

# New Stability Analysis of a Single Link TCP Vegas Model

PooGyeon Park\*, Doo Jin Choi\*, and Yoon Jong Choi\*

\*Division of Electrical and Computer Engineering, POSTECH, Pohang, Kyungbuk 790-784, Korea  
 (Tel: +82-54-279-5588; Fax: +82-54-279-2903; Email: {ppg, chdj, scope75}@postech.ac.kr)

**Abstract:** This paper provides a new approach to analyze the stability of TCP Vegas, which is a kind of feedback-based congestion control algorithm. Whereas the conventional approaches use the approximately linearized model of the TCP Vegas along equilibrium points, this approach uses the exactly characterized dynamic model to get a new stability criterion via a piecewise and delay-dependent Lyapunov-Krasovskii function. Especially, the resulting stability criterion is formulated in terms of linear matrix inequalities (LMIs). Using the new criterion, this paper shows that the current TCP Vegas algorithm is stable in the sufficiently wide region of network delay and link capacity.

**Keywords:** Network congestion control, TCP Vegas, stability, linear matrix inequalities(LMIs).

## 1 INTRODUCTION

A feedback-based congestion control algorithm is essential to realize efficient data transfer services in packed-switched networks. TCP (Transmission Control Protocol), a category of feedback-based congestion control mechanisms, has been widely used in the current Internet. In 1981, RFC(Request for Comments)-793 [1] introduced the basic structure of the TCP, which is a window-based flow control mechanism to pace the transmission of packets. And, RFC-1122 [2] presented the second version of TCP called TCP Tahoe, which includes a congestion avoidance scheme and a fast retransmission additionally in 1989. Furthermore, RFC-2001 [3] presented the third version of TCP called TCP Reno, which includes a fast recovery scheme additionally to the TCP Tahoe in 1997. Now, the TCP Reno is adopted as a standard TCP algorithm by RFC.

Recently, another version of TCP called TCP Vegas has been proposed by Brakmo *et. al.* [4], which can achieve better performance than the TCP Reno. The TCP Vegas has the following advantages over the existing TCP Reno: (1) a new time-out mechanism, (2) an improved congestion avoidance mechanism, and (3) a modified slow-start mechanism. In particular, the congestion avoidance mechanism of the TCP Vegas controls the number of on-the-fly packets in the network. More specially, the TCP Vegas measures an RTT (Round Trip Time), which is elapsed time from a packet transmission to the receipt of its corresponding ACK (Acknowledgement) packet. It then uses the measured RTT as feedback information from the network. Namely, if the measured RTT is getting large, the source host of the TCP Vegas conjectures that the network is falling into congestion. Then, the window size is throttled. If the measured RTTs become small, on the other hand, the source host recognizes that the network is relieved from the congestion and thus increases the window size again. In the TCP Vegas, it is not necessary for the source host to wait for a packet loss in the network to detect congestion. This is an advantage of the TCP Vegas over other early versions of TCP. With this mechanism, the window size of a source host is expected to converge to a

constant value in steady state. The simulation and experimental results show that the congestion control mechanism of the TCP Vegas leads to 37-71 % higher throughput than that of the TCP Reno.

The dynamics of the TCP Vegas has been analytically investigated by several papers [5], [6], [7]. In those papers, the evolution of a window size has been approximated by a fluid model and the throughput of each connection has been obtained. However, the stability of the TCP Vegas has not been investigated at all, even though the TCP Vegas is essentially a feedback congestion control and thus a stable operation of the control mechanism is extremely important.

More recently, some people have tried to analyze the stability of TCP Vegas model based on the dynamic TCP model. But, since the dynamic TCP model is nonlinear time-varying system with time-delay, people imposed some critical assumptions such as no-delay, simple linear system, and so on, for the dynamic TCP model. And most of them presented some results based on the discrete-time versions of the dynamic TCP Vegas model [8], [9], [10]. Especially, stability analysis for continuous-time dynamics of TCP Vegas has been introduced by [11], [12]. They used a classical stability method for the loop gain of the dynamic TCP Vegas model in the frequency-domain. Furthermore, based on the proposed stability analysis method for the dynamic TCP Vegas, they presented a modified TCP Vegas algorithm. But, since they developed the new stability analysis method based on an approximated dynamic model of TCP Vegas, in which they approximated the  $sgn(\cdot)$  function with a smooth  $\tan^{-1}(\cdot)$  function, they did not sufficiently describe the stability of the dynamic TCP Vegas.

