Unscented Kalman Filter For Aircraft Sensor Fault Detection

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Abstract: To prevent the critical situation due to the fault in the aircraft sensor system, the fault tolerant system with triple or quadruple redundancy can be made. However, if the faults are occurred in two or more than sensors simultaneously, the conventional fault detection process, such as cross-channel monitoring, may give the wrong fault alarm. For this case, we can detect the fault by estimating the state vector based on the system dynamics model, which is nonlinear for aircraft. In this paper, we propose the unscented Kalman filter to estimate the nonlinear state vector. This filter utilizes the so-called unscented transformation of sigma points featured the statistical characteristics of the random variable. For verification, we perform the simulations for F-16 aircraft with accelerometers, gyros, GPS and air data system.

Keywords: fault detection and isolation, unscented kalman filter, inertial measurement unit, redundant system

1. INTRODUCTION

When the sensor system for an aircraft breaks down unexpectedly, this malfunction may give rise to degrade the aircraft performance and fall into a dangerous situation. To prevent this critical situation, the sensor system should be configured with a redundant system, which is consisted of a triple or quadruple of the same sensors. For this redundant system, the cross-channel monitoring (CCM) technique is useful for fault detection and isolation (FDI). If the difference between a signal from the multiple sensors and their mean value are larger than the prescribed threshold value over the prescribed confirmation time, we can convince that the sensor should be failed. Then we should reconfigure the system except the failure so that we may recover the performance degradation and the safety.

However, there are some cases to cause a wrong fault alarm with CCM technique. If the only two sensors are available, it cannot decide which sensor is out of order. In addition, though the system is a triple or quadruple, if the failure happens to both sensors at the same time, it may yield the wrong fault alarm.

The alternative of fault detection is to use the estimated state variables for the dynamic system. If the measurement residual, which is defined as the difference between the estimated and measured state variables, is larger than the prescribed threshold value over the prescribed confirmation time, then the fault alarm is given.

In general, the standard Kalman filter for a linear dynamic system or the extended Kalman filter (EKF) for a nonlinear dynamic system has been used as a state estimator. It is important to note that EKF gives rise to the truncation error induced by neglecting the second and higher order terms of the Taylor-expanded non-linearity. Moreover, it is difficult to implement EKF because of the derivation of the Jacobian matrices and to tune the Kalman gain matrix and system parameters practically. Sometimes EKF can yield highly unstable filter if the assumption of local linearity around the equilibrium point is violated. In most practical cases, the linearization of EKF introduces the significant biases or errors.

In this paper, we use the unscented Kalman filter (UKF) as a state estimator [1-3]. UKF is an alternative Kalman filter based on the unscented transformation. The main idea of this transformation is that the sigma points, which represent the statistical characteristics of a given stochastic inputs, are transformed through the nonlinear function, namely, nonlinear dynamic system, and then the statistical characteristics for the transformed sigma points represent those of the method. Unlike to EKF, the convergence is not an issue in this method. Also, the high order information of the state variables can be computed with the small umber of transformed sigma points. In these aspects, UKF compensates the drawbacks of EKF.

In this paper, we derive the nonlinear 6-DOF aircraft dynamic model and design the state estimator with UKF. To verify the proposed method, we perform the simulation for the inertial measurement unit (IMU) and discuss simulation results by comparing with those of EKF.

2. NONLINEAR AIRCRAFT MODEL

In general, if we assume that the x-z plane is a plane of mass symmetry and the resultant torque from engine is zero, the aircraft dynamic equations of motion in body frame are given by

$$\dot{\vec{x}}_1 = \vec{f}_1(\vec{x}_1, \vec{x}_2) + \vec{g}(\vec{x}_1, \vec{x}_2, \vec{u}) \tag{1}$$

$$\dot{\vec{x}}_2 = \vec{f}_2(\vec{x}_1, \vec{x}_2) \tag{2}$$

where $\vec{x}_1$ and $\vec{x}_2$ are the dynamics and kinematics state vectors, respectively, defined as

