Unscented Filtering Approach to Magnetometer-Only Orbit Determination

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Abstract: The basic difference between the EKF(Extended Kalman Filter) and UKF(Unscented Kalman Filter) stems from the manner in which Gaussian random variables(GRV) are represented for propagating through system dynamics. In the EKF, the state distribution is approximated by a GRV, which is then propagated analytically through the first-order linearization of the nonlinear system. This can possibly introduce large errors in the true posterior mean and covariance of the transformed GRV, which may lead to sub-optimal performance and sometimes divergence of the filter. However, the UKF addresses this problem by using a deterministic sampling approach. The state distribution is also approximated by a GRV, but is now represented using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the GRV, and UKF captures the posterior mean and covariance accurately up to the 2nd order(Taylor series expansion) for any nonlinearity. This paper utilizes the UKF to determine spacecraft orbit when only magnetometer is available. Several catastrophic failures of spacecraft in orbit have been attributed to failures of the spacecraft mission. Recently studies on contingency-major sensor failure cases- have been performed. For mission success, contingency design or plan should be implemented in case of a major sensor failure. Therefore the algorithm presented in this paper can be used for a spacecraft without GPS or contingency design in case of GPS failure.

Keywords: Unscented filtering, Kalman filter, Orbit determination, Magnetometer, Orbit dynamics

1. INTRODUCTION

Typically GPS receiver or ranging signal are used to determine orbital parameters of spacecraft. The GPS receiver is somewhat expensive but provides relatively accurate orbital information. Ranging signal also can be used for orbit determination of spacecraft on ground. But it is hard to use ranging signal on-board real-time processing.

Due to its reliability and low cost, magnetometer has been the focus of many studies in the recent past[1-5]. Emphasis has been place on using the magnetometer alone, without any additional input to estimate the spacecraft trajectory.

Studies have been performed to determine the spacecraft orbital information using magnetometer via Batch filter[3] and especially EKF(Extended Kalman Filter) [1-2, 4].

No explicit Jacobian or Hessian calculations are required for the UKF. The computation complexity of the UKF is the same order as that of the EKF.

The basic difference between the EKF(Extended Kalman Filter) and UKF stems from the manner in which Gaussian random variables(GRV) are represented for propagating through system dynamics. In the EKF, the state distribution is approximated by a GRV, which is then propagated analytically through the first-order linearization of the nonlinear system. This can possibly introduce large errors in the true posterior mean and covariance of the transformed GRV, which may lead to sub-optimal performance and sometimes divergence of the filter. However, the UKF addresses this problem by using a deterministic sampling approach. The state distribution is also approximated by a GRV, but is now represented using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the GRV, and UKF captures the posterior mean and covariance accurately up to the 2nd order(Taylor series expansion) for any nonlinearity.

This paper utilizes the UKF to determine spacecraft orbit when only magnetometer is available. Several catastrophic failures of spacecraft in orbit have been attributed to failures of the spacecraft mission. Recently studies on contingency-major sensor failure cases- have been performed. For mission success, contingency design or plan should be implemented in case of a major sensor failure. Therefore the algorithm presented in this paper can be used for a spacecraft without GPS or contingency design in case of GPS failure.

2. UNSCENTED FILTERING

In this section the UKF algorithm is reviewed. The filter algorithm is derived for the following state-space model.

$$\boldsymbol{x}_{k+1} = \boldsymbol{F}(\boldsymbol{x}_k, \boldsymbol{u}_k, \boldsymbol{v}_k) \tag{1a}$$

$$\boldsymbol{y}_k = \boldsymbol{H}(\boldsymbol{x}_k, \boldsymbol{n}_k) \tag{1b}$$

where $\boldsymbol{x}_k$ is the nx1 state vector and $\boldsymbol{y}_k$ is the mx1 measurement vector. Note that a continuous-time model can always be expressed in the form of Eq. (1a) through an appropriate numerical integration scheme. We assume that the process noise $\boldsymbol{v}_k$ and measurement noise $\boldsymbol{n}_k$ are given by $p(\boldsymbol{v}_k) = N(0, \boldsymbol{R}^{\boldsymbol{v}})$ and respectively.