In this paper, we fully account the dynamics of single link TCP Vegas model. Then, we represent the TCP Vegas model with a piecewise linear system with time-delay. And we provide a new stability analysis criterion associated with a piecewise and delay-dependent Lyapunov-Krasovskii function for the system. Especially, this paper formulates the resulting stability criterion as a convex optimization problem in terms of linear matrix inequalities (LMIs). Based on the new criterion and the previous method Choe and Low [12], this paper shows that the current TCP Vegas algorithm can become

stable in the sufficiently wide region of network delay and link capacity. This paper is organized as follows. Section 2 describes the TCP Vegas model. And Section 3 presents a new stability criterion in terms of LMIs. Section 4 compares the stable regions obtained by the proposed new stability criterion and the loop gain method of Choe and Low [12]. The notation of this paper is fairly standard.

## 2 TCP VEGAS NETWORK MODEL

Before presenting a new model of the TCP Vegas, we shall briefly explain the general TCP Vegas network model. Generally, a network is modeled as a set of  $L$  links with finite capacities  $c = (c_l, l \in L)$ . They are shared by a set of  $N$  sources indexed by  $r$ . Each source  $r$  uses a set of links defined by the  $L \times N$  routing matrix

$$R_{lr} = \begin{cases} 1 & \text{if source } r \text{ uses link } l \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Associated with each link  $l$  is a congestion measure  $p_l(t)$  we will call ‘price’;  $p_l(t)$  is the scaled queueing delay at link  $l$ . Each source  $r$  maintains a rate  $x_r(t)$  in packets/sec. At time  $t$ , we assume source  $r$  observes the aggregate price in its path

$$q_r(t) = \sum_l R_{lr} p_l(t - \tau_{lr}^b) \quad (2)$$

and link  $l$  observes the aggregate source rate

$$y_l(t) = \sum_r R_{lr} x_r(t - \tau_{lr}^f) \quad (3)$$

where  $\tau_{lr}^f$  and  $\tau_{lr}^b$  denote the forward and backward RTTs of the equilibrium RTT  $T_r^*$  such as

$$T_r^* = \tau_{lr}^f + \tau_{lr}^b, \quad \forall l \in L \quad (4)$$

Then, Low *et al.* [13] models TCP Vegas, with its associated queue management, as the following dynamical system :

$$\dot{x}_r(t) = T_r^{-2}(t) \text{sgn}(x_r(t)q_r(t) - \alpha_r d_r) \quad (5)$$

$$\dot{p}_l(t) = \begin{cases} \frac{1}{c_l}(y_l(t) - c_l) & \text{if } p_l(t) > 0 \\ \frac{1}{c_l}(y_l(t) - c_l)^+ & \text{if } p_l(t) = 0 \end{cases} \quad (6)$$

where  $(z)^+ = \max\{0, z\}$ ,  $\text{sgn}(z) = 1$  if  $z > 0$ ,  $-1$  if  $z < 0$ , and  $0$  if  $z = 0$ . Here,  $\alpha_r$  is a Vegas protocol parameter, and  $d_r$  is the round trip propagation delay of source  $r$ . Price  $p_l(t)$  is the queueing delay at link  $l$  and  $q_r(t)$  is the end-to-end queueing delay of source  $r$ . Round trip time of source  $r$  is defined as

$$T_r(t) = d_r + q_r(t) \quad (7)$$

with equilibrium value  $T_r^*$  defined in (4). An interpretation of TCP Vegas algorithm is that each source  $r$  adjusts its rate to maintain  $\alpha_r d_r$  number of its own packets buffered in the queues in its path. The link algorithm (6) is automatically carried out by the buffer process. The source algorithm (5) increments or decrements the ‘‘window’’ by 1 packet per round trip time, according as the number  $x_r(t)q_r(t)$  of packets buffered in the links is smaller or bigger than  $\alpha_r d_r$ .

