Control Of Flexible Multi-Body System

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Abstract: An alternative optimal control law formulation is introduced and compared with two different control law, a conventional linear quadratic regulator and the control law based on game theory. This formulation eliminates the undesired modes of the system by the projection of a controller onto the subspace orthogonal to that of the bad modes. In conventional LQR control law, the control performance can be improved only by using proper weighting matrices in performance index, normally, with high cost. The control law formulation by game theory may provide various ways to obtain the desired performance. The control law modified by the elimination of bad modes provides efficient ways to get rid of an undesired performance since it eliminates the exact modes which cause the bad control performance.

Keywords: Multi-body Dynamics, Optimal Control Theory, Flexible body

1. INTRODUCTION

A conventional approach for designing an optimal control law for the control of a multi-body structure is to use linear quadratic regulator (LQR) theory with a properly chosen performance index. The LQR approach is well developed and widely used. However, if the system to be controlled has flexible elements, especially if it has very high frequency vibration mode, then LQR is not a suitable method to get a desired performance.

Cho et al.[1], and Fitz-coy et al.[2] proposed an alternative optimal control law formulation based on N-player, nonzero-sum, linear quadratic, differential game theory. In this formulation, control law can be obtained by the appropriate selection of control strategy for the target system.

In this paper, the system is uncontrollable for some value of structural parameters (mass properties and elastic constant), since this parameters affect the system matrix in linear system by which the controllability matrix is made. The analysis of the closed-loop poles may give an identification of the bad modes. The investigation of pole location of the closed-loop system gives the bad modes which should be dealt with to obtain the desired result.

We introduce an alternative way that is based on a projection method to solve this problem.

2. SYSTEM DYNAMICS

A simple multi-body model is shown in Fig. 1. The model consists of three bodies. Two (body 1 and body 2) are rigid and one (body 3) is flexible. The centers of mass of the labeled bodies, Ci , i = 1,2, and 3, are located at the geometric centers of the respective bodies. For the purpose of gaining a better insight into the controlled dynamics, only planar motion is considered. The flexible body is modeled as a "pined - free" Euler-Bernoulli beam.

The state vector of this system is

$$\boldsymbol{x} = (Y, \Theta_1, \Theta_2, \Theta_3, q, \dot{Y}, \omega_1, \omega_2, \omega_3, \dot{q})^T \ . \tag{1}$$

Here, ω1, ω2, and ω3 are the total angular rates of the respective bodies. The output states are expressed as

$$\boldsymbol{y} = (Y, \Theta_1, \Theta_2, \Theta_3, w) \ . \tag{2}$$

In the output, w represents the total transverse displacement of the flexible body and is approximated by

$$w = \sum_{j}^{n} [\, \phi_j(x_3) q_j(t) \,] \ , \tag{3}$$

where $\phi_j(x_3)$ is j^{th} mode shape function and $q_j(t)$ is generalized coordinates. Eq. (3) comes from the uniform beam equations commonly used in vibration theory[3].

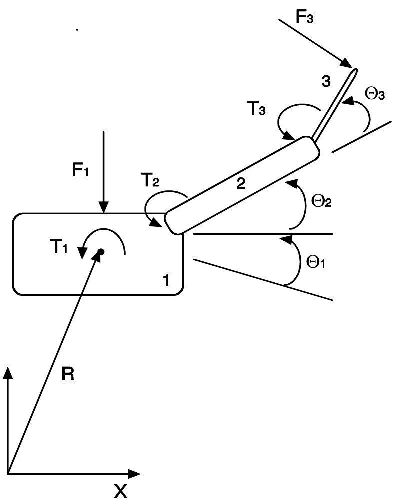

Fig. 1 Flexible Multi-Body System.

3. CONTROL LAW MODIFICATION

3.1 Conventional Optimal Control Law

The performance index for the LQR theory is defined as

$$J = \int_0^\infty \left(\boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} + \boldsymbol{u}^T \boldsymbol{R} \boldsymbol{u} \right) dt \ , \tag{4}$$

where **Q** is a positive semi-definite weighting matrix, and R is a positive definite matrix[4].