If we make the basic assumption that all densities remain Gaussian, then the Bayesian recursion can be greatly simplified. In this case, only the conditional mean $\hat{\boldsymbol{x}}_k = E[\boldsymbol{x}_k \mid \boldsymbol{Y}_0^k]$ and covariance $\boldsymbol{P}_{\boldsymbol{x}_k}$ need to be evaluated. It is straightforward to show that this leads to the recursive estimation, i.e., Kalman filter update equations.

$$\hat{\boldsymbol{x}}_k = \hat{\boldsymbol{x}}_k^- + \boldsymbol{K}_k \tilde{\boldsymbol{y}}_k \tag{2a}$$

$$\boldsymbol{P}_{\boldsymbol{x}_k} = \boldsymbol{P}_{\boldsymbol{x}_k}^- - \boldsymbol{K}_k \boldsymbol{P}_{\tilde{\boldsymbol{y}}_k} \boldsymbol{K}_k^T \tag{2b}$$

where $\hat{\boldsymbol{x}}_k^-$ and $\boldsymbol{P}_{\boldsymbol{x}_k}^-$ are pre-update state estimate and covariance, respectively, and $\hat{\boldsymbol{x}}_k$ and $\boldsymbol{P}_{\boldsymbol{x}_k}$ are the post-update state estimate and covariance, respectively. The innovation $\tilde{\boldsymbol{y}}_k$ is given by

$$\tilde{\boldsymbol{y}}_k = \boldsymbol{y}_k - \hat{\boldsymbol{y}}_k^- \tag{3}$$

The gain $\boldsymbol{K}_k$ is computed by

$$\boldsymbol{K}_k = \boldsymbol{P}_{\boldsymbol{x}_k \boldsymbol{y}_k} \boldsymbol{P}_{\tilde{\boldsymbol{x}}_k \tilde{\boldsymbol{y}}_k} \tag{4}$$

The UKF addresses the approximation issues of the EKF. The state distribution is again represented by a GRV, but is now specified using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the GRV, and when propagated through the true non-linear system, captures the posterior mean and covariance accurately up to the 2nd order of Taylor series expansion for any nonlinearity.

Consider propagating a random variable $\boldsymbol{x}$ with dimension L through a nonlinear transformation, i.e., Unscented Transformation(UT), $\boldsymbol{y} = f(\boldsymbol{x})$. Assume $\boldsymbol{x}$ has mean $\bar{\boldsymbol{x}}$ and covariance $\boldsymbol{P}_{\boldsymbol{x}}$. The general formulation for the propagation equations are given as follows and yields a matrix χ of 2L + 1 sigma vectors.

$$\chi_0 = \bar{\boldsymbol{x}}$$

$$\chi_i = \bar{\boldsymbol{x}} + (\sqrt{(L+\lambda)\boldsymbol{P}_x})_i \ , \quad i = 1,...,L$$

$$\chi_i = \bar{\boldsymbol{x}} - (\sqrt{(L+\lambda)\boldsymbol{P}_x})_{i-L} \ , \quad i = L+1,...,2L \tag{5}$$

where is $\lambda = \alpha^2(L+\kappa) - L$ a scaling parameter. The constant α determines the spread of the sigma points around $\bar{\boldsymbol{x}}$. The constant κ is a secondary scaling parameter and is used to incorporate prior knowledge of the distribution of $\boldsymbol{x}$. $(\sqrt{(L+\lambda)\boldsymbol{P}_x})$ is the ith column of the matrix square root. These sigma points are propagated through the nonlinear function,

$$\varphi_i = f(\chi_i) \ , \quad i = 0,...,2L \tag{6}$$

and mean and covariance for $\boldsymbol{y}$ are approximated using a weighted sample mean and covariance of the posterior sigma points,

$$\bar{\boldsymbol{y}} \approx \sum_{i=0}^{2L} W_i^{(m)} \varphi_i \tag{7a}$$

$$\boldsymbol{P}_y \approx \sum_{i=0}^{2L} W_i^{(c)} \{\varphi_i - \bar{\boldsymbol{y}}\}\{\varphi_i - \bar{\boldsymbol{y}}\}^T \tag{7b}$$