In this paper, we consider a single link of capacity  $c$  shared by  $N$  homogeneous sources with round trip propagation delay  $d$  and thus  $q_r(t)$  and  $y(t)$  in the TCP model (5) and (6) have the following form:

$$q_r(t) = p(t - \tau_r^b), \quad y(t) = N x_r(t - \tau_r^f).$$

And since  $N$  homogeneous source sharing a single link is equivalent to the large 1 source of a single link, we can consider the above model as a Vegas model of a single link of capacity  $c$  shared by 1 homogeneous source with round trip propagation delay  $d$  as follows

$$\dot{x}(t) = -T^{-2}(t) \text{sgn}(x(t)p(t - \tau^b) - \alpha d) \quad (8)$$

$$\dot{p}(t) = \begin{cases} (x(t - \tau^f) - c)/c & \text{if } p(t) > 0 \\ (x(t - \tau^f) - c)^+/c & \text{if } p(t) = 0 \end{cases} \quad (9)$$

where  $\alpha d = N \alpha_r d_r$ . Then, we assume followings for simplicity without loss of generality.

- Assumption 1 : Since  $\tau_b \ll \tau_f$  in many case, we can shift the backward time-delay term to the forward time-delay term

$$\tau_b = 0, \quad \tau_f = T^* \quad (10)$$

where  $T^* = d + \frac{\alpha d}{c}$  is equilibrium RTT.

- Assumption 2 : We limit  $x(t)$  and  $p(t)$  with any physical bounds  $\bar{x}$  and  $\bar{p}$  such as

$$0 \leq x(t) \leq \bar{x}, \quad 0 < p(t) \leq \bar{p} \quad (11)$$

- Assumption 3 : RTT has the following upper and lower bounds

$$d \leq T(t) \leq d + \bar{p} \quad (12)$$

## 3 NEW STABILITY ANALYSIS

We shall first redefine the variables  $x(t)$  and  $p(t)$  with the following variables:

$$\eta(t) = x(t) - c, \quad (13)$$

$$\mu(t) = p(t) - \alpha d/c. \quad (14)$$

In this case, the physical constraints shown in (11) the assumption 3 can be written as

$$m^T(t)M_1 m(t) < 0, \quad m^T(t)M_2 m(t) < 0, \quad (15)$$

where the extended state  $m(t)$ ,  $M_1$ , and  $M_2$  denote

$$m(t) \triangleq \begin{bmatrix} \eta(t) & \mu(t) & 1 \end{bmatrix}^T \quad (16)$$

$$M_1 \triangleq \begin{bmatrix} 2 & 0 & -\bar{x} + 2c \\ 0 & 0 & 0 \\ -\bar{x} + 2c & 0 & 2(c^2 - \bar{x}c) \end{bmatrix}, \quad (17)$$

$$M_2 \triangleq \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & -\bar{p} + 2\alpha d/c \\ 0 & -\bar{p} + 2\alpha d/c & 2(\alpha^2 d^2/c^2 - \bar{p}\alpha d/c) \end{bmatrix} \quad (18)$$

Furthermore, we can rewrite the model (8) and (9) with a piecewise linear time-varying delayed system as follows: for  $m(t) \in S_i$  and for  $i = 1, 2$

$$\dot{m}(t) = A_i(t)m(t) - B \int_{t-T^*}^t \dot{m}(\alpha) d\alpha, \quad (19)$$

where the system matrices  $(A_i(t), B)$  are defined as

$$A_i(t) \triangleq \begin{bmatrix} 0 & 0 & (-1)^i T^{-2}(t) \\ 1/c & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B \triangleq \begin{bmatrix} 0 & 0 & 0 \\ 1/c & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

and the switching zones  $S_i$ , resulted from the switching function  $x(t)p(t) - \alpha d$ , denote

$$S_i \triangleq \left\{ m(t) \mid m^T(t) M_{3i} m(t) < 0 \right\}, \quad (20)$$

$$M_{3i} \triangleq (-1)^i \begin{bmatrix} 0 & 1 & \alpha d/c \\ 1 & 0 & c \\ \alpha d/c & c & 0 \end{bmatrix}.$$