$$\vec{x}_1 = \begin{bmatrix} \vec{v} \\ \vec{\omega} \end{bmatrix} = \begin{bmatrix} (u, v, w) \\ (p, q, r) \end{bmatrix} \tag{3}$$

$$\vec{x}_2 = \begin{bmatrix} q \\ \vec{p} \end{bmatrix} = \begin{bmatrix} (q_1, q_2, q_3, q_4) \\ (x, y, z) \end{bmatrix} \tag{4}$$

where (u, v, w) is the velocity of the center of mass with respect to ground frame, (p, q, r) is the angular velocity with respect to the ground frame, (q_1, q_2, q_3, q_4) is the quaternion of the body frame with respect to the ground frame and (x, y, z) is the location of the aircraft in the ground frame.

The dynamics terms of Eq. (1) are the nonlinear vectors expressed as

$$\vec{f}_1 = \begin{bmatrix} \vec{v} \times \vec{\omega} + m^{-1} A_q \vec{F}_g \\ J\vec{\omega} \times \vec{\omega} \end{bmatrix} \tag{5}$$

$$\vec{g} = \begin{bmatrix} m^{-1}\left(\vec{F}_t + \vec{F}_a\right) \\ J^{-1}\vec{T}_a \end{bmatrix} \tag{6}$$

where m is the mass of the aircraft, J is the moment of inertia of the aircraft, $\vec{F}_g$ is the gravitational acceleration about the center of mass in the ground frame, $\vec{F}_t$ is the thrust about the center of mass in the body frame, $\vec{F}_a$ is the aerodynamic force about the center of mass in the body frame, $\vec{T}_a$ is the aerodynamic moment about the center of mass in the body frame, A_q is the ground to body direction cosine matrix defined as

$$A_q = \left(q_4^2 - \vec{q}\cdot\vec{q}\right)I + 2\vec{q}\vec{q}^T - 2q_4[q_\times] \tag{7}$$

where $[q_\times]$ is the skew-symmetric vector product matrix defined as

$$[q_\times] = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \tag{8}$$

The aerodynamic coefficients are obtained from the wind tunnel test. Then the aerodynamic force and moment are computed according to the corresponding flight condition, namely, the state vector and control surface deflection angle.

The kinematics part of Eq. (2) can be written by

$$\vec{f}_2 = \begin{bmatrix} \frac{1}{2}\Omega_\omega q \\ A_q^T\vec{v} \end{bmatrix} \tag{9}$$

where Ω_ω is the skew-symmetric quaternion product matrix defined as

$$\Omega_\omega = \left[\begin{array}{c|c} -[\omega_\times] & \vec{\omega} \\ \hline -\vec{\omega}^T & 0 \end{array}\right] \tag{10}$$

where $[\omega_\times]$ is the skew-symmetric vector product matrix for the angular rate vector.

We can use several sensors to measure or compute the state vector of the aircraft. The inertial measurement unit (IMU) including accelerometers and gyros along each axis measures the applied acceleration except the gravitational acceleration and the applied angular rate. The attitude heading and reference system (AHRS) computes the attitude and heading angles of the aircraft body frame with respect to the ground frame. These angles can be easily expressed as the quaternion. The global positioning system (GPS) produces the position information of the aircraft with respect to the ground frame. The air data system outputs not only the atmospheric temperature but also static and dynamic pressure. The measurements from these sensors have the high frequency errors inevitably. Moreover, they may include the low frequency errors such as bias, drift, misalignment, etc. For simplicity, however, we assume that the low frequency errors have been compensated perfectly. Then the measurement equations are given by

$$\vec{y} = \begin{bmatrix} \left(\vec{v}\times\vec{\omega} + m^{-1}\left(\vec{F}_t + \vec{F}_t\right)\right)/g_0 \\ \vec{\omega} \\ q \\ \vec{p} \end{bmatrix} + \begin{bmatrix} \vec{n}_a \\ \vec{n}_\omega \\ \vec{n}_q \\ \vec{n}_p \end{bmatrix} \tag{11}$$

where $\vec{n}_a$, $\vec{n}_\omega$, $\vec{n}_q$ and $\vec{n}_p$ are the white noise vector with the zero mean and the covariance of R_a, R_ω, R_q and R_p, respectively.