with weights W_i given by

$$W_0^{(m)} = \lambda/(L+\lambda)$$

$$W_0^{(c)} = \lambda/(L+\lambda) + (1 - \alpha^2 + \beta)$$

$$W_i^{(m)} = W_i^{(c)} = 1/\{2(L+\lambda)\} \ , \quad i = 1,...,2L \tag{8}$$

The Unscented Filtering is a straight- forward extension of the UT to the recursive estimation in Eq. (2), where the state random variable is redefined as the concatenation of the original state and noise variables: $\boldsymbol{x}^a = [\boldsymbol{x}_k^T, \boldsymbol{v}_k^T, \boldsymbol{n}_k^T]^T$. The selection scheme of sigma points is applied to this new augmented state random variable to compute the corresponding sigma matrix, χ_k^a. The Unscented Filtering equations are given as follows:

Initialize with:

$$\hat{\boldsymbol{x}}_0 = E[\boldsymbol{x}_0]$$

$$\boldsymbol{P}_0 = E[(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)(\boldsymbol{x}_0 - \hat{\boldsymbol{x}}_0)^T]$$

$$\boldsymbol{P}_0^a = E[(\boldsymbol{x}_0^a - \hat{\boldsymbol{x}}_0^a)(\boldsymbol{x}_0^a - \hat{\boldsymbol{x}}_0^a)^T] = \begin{pmatrix} \boldsymbol{P}_0 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}^{\boldsymbol{v}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{R}^{\boldsymbol{n}} \end{pmatrix}$$

For $k \in \{1,...,\infty\}$,
Compute sigma points:

$$\chi_{k-1}^a = [\hat{\boldsymbol{x}}_{k-1}^a \quad \hat{\boldsymbol{x}}_{k-1}^a + \gamma\sqrt{\boldsymbol{P}_{k-1}^a} \quad \hat{\boldsymbol{x}}_{k-1}^a - \gamma\sqrt{\boldsymbol{P}_{k-1}^a}]$$

Time update equations:

$$\chi_{k|k-1}^{\boldsymbol{x}} = \boldsymbol{F}[\chi_{k-1}^{\boldsymbol{x}}, \boldsymbol{u}_{k-1}, \chi_{k-1}^{\boldsymbol{v}}]$$

$$\hat{\boldsymbol{x}}_k^- = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k|k-1}^{\boldsymbol{x}} - \hat{\boldsymbol{x}}_k^-][\chi_{i,k|k-1}^{\boldsymbol{x}} - \hat{\boldsymbol{x}}_k^-]^T$$

$$\varphi_{k|k-1} = \boldsymbol{H}[\chi_{k|k-1}^{\boldsymbol{x}}, \chi_{k|k-1}^{\boldsymbol{n}}]$$

$$\hat{\boldsymbol{y}}_k^- = \sum_{i=0}^{2L} W_i^{(m)} \varphi_{i,k|k-1}$$

Measurement update equations:

$$\boldsymbol{P}_{\tilde{\boldsymbol{y}}_k \tilde{\boldsymbol{y}}_k} = \sum_{i=0}^{2L} W_i^{(c)} [\varphi_{i,k|k-1} - \hat{\boldsymbol{y}}_k^-][\varphi_{i,k|k-1} - \hat{\boldsymbol{y}}_k^-]^T$$

$$\boldsymbol{P}_{\tilde{\boldsymbol{x}}_k \tilde{\boldsymbol{y}}_k} = \sum_{i=0}^{2L} W_i^{(c)} [\chi_{i,k|k-1} - \hat{\boldsymbol{x}}_k^-][\varphi_{i,k|k-1} - \hat{\boldsymbol{y}}_k^-]^T$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{\boldsymbol{x}_k \boldsymbol{y}_k} \boldsymbol{P}_{\tilde{\boldsymbol{y}}_k \tilde{\boldsymbol{y}}_k}^{-1}$$

$$\hat{\boldsymbol{x}}_k = \boldsymbol{x}_k^- + \boldsymbol{K}_k(\boldsymbol{y}_k - \hat{\boldsymbol{y}}_k^-)$$

$$\boldsymbol{P}_k = \boldsymbol{P}_k^- - \boldsymbol{K}_k \boldsymbol{P}_{\tilde{\boldsymbol{y}}_k \tilde{\boldsymbol{y}}_k} \boldsymbol{K}_k^T$$

where, $\boldsymbol{x}^a = [\boldsymbol{x}^T, \boldsymbol{v}^T, \boldsymbol{n}^T]^T$, $\gamma = \sqrt{(L+\lambda)}$, $\chi^a = [(\chi^{\boldsymbol{x}})^T, (\chi^{\boldsymbol{v}})^T, (\chi^{\boldsymbol{n}})^T]^T$, λ =composite scaling parameter, L=dimension of augmented state, $\boldsymbol{R}^{\boldsymbol{v}}$ =process noise covariance, $\boldsymbol{R}^{\boldsymbol{n}}$ =measurement noise covariance.