We shall now consider the stability of the system above. Since the system is piecewise and time-delayed, we shall suggest the following piecewise and delay-dependent Lyapunov-Krasovskii functional, which is modified from the functional introduced by Park [14],

$$V(m(t - \alpha), \alpha \in [0, T^*]) = V_1 + V_2 + V_3, \quad (21)$$

where, for  $m_i \in S_i$  and for  $i = 1, 2$

$$V_1 \triangleq m^T(t) P_i m(t),$$

$$V_2 \triangleq \int_{-T^*}^0 \int_{t+\beta}^t \dot{m}^T(\alpha) Z \dot{m}(\alpha) d\alpha d\beta,$$

$$V_3 \triangleq \int_{t-T^*}^t m^T(\alpha) Q m(\alpha) d\alpha.$$

For the functional above, we shall impose the following condition for  $V_1$  in order to achieve the radial unboundedness of  $V_1$  with respect to  $m(t)$ : for any nonzero vector  $m(t) \in S_i$  yielding the condition (11) in the assumption 3, or the condition (15),

$$V_1(t) > 0, \quad (22)$$

which can be written as follows: there exist nonnegative scalars  $\lambda_{1i}$ ,  $\lambda_{2i}$ , and  $\lambda_{3i}$  such that, for any nonzero  $m(t)$ ,

$$P_i + \lambda_{1i} M_1 + \lambda_{2i} M_2 + \lambda_{3i} M_{3i} > 0. \quad (23)$$

With this condition, the positive definiteness of  $Z$  and  $Q$ , or

$$Q > 0, \quad Z > 0, \quad (24)$$

says that  $V(x(t - \alpha), \alpha \in [0, T^*])$  is radially unbounded with respect to  $x(t)$ .

As mentioned in the work [15], [16], when one uses a type of piecewise Lyapunov function for the analysis of piecewise continuous-time systems, one should consider the decreasing property of the piecewise Lyapunov function at the switching instant. Similarly, when a *piecewise* Lyapunov-Krasovskii functional is considered, an important condition that should be checked out is the variation of the Lyapunov-Krasovskii functional on the switching surface constructed by the switching function  $(x(t)p(t) - \alpha d)$ . In this model,

there are only two switching zones related to one switching surface and thus we simply need to have the following continuity condition:

$$m^T(t) P_1 m(t) = m^T(t) P_2 m(t) \quad (25)$$

$$\text{for } m^T(t) M_{31} m(t) = m^T(t) M_{32} m(t) = 0,$$

which can be written with the existence of a real number  $\omega$  such that

$$P_2 = P_1 + \omega M_{31}, \quad \omega \in \mathcal{R}. \quad (26)$$

Finally what we need for the stability is that its derivative  $\dot{V}$  in each switching zone  $S_i$  should be strictly negative for nonzero  $\{m(t - \alpha), \alpha \in [0, T^*]\}$  under the constraints (15). We shall present such conditions in the following theorem.

**Theorem 1:** Suppose the TCP Vegas satisfies the assumptions 1, 2, and 3. Then the piecewise linear time-varying delayed model (19) with the equilibrium point  $(x^*, p^*) = (c, \alpha d/c)$  is asymptotically stable, if there exist matrices  $X = X^T$ ,  $Y$ ,  $Z = Z^T$ ,  $P = P^T$ ,  $Q = Q^T$ , and scalar variables  $\lambda_{1i}$ ,  $\lambda_{2i}$ ,  $\lambda_{3i}$ ,  $\mu_{1i}$ ,  $\mu_{2i}$ ,  $\mu_{3i}$ ,  $\omega$ , for all  $i = 1, 2$ ,  $j = 1, 2$  satisfying

$$P_i + \lambda_{1i} M_1 + \lambda_{2i} M_2 + \lambda_{3i} M_{3i} > 0, \quad (27)$$

$$\begin{bmatrix} (1, 1)_{ij} & (1, 2)_i & T^* A_{ij}^T Z \\ (1, 2)_i^T & -Q & -T^* B^T Z \\ T^* Z A_{ij} & -T^* Z B & -T^* Z \end{bmatrix} < 0, \quad (28)$$