3. UNSCENTED KALMAN FILTER

The unscented Kalman filter (UKF) proposed by Simon Julier et al. is an approximation method for the state distribution of the nonlinear model [1-3]. This method utilizes the so-called unscented transformation, which is a nonlinear transformation. The transformation uses a finite number of sampled sigma points to characterize the statistics of true distribution. The UKF propagate the statistics of the sigma points though the nonlinear system model and then compute the statistics of the transformed sigma points. Because the sigma points are transformed through the true nonlinear model, we can obtain more accurate mean and covariance of the state distribution. In this section, we describe the unscented transformation and summary the unscented Kalman filter in brief

3.1 Unscented transformation

Given a random variable $\vec{x} \in \Re^{n\times 1}$ with the mean $\vec{m}_x$ and covariance P_{xx}, the problem addressed in the unscented transformation is to compute the mean $\vec{m}_y$ and covariance P_{yy} of the second random variable $\vec{y} = \vec{f}(\vec{x}) \in \Re^{m\times 1}$. The first step is to approximate the n-by-1 random variable $\vec{x}$ with the 2n-by-1 weighted sigma points given by

$$\vec{X}_0 = \vec{m}_x \tag{12}$$

$$\vec{X}_i = \vec{m}_x + \left(\sqrt{(n+\kappa)P_{xx}}\right)_i \tag{13}$$

$$\vec{X}_{i+n} = \vec{m}_x - \left(\sqrt{(n+\kappa)P_{xx}}\right)_i \tag{14}$$

where κ is a scaling parameter to tune the higher order moments of the approximation so as to reduce the overall errors. For matrix square root, we can use numerically efficient and stable methods such as Cholesky decomposition. The second step is to propagate the sigma points through the nonlinear function

$$\vec{Y}_i = \vec{f}\left(\vec{X}_i\right) \tag{15}$$

The third step is to compute the mean and covariance from the transformed sigma points

$$\vec{m}_y = \sum_{i=0}^{2n} W_i\vec{Y}_i \tag{16}$$

$$P_{yy} = \sum_{i=0}^{2n} W_i\left[\vec{Y}_i - \vec{m}_y\right]\left[\vec{Y}_i - \vec{m}_y\right]^T \tag{17}$$

where W_i is the weight given by

$$W_0 = \frac{\kappa}{n+\kappa} \tag{18}$$

$$W_i = \frac{1}{2(n+\kappa)} \tag{19}$$

$$W_{i+n} = \frac{1}{2(n+\kappa)} \tag{20}$$

Although κ can be selected as to be positive or negative, it is obvious that a negative choice of κ can make a non-positive semi-definite of P_{yy}. For Gaussian distributions, we can select $n+\kappa = 3$ heuristically.

3.2 Unscented Kalman filter

The unscented transformation described in the previous section can be applied to the Kalman filter directly. For simplicity, we assume that the measurement noise is additive.

Consider the system and measurement models given by

$$\vec{x}_{k+1} = \vec{f}\left(\vec{x}_k, \vec{u}_k, \vec{v}_k, k\right) \tag{21}$$

$$\vec{y}_k = \vec{g}\left(\vec{x}_k, \vec{u}_k, k\right) + \vec{w}_k \tag{22}$$

where $\vec{v}_k$ and $\vec{w}_k$ are the process and measurement noise vectors with zero-mean and covariance

$$E\left[\vec{v}_i \vec{v}_j^T\right] = \delta_{ij} Q_i, \quad \text{for all } i \text{ and } j \tag{23}$$

$$E\left[\vec{w}_i \vec{w}_j^T\right] = \delta_{ij} R_i, \quad \text{for all } i \text{ and } j \tag{24}$$

$$E\left[\vec{v}_i \vec{w}_j^T\right] = 0, \quad \text{for all } i \text{ and } j \tag{25}$$