The state-space representation of the linearized system is defined as

$$\dot{x} = Ax + Bu\,. \tag{5}$$

The control inputs are

$$u = (F_1, T_1, T_2, T_3, F_3)^{\mathrm{T}}\,. \tag{6}$$

The LQR control law is expressed as

$$u = R^{-1}B^T Px = Kx\,, \tag{7}$$

where P is the solution to the Algebraic Riccati equation.[4]

3.2 Control Law Modification-"Game Theory"

In "game theory", a linear dynamic system can be represented in the state space form [5,6]

$$\dot{x} = Ax + B_1 u_1 + B_2 u_2\,, \tag{8}$$

$$y_1 = C_1 x_1\,, \tag{9}$$

$$y_2 = C_2 x_2\,. \tag{10}$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ represent separate control inputs to the plant by players 1 and 2 respectively; **x** represents the plant's state (current condition of the game); $\mathbf{y_1}$ and $\mathbf{y_2}$ represent the plant outputs; and $\mathbf{C_1}$ and $\mathbf{C_2}$ are observation matrices. The nonzero-sum differential game is formulated by allowing the controllers (players) to minimize quadratic performance indices J_1 and J_2, defined as

$$J_1 = \int_0^\infty (y_1^T Q_1^* y_1 + u_1^T R_{11} u_1 + u_2^T R_{12} u_2)dt\,, \tag{11}$$

$$J_2 = \int_0^\infty (y_2^T Q_2^* y_2 + u_1^T R_{21} u_1 + u_2^T R_{22} u_2)dt\,, \tag{12}$$

where $\mathbf{Q_1^*}$, $\mathbf{Q_2^*}$, $\mathbf{R_{12}}$, $\mathbf{R_{21}}$, are positive semi-definite matrices, and $\mathbf{R_{11}}$ and $\mathbf{R_{22}}$ are positive definite matrices. The advantage of using output vectors $\mathbf{y_1}$ and $\mathbf{y_2}$ instead of the state vector x is that more attention can be given to minimizing particular combinations of the state variables rather than to minimizing all individual states. Equation (2) can be used to write the following conventional forms of the performance indices:

$$J_1 = \int_0^\infty (x^T Q_1 x + u_1^T R_{11} u_1 + u_2^T R_{12} u_2)dt\,, \tag{13}$$

$$J_2 = \int_0^\infty (x^T Q_2 x + u_1^T R_{21} u_1 + u_2^T R_{22} u_2)dt\,, \tag{14}$$

where $\mathbf{Q_1} = \mathbf{C_1^T Q_1^* C_1}$ and $\mathbf{Q_2} = \mathbf{C_2^T Q_2^* C_2}$. The controls that satisfy the Nash strategy are linear functions of the state variables[5,6], that is,

$$u_1 = R_{11}^{-1} B_1^T P_1 x\,, \tag{15}$$

$$u_2 = R_{22}^{-1} B_2^T P_2 x\,, \tag{16}$$

where $\mathbf{P_1}$ and $\mathbf{P_2}$ are solutions to the coupled Algebraic Riccati Equations (ARE's).

$$\begin{aligned} &-P_1 A - A^T P_1 - Q_1 \\ &\quad + P_1 B_1 R_{11}^{-1} B_1^T P_1 + P_1 B_2 R_{22}^{-1} B_2^T P_2 \\ &\quad + P_2 B_2 R_{22}^{-1} B_2^T P_1 - P_2 B_2 R_{22}^{-1} R_{12} R_{22}^{-1} B_2^T P_2 = 0 \end{aligned}\,, \tag{17}$$

$$\begin{aligned} &-P_2 A - A^T P_2 - Q_2 \\ &\quad + P_2 B_2 R_{22}^{-1} B_{21}^T P_2 + P_2 B_1 R_{11}^{-1} B_1^T P_1 \\ &\quad + P_1 B_2 R_{11}^{-1} B_1^T P_2 - P_1 B_1 R_{11}^{-1} R_{21} R_{11}^{-1} B_1^T P_1 = 0 \end{aligned}\,, \tag{18}$$

The coupled ARE's may be solved by using a Newton-Raphson-Kantorovich (NRK) algorithm[1,2].

3.3 Control Law Modification-"Bad Mode Elimination"

In many cases, vibrations of f1exible structure cause a controllability problem which derives mainly from the high frequency of bad modes. In our example, the system is uncontrollable for some value of structural parameters (mass properties and elastic constant), since this parameters affect the system matrix A and B in eq.(5) by which the controllability matrix is made. The analysis of the closed-loop poles may give an identification of the bad modes.

From eqs.(5)and (7), closed-loop system is expressed by

$$\dot{x} = A^* x\,, \tag{19}$$

where $\mathbf{A^* = A\text{-}BK}$. The investigation of pole location of the closed-loop system in the z-plane gives the bad modes which should be dealt with to obtain the desired result.

A method which can be used to eliminate the undesired mode by projection of the controller onto the subspace orthogonal to that of the bad modes, such as

$$Bu^* = Bu - \frac{b^T Bb}{b^T b} u\,, \tag{20}$$

where $\mathbf{u^*}$ is new control law and **b** is the eigenvector of a corresponding bad mode. Then $\mathbf{u^*}$ can be written as

$$u^* = (B^T B)^{-1} B^T \left[Bu - \frac{b^T Bb}{b^T b} u \right]. \tag{21}$$

Table 1 Closed-loop Poles for LQR.