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0, \quad Q > 0, \quad \lambda_{1i} > 0, \quad \lambda_{2i} > 0, \quad \lambda_{3i} > 0, \quad (29)$$

$$\mu_{1i} > 0, \quad \mu_{2i} > 0, \quad \mu_{3i} > 0, \quad (30)$$

where  $M_1$ ,  $M_2$  and  $M_{3i}$  were defined in (17), (18) and (20), respectively, and

$$P_1 = P, \quad (31)$$

$$P_2 = P + \omega M_{31}, \quad \omega \in \mathcal{R}, \quad (32)$$

$$(1, 1)_{ij} = P_i A_{ij} + A_{ij}^T P_i + T^* X - \mu_{1i} M_1 - \mu_{2i} M_2 - \mu_{3i} M_{3i}, \quad (33)$$

$$(1, 2)_i = -P_i B + Y + Q, \quad (34)$$

$$A_{ij} = \begin{bmatrix} 0 & 0 & -a_{ij} \\ 1/c & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad (35)$$

$$a_{11} = -a_{21} = d^{-2}, \quad a_{12} = -a_{22} = (d + \bar{p})^{-2}.$$

**Proof:** We have already generated the conditions (27), (31), and (32) in (23) and (26). What I need to show is that the following time derivative of  $V(m(t - \alpha), \alpha \in [0, T^*])$  in each switching zone  $S_i$  should be strictly negative for nonzero  $\{m(t - \alpha), \alpha \in [0, T^*]\}$  under the constraints (15), which will result in the condition (28).

$$\begin{aligned}
\dot{V} &= \dot{V}_1 + \dot{V}_2 + \dot{V}_3, \\
&= 2 \left( A_i(t)m(t) - B \int_{t-T^*}^t \dot{m}(\alpha) d\alpha \right)^T P_i m(t) \\
&\quad - \int_{-T^*}^0 \dot{m}^T(t+\alpha) Z \dot{m}(t+\alpha) d\alpha \\
&\quad + T^* \dot{m}^T(t) Z \dot{m}(t) + m^T(t) Q m(t) \\
&\quad - m^T(t - T^*) Q m(t - T^*) \\
&= \begin{bmatrix} m(t) \\ \bar{m}(t) \end{bmatrix}^T \begin{bmatrix} X_{11}(i, t) & X_{12}(i, t) \\ X_{12}^T(i, t) & X_{22} \end{bmatrix} \begin{bmatrix} m(t) \\ \bar{m}(t) \end{bmatrix} \\
&\quad - \int_{t-T^*}^t \dot{m}^T(\alpha) Z \dot{m}(\alpha) d\alpha.
\end{aligned}$$

where  $X_{11}(t)$ ,  $X_{12}(t)$ ,  $X_{22}$ , and  $\bar{m}(t)$  denote

$$\begin{aligned}
X_{11}(i, t) &\triangleq P_i A_i(t) + A_i^T(t) P_i + T^* A_i^T(t) Z A_i(t), \\
X_{12}(i, t) &\triangleq -P_i B + Q - T^* A_i^T(t) Z B, \\
X_{22} &\triangleq -Q + T^* B^T Z B, \\
\bar{m}(t) &\triangleq \int_{t-T^*}^t \dot{m}(\alpha) d\alpha.
\end{aligned}$$

Now we shall add the following term to the above equation

$$\begin{aligned}
0 &\leq \int_{t-T^*}^t \begin{bmatrix} m(t) \\ \dot{m}(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \begin{bmatrix} m(t) \\ \dot{m}(\alpha) \end{bmatrix} d\alpha \\
&= \begin{bmatrix} m(t) \\ \bar{m}(t) \end{bmatrix}^T \begin{bmatrix} T^* X & Y \\ Y^T & 0 \end{bmatrix} \begin{bmatrix} m(t) \\ \bar{m}(t) \end{bmatrix} \\
&\quad + \int_{t-T^*}^t \dot{m}^T(\alpha) Z \dot{m}(\alpha) d\alpha.
\end{aligned}$$

Therefore, we can get the following condition.

$$\dot{V} \leq \begin{bmatrix} m(t) \\ \bar{m}(t) \end{bmatrix}^T \Pi \begin{bmatrix} m(t) \\ \bar{m}(t) \end{bmatrix}.$$

where

$$\Pi = \begin{bmatrix} X_{11}(i, t) + T^* X & X_{12}(i, t) + Y \\ X_{12}^T(i, t) + Y^T & X_{22} \end{bmatrix}$$

The proof will complete if we find a sufficient condition that makes the middle matrix block negative definite for all the states in each zone satisfying the condition (15). One sufficient condition can be chosen via the  $S$ -procedure as follows: there are nonnegative scalar variables  $\mu_{1i}$ ,  $\mu_{2i}$ , and  $\mu_{3i}$  such that, for all  $i = 1, 2$ ,

$$0 > \begin{bmatrix} (1, 1) & X_{12}(i, t) + Y \\ X_{12}^T(i, t) + Y^T & X_{22} \end{bmatrix},$$

where

$$(1, 1) = X_{11}(i, t) + T^* X - \mu_{1i} M_1 - \mu_{2i} M_2 - \mu_{3i} M_{3i}$$