The Kalman filter has the two-step structure of the prediction and update processes. First, the prediction process of Kalman filter is given by

$$\hat{\vec{x}}_{k+1|k} = E\left[\vec{f}\left(\vec{x}_k, \vec{u}_k, \vec{v}_k\right) \middle| \left(\vec{y}_1, \cdots, \vec{y}_k\right)\right] \tag{26}$$

$$P_{k+1|k} = E\left[\left(\vec{x}_{k+1} - \hat{\vec{x}}_{k+1|k}\right)\left(\vec{x}_{k+1} - \hat{\vec{x}}_{k+1|k}\right)^T \middle| \left(\vec{y}_1, \cdots, \vec{y}_k\right)\right] \tag{27}$$

where the subscript $(\cdot)_{k+1|k}$ means the a priori quantity. It is obvious that $\vec{f}$ and $\vec{g}$ are nonlinear and thus it is hard to compute these estimation of prediction process. Next, the update process of Kalman filter is given by

$$\hat{\vec{x}}_{k+1|k+1} = \hat{\vec{x}}_{k+1|k} + K_{k+1} \vec{\delta}_{k+1} \tag{28}$$

$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1} P^{yy}_{k+1|k} K^T_{k+1} \tag{29}$$

$$K_{k+1} = P^{xy}_{k+1|k} \left[P^{yy}_{k+1|k}\right]^{-1} \tag{30}$$

where the subscript $(\cdot)_{k+1|k+1}$ means the a posteriori quantity, K_{k+1} is the Kalman gain matrix, $\vec{\delta}_{k+1} = \vec{y}_{k+1} - \hat{\vec{y}}_{k+1|k}$ is the measurement residual and $P^{yy}_{k+1|k}$ is the innovation covariance. It is important to note that the update process can be obtained from the estimated mean and covariance of $\vec{x}_{k+1}$ and $\vec{y}_{k+1}$. Thus the unscented Kalman filter can be derived by applying the unscented transformation to estimate the mean and covariance of $\vec{x}_{k+1}$ and $\vec{y}_{k+1}$.

The moments of the state and measurement vectors can be derived as follows:

1) Mean $\hat{\vec{x}}_{k+1|k}$ and covariance $P_{k+1|k}$

Since the state model is the nonlinear function of the state and process noise vectors, however, the new state vector is augmented with the state and process noise vectors.

$$\vec{x}^a_k = \begin{bmatrix} \vec{x}_k \\ \vec{v}_k \end{bmatrix} \tag{31}$$

Then its mean and covariance become

$$\hat{\vec{x}}^a_{k|k} = \begin{bmatrix} \hat{\vec{x}}^a_{k|k} \\ 0 \end{bmatrix} \tag{32}$$

$$P^a_{k|k} = \begin{bmatrix} P_{k|k} & P^{xv}_{k|k} \\ P^{xv}_{k|k} & Q_k \end{bmatrix} \tag{33}$$

where $P^{xv}_{k|k}$ is the correlation matrix between the error state and process noise vectors. The sigma points for the augmented state vector are given by

$$\vec{X}^a_{i,k} = \begin{bmatrix} \vec{X}^x_{i,k} \\ \vec{X}^v_{i,k} \end{bmatrix} \quad \text{for } i = 0, \cdots, 2n^a \tag{34}$$

where $n^a = n + q$ is the dimension of the augmented state vector. They are generated by $\hat{\vec{x}}^a_{k|k}$, $\hat{\vec{x}}^a_{k|k} + \left(\sqrt{(n^a + \kappa) P^a_{k|k}}\right)_i$ and $\hat{\vec{x}}^a_{k|k} - \left(\sqrt{(n^a + \kappa) P^a_{k|k}}\right)_i$. Then the state propagation process becomes

