0 \\ 0.0 & -1.6 & 0.2 & 0.0 \\ 0.0 & 0.9 & -1.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & -1.7 \end{bmatrix},$$

$$C = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 1.0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \\ 1.0 \end{bmatrix}, \quad \eta_s = \begin{bmatrix} 1.0 \\ 1.0 \\ 0.0 \end{bmatrix}$$

Case 1 : FDI problem of sensor fault

Step 1 : The matrices $E^\sharp$ and $C^\sharp$ are defined as follows

$$E^\sharp = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix}$$

$$C^\sharp = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.5 & 0.5 & -0.5 \\ -0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 \end{bmatrix}$$

Step 2 : The matrix R_a is obtained from (3)

$$R_a = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \end{bmatrix}$$

Step 3 : The matrices ${E_s}^\sharp$ and ${C_s}^\sharp$ are defined as follows

$${E_s}^\sharp = \begin{bmatrix} 0.71 & 0.00 & 0.00 & -0.00 \\ -0.00 & 0.41 & 0.41 & -0.41 \\ 0.41 & 0.33 & -1.08 & -0.33 \\ 0.58 & 0.47 & -0.53 & 0.53 \end{bmatrix}$$

$${C_s}^\sharp = \begin{bmatrix} 0.00 & 0.00 & -0.00 \\ 0.41 & 0.41 & -0.41 \\ 0.33 & -1.08 & -0.33 \\ 0.47 & -0.53 & 0.53 \end{bmatrix}$$

Step 4 : From (24), the matrix V_s is calculated as

$$V_s = \begin{bmatrix} -0.00 & 0.00 & 0.00 \\ -0.00 & 0.00 & 0.00 \\ 0.71 & -0.71 & -0.71 \\ 0.50 & -0.50 & 0.50 \end{bmatrix}$$

Step 5 : The matrix λ_s is obtained from (31)

$$\lambda_s = \begin{bmatrix} 4.7784 & -0.1444 & -0.5069 \\ -0.4310 & 5.6820 & -0.2294 \\ -0.9985 & 0.0573 & 4.7343 \\ 6.4397 & -0.2867 & 5.9482 \end{bmatrix}$$

Case 2 : FDI problem of actuator fault

Step 1 : The matrices ${E_a}^\sharp$ and ${E_a}^\sharp$ which are satisfied with Lemma 2 are calculated as

$${E_a}^\sharp = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ -0.5 & 0.5 & -0.5 & 1.5 \end{bmatrix}$$

$${C_a}^\sharp = \begin{bmatrix} 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.5 & -0.5 & 0.5 \end{bmatrix}$$

Step 2 : The matrix λ_a is obtained from (45)

$$\lambda_a = \begin{bmatrix} 11.4873 & 1.2881 & -2.7568 \\ 1.4492 & 13.1728 & -2.5820 \\ 0.0134 & -2.7431 & 22.0199 \\ 13.5603 & 0.1611 & 16.3306 \end{bmatrix}$$

Based on the above matrices, the simulation was done for the numerical descriptor system with faults as Fig. 1. Here, we assume that the actuator has input trouble of sine curve and the sensor possesses bias problem.

Fig. 2 and 3 show the reconstructed sensor fault(solid line) and reconstructed actuator fault(solid line) respectively. From the simulation results of Fig. 2 and 3, we can see that the reconstructed faults almost correspond with the faults of Fig. 1.

6. CONCLUSIONS

This paper proposed the application of sliding mode observer to the problem of fault detection and reconstruction for descriptor systems with both actuator and sensor faults. In detecting and reconstructing the faults simultaneously, first, we considered for sensor fault. The detection of the sensor fault was achieved from the design of the matrix which eliminates the influence of actuator fault. Secondly, for the system excepted the actuator fault, the sliding mode observer which adds the general full-order observer to the feedforward injection map and the feedforward compensation signal was designed. The reconstruction of sensor fault was achieved from the feedforward signals and the output error between the system and the sliding mode observer.

The detection and reconstruction of actuator fault were achieved with the reconstructed sensor fault and by eliminating the differential term of sensor fault. Finally, from the numerical simulation, we proved that the reconstructed faults almost correspond with the real faults.