Since  $A_i(t)$  is time-varying, however, we cannot provide feasible variables easily. Let us use the Schur complement technique to modify the condition into the following:

$$0 > \begin{bmatrix} \bar{X}_{11}(i, t) & -P_i B + Q + Y & T^* A_i^T(t) Z \\ -B^T P_i + Q + Y^T & -Q & -T^* B^T Z \\ T^* Z A_i(t) & -T^* Z B & -T^* Z \end{bmatrix},$$

where  $\bar{X}_{11}(i, t)$  denotes

$$\begin{aligned}
\bar{X}_{11}(i, t) &\triangleq P_i A_i(t) + A_i^T(t) P_i + T^* X \\
&\quad - \mu_{1i} M_1 - \mu_{2i} M_2 - \mu_{3i} M_{3i}.
\end{aligned}$$

In this case, we can now use a nice property of  $A_i(t)$  that  $A_i(t)$  at any time belongs to a convex hull of  $\{A_{ij}\}_{j=1,2}$  for each  $i$ , i.e.  $A_i(t) \in \text{cov}(A_{i1}, A_{i2})$ , which results in the condition (28). Therefore, the proof completes.  $\blacksquare$

## 4 COMPARISONS

In this section, we demonstrate the proposed method with respect to the stability condition of Choe and Low [12] for the dynamic TCP Vegas model. We directly apply the proposed method to Example 1 of Choe and Low [12]. Consider a single link of capacity  $c$  shared by  $N$  homogeneous sources with round trip propagation delay  $d$ . And all sources have the same target queue length  $\alpha$ . For this example, Choe and Low [12] presented the following stability condition based on their loop gain method in the frequency domain.

$$cd < \left(\frac{\pi}{2} - 1\right) \alpha N \quad (36)$$

For given  $\alpha = 20$  packets and  $N = 100$  sources, we plotted the stable regions by (36) and Theorem 1 in Figure 1 and 2, respectively. From now on, people have accepted that TCP Vegas is a greatly stable TCP algorithm, but people did not prove the great stability because a good analysis method was not developed for the nonlinear time-varying TCP model with time-delay. By means of applying the new proposed method for this example, we show that the current TCP Vegas algorithm is sufficiently stable in the wide bandwidth(link capacity)-delay region.

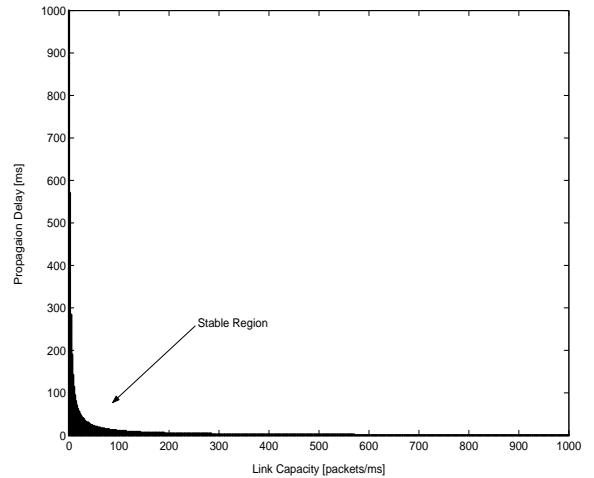


Fig. 1. The stable bandwidth-delay region of Choe and Low

## 5 CONCLUSION AND FURTHER WORKS

We have presented a new stability analysis method for the single link dynamic TCP Vegas model. First, this paper described the nonlinear(switced) time-varying TCP Vegas model with time-delay by a piecewise linear time-invariant