Pole	Damping Ratio	Frequency
-6.86e-001 + 6.86e-001i	7.07E-01	9.70E-01
-6.86e-001 - 6.86e-001i	7.07E-01	9.70E-01
-7.72e-001 + 3.56e+001i	**2.17E-02**	**3.56E+01**
-7.72e-001 – 3.56e+001i	**2.17E-02**	**3.56E+01**
-9.97e-001 + 9.97e-001i	7.07E-01	1.41E+00
-9.97e-001 - 9.97e-001i	7.07E-01	1.41E+00
-3.39e+000 + 3.39e+000i	7.07E-01	4.80E+00
-3.39e+000 - 3.39e+000i	7.07E-01	4.80E+00
-5.28e+000 + 5.27e+000i	7.07E-01	7.46E+00
-5.28e+000 - 5.27e+000i	7.07E-01	7.46E+00

An example of the uncontrollable closed-loop poles, damping ratio, and damping frequency for the system in Fig. 1 are given in Table 1. The bad modes of this system are the third and 4th modes, which have high frequencies and low damping.

4. SIMULATION RESULTS

Results obtained by using the "game theory" and "bad mode elimination method" are compared with the result of LQR control law in this section. The maneuver to be controlled is the simultaneous rotation of Body 2 by 5 degrees relative to body 1 and body 3 by 5 degrees relative to body 2. The attitude of body 1 is to be maintained as close to being fixed as possible with the given controller. Control is via three torques and two forces. Torque T_1 and force F_1 act on body 1; torques T_2 and T_3 act between bodies 1 and 2, and bodies 2 and 3, respectively. Force F_3 acts on the tip of the flexible body 3. The final time is open. Specified performance criteria are $|\Theta_1| < 0.1$ deg, and Θ_2 and Θ_3 both within 0.1 deg of 5 deg in 10 sec.

In Figs. 2 and 3, the output states generated by using LQR control theory and game theory are compared. Strategy set for the game theory formulation is selected as $\mathbf{x_1} = [Y, \Theta_1, w]^T$ and $\mathbf{x_2} = [\Theta_2, \Theta_3]^T$. The weighting matrices are provided in Table 2. As we expected, the amount of deformation of flexible body is much smaller when game theory is used.

Table 2 Weighting Matrices

$$Q^* = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 500 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix} \times 10^6$$

$$R = \begin{bmatrix} 150 & 0 & 0 & 0 & 0 \\ 0 & 30 & 0 & 0 & 0 \\ 0 & 0 & 60 & 0 & 0 \\ 0 & 0 & 0 & 120 & 0 \\ 0 & 0 & 0 & 0 & 900 \end{bmatrix}$$

$$Q_1^* = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 500 & 0 \\ 0 & 0 & 5 \end{bmatrix} \times 10^6 \qquad Q_2^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times 10^5$$

$$R_{11} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix} \qquad R_{22} = \begin{bmatrix} 120 & 0 \\ 0 & 120 \end{bmatrix}$$

$$R_{21} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad R_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figs. 4 and 5 shows that the effect of the bad mode elimination of the control law. It can be seen that the amount of deformation of flexible body is much smaller than by using LQR. However, the damping is too low to get a good steady state results in the desired time. In order to get an acceptable steady state output, the control laws are switched at the appropriate time. In Figs. 6 and 7, the modified control law is used for the first five second, and then the control law is switched to the LQR control law.

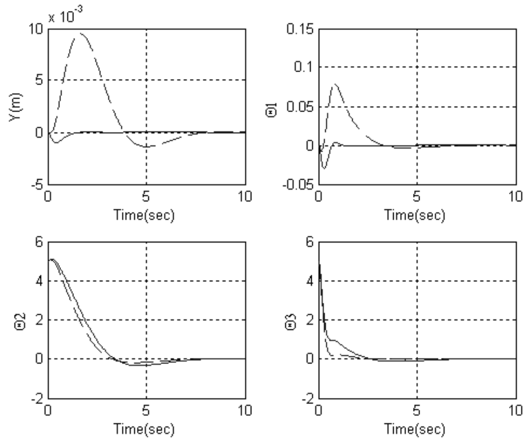

Fig. 2 Time History of Output States.
(LQR: dashed line, Game theory: solid line)