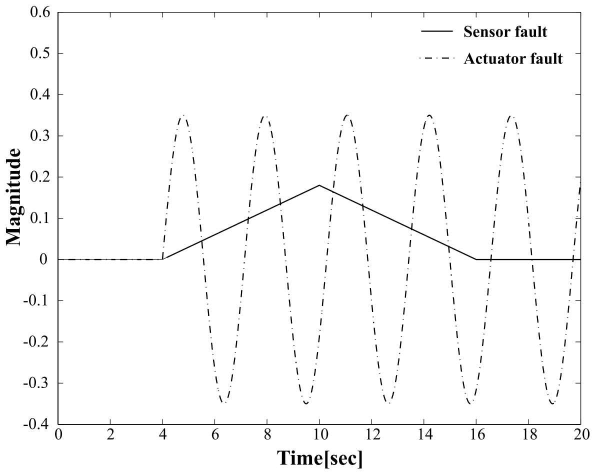

Fig. 1. The actuator fault and sensor fault

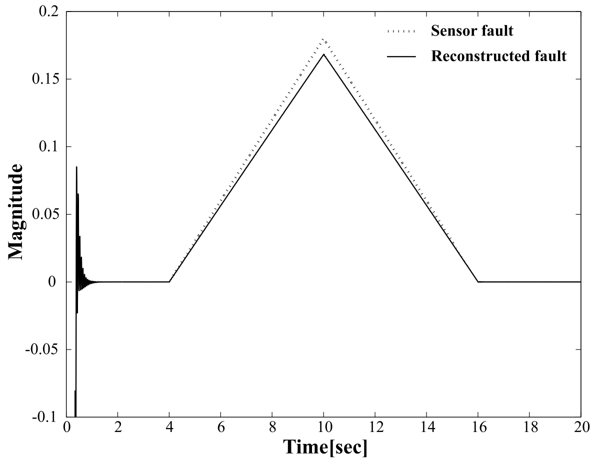

Fig. 2. The result of the reconstructed sensor fault

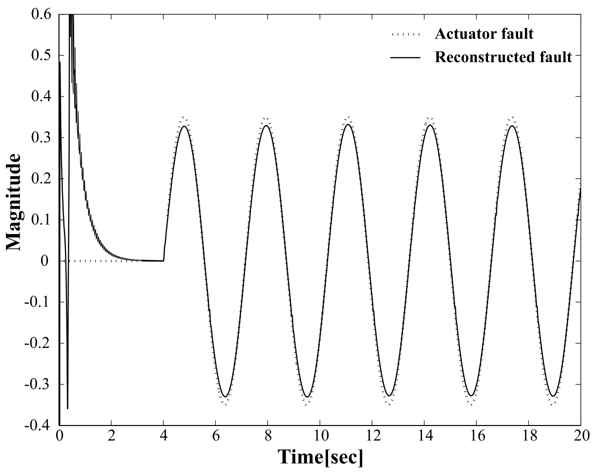

Fig. 3. The result of the reconstructed actuator fault

References

[1] K. Watanabe and D. M. Himmelblau, " Instrument Fault Detection in Systems with Uncertainties", *International Journal of System Science*, Vol.13, pp. 137-158, 1982.

[2] J. Chen and H. Y. Zhang, " Robust Detection of Faulty Actuators via Unknown Input Observers", *International Journal of System Science*, Vol.22, pp. 1829-1839, 1991.

[3] J. Chen, R. J. Patton and J. U. Zhang, " Design of Unknown Input Observers and Robust Fault Detection Filters", *Intrnational Journal of Control*, Vol.63, No.1, 85-105, 1996.

[4] D. G. Luenberger, " An Introduction to Observers", *IEEE Tran. on Automatic Control*, Vol.AC-16, No.8, pp.596-602, 1971.

[5] V. I. Utkin, Sliding Modes in Control and Optimization (Berlin : Springer - Verlag), 1992.

[6] A. J. Koshkouei, A. S. I. Zinober, " Sliding Mode Controller - Observer Design for SISO Linear Systems", *International Journal of Systems Science*, Vol.29, No.12, pp. 1363-1373, 1998.

[7] C. Edwards, S. K. Spurgeon and R. J. Patton," Sliding Mode Observers for Fault Detection and Isolation", *Automatica*, Vol.36, pp. 541-553, 2000.

[8] M. M. Fahmy and J.O'reilly," Observers for Descriptor Systems", *International Journal of Control*, Vol. 49, No.6, 2013-2028, 1989.

[9] S. Kawaji," Design of Observer for Linear Descriptor Systems", *IFAC 11th Triennial World Congress*, pp. 241-245, 1990.

[10] T. K. Yeu and S. Kawaji, " Decentralized Observer-Based Control Scheme for Interconnected Descriptor System with Unknown Inputs", *ISIE2001*, pp. 1702-1707, 2001.

[11] T. K. Yeu and S. Kawaji, " Fault Detecting and Isolation for Descriptor Systems Using Sliding Mode Observer ", *Proc. of the 40th IEEE Conference on Decision and Control*, Florida, pp. 596-597, 2001.