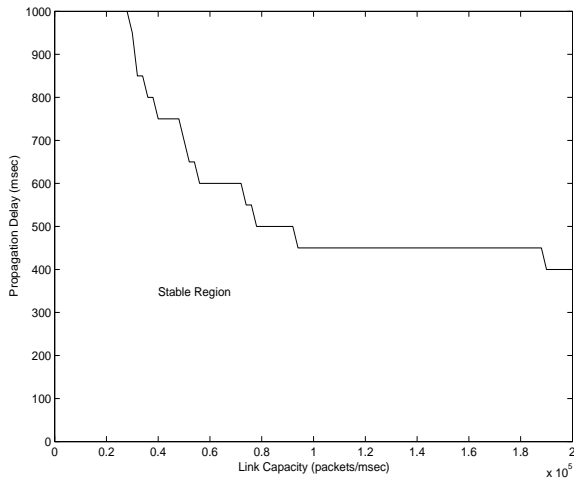


Fig. 2. The stable bandwidth-delay region by Theorem 1

dynamic system with time-delay. Then, a piecewise and delay-dependent Lyapunov-Krasovskii function was applied for the system. Consequently, this paper successfully developed new stability criterion for the single link dynamic TCP Vegas model in terms of LMIs. By an example, we showed that the proposed criterion performs much better than Choe and Low [12] in the aspect of the exact describing the great stability of TCP Vegas model. Since we only treated the stability analysis of the single link TCP Vegas model, we will extend the proposed method to the general multi-link TCP Vegas model in the near future. Furthermore, if we apply the proposed method for performance-oriented problems of dynamic TCP Vegas model such as guaranteed cost, saturation, tracking, and their combinations, we will examine closely the other problems of TCP network system. And we believe that the proposed method will be the basis for developing an advanced TCP algorithm.

## REFERENCES

- [1] J. Ed. Postel, "Transmission control protocol : DARPA internet program protocol specification," RFC-793, September 1981.
- [2] R. Braden, "Requirements for Internet hosts - communication layers," RFC-1122, October 1989.
- [3] W. Stevens, "TCP slow start, congestion avoidance, fast retransmit, and fast recovery algorithms," RFC-2001, January 1997.
- [4] L. S. Brakmo, S. W. O'Malley, and L. L. Peterson, "TCP Vegas: new techniques for congestion detection and avoidance," *Proceedings of ACM SIGCOMM*, October 1994.
- [5] O. A. Hellal and E. Altman, "Analysis of TCP Vegas and TCP Reno," *Proceedings of IEEE ICC'97*, June 1997.
- [6] A. Kumar, "Comperative performance analysis of versions of TCP in a local network with a lossy link," *IEEE/ACM Transaction on Networking*, vol.6, pp.485-498, August 1998.
- [7] J. Mo, R. J. La, V. Anantharam, and J. Walrand,

- "Analysis and comparison of TCP Reno and TCP Vegas," *Proceedings of IEEE INFOCOM*, March 1999.
- [8] H. Ohsaki, M. Murata, T. Ushio, and H. Miyahara, "Stability analysis of window-based flow control mechanism in TCP/IP networks," *Proceedings of the 1999 IEEE International Conference on Control Applications*, Hawaii, USA, August 22-27, 1999.
- [9] G. Hasegawa, M. Murata, and H. Miyahara, "Fairness and stability of congestion control mechanism of TCP", *Proceedings of 11th ITC Special Seminar*, October, 1998.
- [10] K. Takagaki, H. Ohsaki, and M. Murata, "Stability analysis of a window-based flow control mechanism for TCP connections with differential propagation delays," *Proceedings of INET 2000*, July, 2000.
- [11] Z. Wang and F. Paganini, "Global stability with time-delay in network congestion control," *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, USA, December 2002.
- [12] D. H. Choe and S. H. Low, "Stabilized vegas," *Proceedings of IEEE INFOCOM*, San Francisco, USA, April, 2003.
- [13] S. H. Low, L. L. Peterson, and L. Wang, "Understanding Vegas: a duality model," *Journal of ACM*, vol.49, no.2, pp.207-235, March 2002.
- [14] P. Park, "A delay-dependent stability criterion for systems with uncertain time-invariant delays," *IEEE Transactions on Automatic Control*, vol.44, no.4, pp.876-877, April 1999.
- [15] M. Johansson and A. Rantzer, "Computation of piecewise quadratic Lyapunov functions for hybrid systems," *IEEE Transactions on Automatic Control*, vol.43, no.4, pp.555-559, April 1998.
- [16] A. Rantzer and M. Johansson, "Piecewise linear quadratic optimal control," *IEEE Transactions on Automatic Control*, vol.45, no.4, pp.629-637, April 2000.