$$\vec{X}^x_{i,k+1} = \vec{f}\left(\vec{X}^x_{i,k}, \vec{X}^v_{i,k}, \vec{u}_k, k\right) \tag{35}$$

$$\hat{\vec{x}}_{k+1|k} = \sum_{i=0}^{2n^a} W_i \vec{X}^x_{i,k+1} \tag{36}$$

$$P_{k+1|k} = \sum_{i=0}^{2n^a} W_i \left[\vec{X}^x_{i,k+1} - \hat{\vec{x}}_{k+1|k}\right]\left[\vec{X}^x_{i,k+1} - \hat{\vec{x}}_{k+1|k}\right]^T \tag{37}$$

2) Mean $\hat{\vec{y}}_{k+1|k}$ and covariance $P^{yy}_{k+1|k}$

Similarly, the estimated measurement vector $\hat{\vec{y}}_{k+1|k}$ and its covariance $P^{yy}_{k+1|k}$ become

$$\vec{Y}_{i,k+1} = \vec{g}\left(\vec{X}_{i,k+1}, \vec{u}_{k+1}, k+1\right) \tag{38}$$

$$\hat{\vec{y}}_{k+1|k} = \sum_{i=0}^{2n^a} W_i \vec{Y}_{i,k+1} \tag{39}$$

$$P^{yy}_{k+1|k} = R_{k+1} + \sum_{i=0}^{2n^a} W_i \left[\vec{Y}_{i,k+1} - \hat{\vec{y}}_{k+1|k}\right]\left[\vec{Y}_{i,k+1} - \hat{\vec{y}}_{k+1|k}\right]^T \tag{40}$$

Since we assume that the measurement noise is additive and independent, the measurement covariance R_{k+1} is added to the estimated covariance.

3) Cross-correlation matrix $P^{xy}_{k+1|k}$

Finally, the cross-correlation matrix $P^{xy}_{k+1|k}$ becomes

$$P^{xy}_{k+1|k} = \sum_{i=0}^{2n^a} W_i \left[\vec{X}^x_{i,k+1} - \hat{\vec{x}}_{k+1|k}\right]\left[\vec{Y}_{i,k+1} - \hat{\vec{y}}_{k+1|k}\right]^T \tag{41}$$

Now we can establish the unscented Kalman filter by substituting Eqs.(36), (37), (40) and (41) into Eqs.(28) ~ (30). For various nonlinear systems, for example, systems with non-additive measurement noise or colored process/measurement noise, we can derive the unscented Kalman filter with a similar manner.

4. FAULT DETECTION AND ISOLATION

In the previous sections, we derived the nonlinear aircraft equations of motion and the unscented Kalman filter for a nonlinear system. Since there is no process noise term in the aircraft system model, the UKF do not use the augmented state vector and thus becomes simpler for this case. In this section, we describe the fault detection process using the measurement residual defined as the difference between the actual and estimated sensor outputs.

The functional block diagram for the proposed FDI process is given in Fig. 1. The aircraft state vector is not only measured from sensors but also estimated by UKF. While the measured state vector includes the high-frequency errors, the estimated state vector is the low-frequency part obtained through UKF. If there is a fault, thus, the measurement residual becomes the high-frequency components. Otherwise, the measurement residual includes both the state estimation errors and the high-frequency errors. Since the former is larger than the latter clearly, we can confirm the fault by checking whether the measurement residual is larger than the threshold value during the confirmation time. Once the fault alarm occurs, the fault sensor should be isolated.