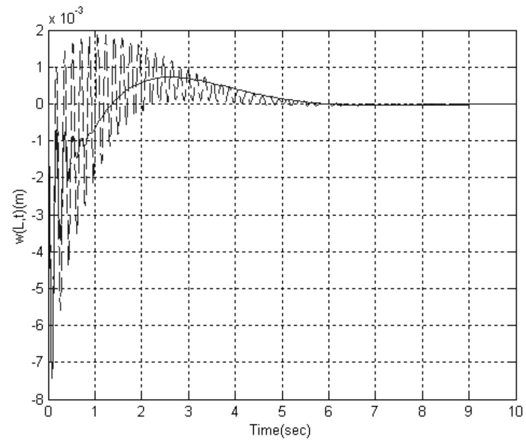

Fig. 3 Time History of Flexible Displacement.
(LQR: dashed line, Game theory: solid line)

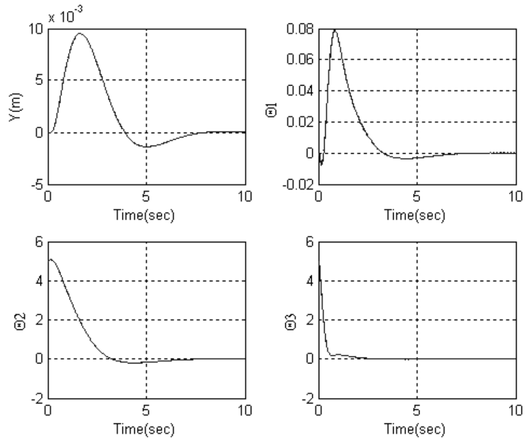

Fig. 4 Time History of Output States.
(LQR: dashed line, Bad mode elimination: solid line)

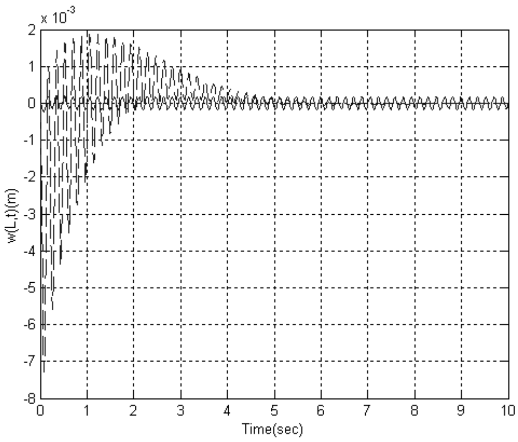

Fig. 5 Time History of Flexible Displacement.
(LQR: dashed line, Game theory: solid line)

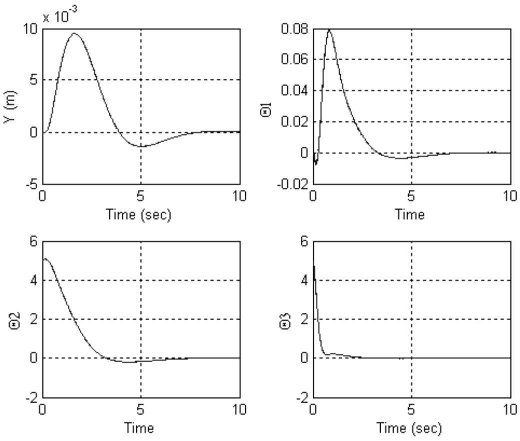

Fig. 6 Time History of Output States.
(LQR: dashed line, Control Switching: solid line)

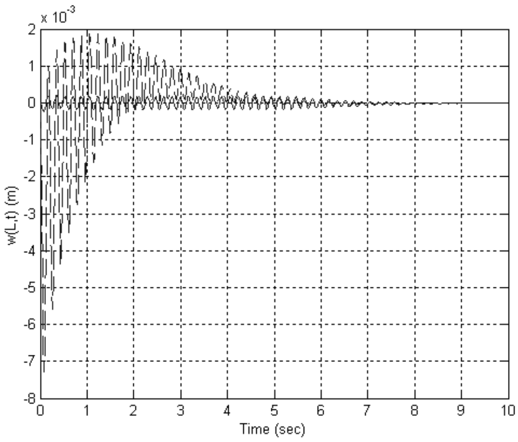

Fig. 7 Time History of Flexible Displacement.
(LQR: dashed line, Game theory: solid line)

5. CONCLUSION

A new control law, which can be used to eliminate the undesired mode by projection of the controller onto the subspace orthogonal to that of the bad modes are suggested.

In control problem, one main emphasis is how much it costs for the desired performance, especially in optimal control problem. In the LQR control law, the magnitude of flexible deformation can be reduced by using proper weighting matrices in performance index. For example, if a control designer gives higher weight to the state or lower weight to the control inputs, then the performance can be improved with high cost. Although the method suggested in this paper also needs higher costs for the desired results, it provides more efficient ways to get rid of an undesired performance since it eliminates the exact modes which cause the bad performance.

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