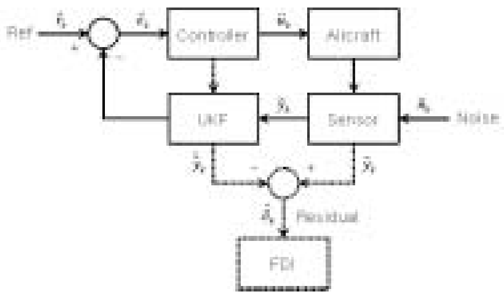

Fig. 1 Functional block diagram for FDI using UKF

For UKF design, the discrete aircraft model can be obtained by applying Euler integration method to the continuous nonlinear system model of Eqs.(1), (2) and (6)

$$\vec{x}_{k+1} = \vec{x}_k + \vec{f}\left(\vec{x}_k, \vec{u}_k\right) \cdot \Delta T$$
$$\vec{y}_{k+1} = \vec{g}\left(\vec{x}_{k+1}\right) + \vec{n}_{k+1}$$

where ΔT is the sampling time. Then the transformed sigma points become

$$\vec{X}_{i,k+1} = \vec{X}_{i,k} + \vec{f}\left(\vec{X}_{i,k}, \vec{u}_k\right) \cdot \Delta T$$
$$\vec{Y}_{i,k+1} = \vec{g}\left(\vec{X}_{i,k+1}, \vec{u}_{k+1}\right)$$

and thus the UKF can be summarized as shown in Table 1. It is obvious that the UKF requires the full-state feedback to estimate the state vector of the nonlinear system model.

5. SIMULATION

For the verification of the proposed FDI method, the simulations are performed for F-16 aircraft model [4]. The sensors are accelerometers, gyros, GPS, and air data system. Three accelerometers and three rate gyros provides a complete 6 degree of freedom solution of aircraft dynamics at a rate of 100Hz. GPS produces the position and ground speed data at a rate of 1Hz. The air data system gives the static and dynamic pressure and the outside temperature at a rate of 1Hz. All sensors are double for a fault tolerant system. It is assumed that the fault in a gyro in yaw axis is occurred during the coordinate turn.

Fig. 2 shows the yaw-axis angular rate measurements. The malfunction is occurred in the second gyro at 40 seconds and the gyro produces an abnormal measurements. Fig. 3 shows the measurement residual between the measured and estimated angular rates when the fault detection and isolation process is not applied. It is obvious from Fig. 3 that the second gyro measurement residual after the fault is biased. If we do not apply the isolation process to the broken gyro, the wrong measurements affect the estimation of the state. Fig. 4 and 5 show the yaw angle and rate estimation errors for the case without FDI and with FDI, respectively.

Table 1 Unscented Kalman filter for nonlinear aircraft model

1) Propagation process

$$\hat{\vec{x}}_{k+1|k} = \sum_{i=0}^{2n} W_i \vec{X}_{i,k+1}$$
$$\vec{y}_{k+1|k} = \sum_{i=0}^{2n} W_i \vec{Y}_{i,k+1}$$
$$P_{k+1|k} = \sum_{i=0}^{2n} W_i \left[\vec{X}_{i,k+1} - \hat{\vec{x}}_{k+1|k}\right]\left[\vec{X}_{i,k+1} - \hat{\vec{x}}_{k+1|k}\right]^T$$

2) Measurement residual

$$\vec{\delta}_{k+1} = \vec{y}_{k+1} - \hat{\vec{y}}_{k+1|k}$$
$$P^{yy}_{k+1|k} = R_{k+1} + \sum_{i=0}^{2n^a} W_i \left[\vec{Y}_{i,k+1} - \hat{\vec{y}}_{k+1|k}\right]\left[\vec{Y}_{i,k+1} - \hat{\vec{y}}_{k+1|k}\right]^T$$
$$P^{xy}_{k+1|k} = \sum_{i=0}^{2n^a} W_i \left[\vec{X}^{x}_{i,k+1} - \hat{\vec{x}}_{k+1|k}\right]\left[\vec{Y}_{i,k+1} - \hat{\vec{y}}_{k+1|k}\right]^T$$

3) Update process

$$K_{k+1} = P^{xy}_{k+1|k}\left[P^{yy}_{k+1|k}\right]^{-1}$$
$$\hat{\vec{x}}_{k+1|k+1} = \hat{\vec{x}}_{k+1|k} + K_{k+1}\vec{\delta}_{k+1}$$
$$P_{k+1|k+1} = P_{k+1|k} - K_{k+1} P^{yy}_{k+1|k} K^T_{k+1}$$

6. CONCLUSION

In this paper, we proposed the aircraft sensor fault detection algorithm using the unscented Kalman filter. This detects the fault signal from sensor by estimating the state vector using the unscented transformation of the sigma points and checking the measurement residual. The advantage of the unscented Kalman filter is that we can estimate the state vector of nonlinear system more accurately than the conventional nonlinear filter such as extended Kalman filter. From the simulation results for the gyro malfunction, we can confirm that all fault signals in sensors can be detected by using the unscented Kalman filter. For the further study, we plan to extend the fault detection and isolation to the sensor including the process noise, such as, bias, drift, misalignment, etc.

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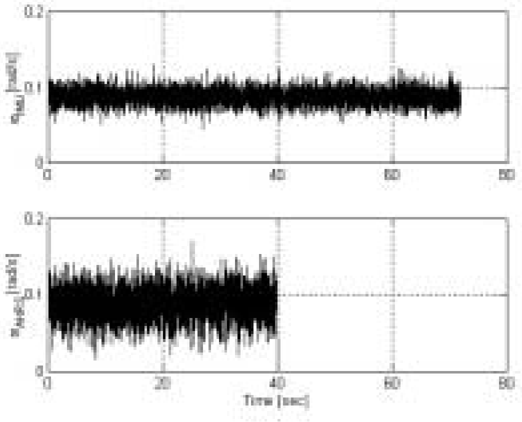

Fig. 2 Yaw axis angular rate measurements from gyros

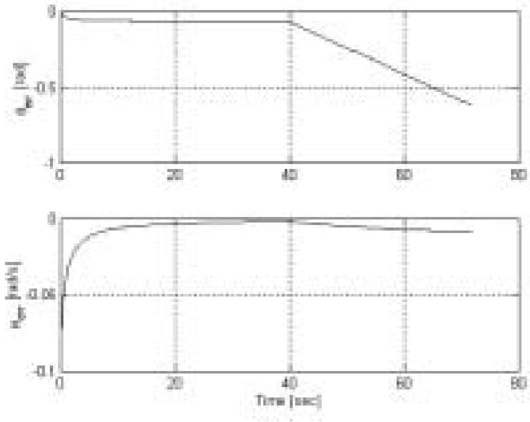

Fig. 4 State estimation error without FDI

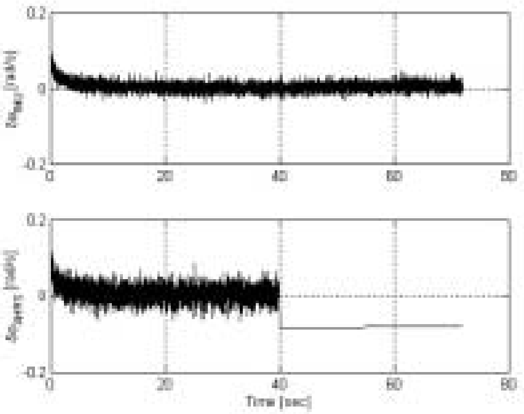

Fig. 3 Measurement residual without FDI

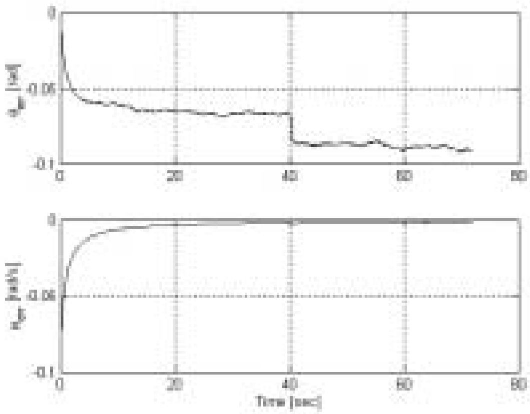

Fig. 5 State estimation error with FDI