3. ORBIT DYNAMICS

There are two types to describe orbit parameters in the satellite orbit determination problems. One is classical six

Keplerian elements, the other is position and velocity vectors. Herein, we define $\boldsymbol{x}(t)$, filter states, as a six-dimensional state vector given by

$$\boldsymbol{x} = \begin{bmatrix} \vec{\boldsymbol{R}} \\ \vec{\boldsymbol{V}} \end{bmatrix}$$

where $\vec{\boldsymbol{R}}$ is the position vector, and $\vec{\boldsymbol{V}}$ is the velocity vector. The effect of and the acceleration of the Earth central gravity force are considered for dynamic equation of satellite.

4. SIMULATIONS

Simulations were carried out using KOMPSAT-1(Korea Multi-Purpose Satellite -1) flight data for real orbit trajectory and inertial magnetic field vectors are simulated using a 10th-order IGRF 2000 model.

Figure 1 thru Figure 4 show the time trajectories of the estimation for each position, x, y and z. Figure 4 shows RSS position estimation error trajectory when initial RSS position error is 3300km. And Figure 5 shows RSS position estimation error trajectory with initial RSS position error of 4500km. Simulation results show that Unscented Filtering algorithm successfully estimates orbit trajectory even when initial RSS error is so large.

A simulation was also performed by assuming that attitude knowledge is available. Attitude knowledge can be easily obtained by using attitude sensors, such as gyros, sun sensors. Figure 6 thru Figure 8 show the time trajectories of position x, y, and RSS position estimation error with initial position error of 7100 km. Simulation result shows that filter convergence time is much faster even when initial position error is larger. According to the simulation results, it is very easy to see that fast convergence of filter together with small steady state error is obtained when attitude knowledge is available.

5. CONCLUSIONS

The problem of magnetometer-only orbit determination is considered. The Unscented Filtering algorithm is utilized to accurately capture the posterior mean and covariance of random variables. Simulation results show that the acceptable performance of Unscented Filtering for orbit determination. Furthermore, filter performance such as convergence time and steady state error will be dramatically improved when the attitude sensors are available. The algorithm presented in this paper can utilized for a low-cost spacecraft(e.g. without GPS) or contingency design when a major sensor is failed(e.g., GPS failure).

REFERENCES

[1] J. Deutschmann and I. Bar-Itzhack, "Comprehensive Evaluation of Attitude and Orbit Estimation Using Real Earth Magnetic Field Data,", *11th AIAA/USU Conference on Small Satellite*, 1997.

[2] G. Shorshi and I. Bar-Itzhack, "Satellite Autonomous Navigation based on Magnetic Field Measurement," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 4, pp. 843-850, July-Aug. 1995.

[3] M. L. Psiaki, L. Huang and S. M. Fox, "Ground Tests of Magnetometer-Based Autonomous Navigation (MAG NAV) for Low-Earth-Orbiting Spacecraft," *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 1, pp. 206-214, Jan.-Feb. 1993

[4] M. Wiegand, "Autonomous Satellite Navigation via Kalman Filtering of Magnetometer Data," *Acta Astronautica*, Vol. 38, No. 408, pp. 395-403, Feb.-April 1996.

[5] M. L. Psiaki, "Autonomous Low-Earth-Orbit Determination From Magnetometer and Sun Sensor Data," *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, pp. 296-304, March-April 1999.

[6] S. J. Julier and J. K. Uhlmann, *A General Method for Approximating Nonlinear Transformations of Probality Distributions*, Technical Report, RRG, Dept. of Engineering Science, University of Oxford, Nov. 1996.

[7] S. J. Julier, J. K. Uhlmann, and H. F. Durrant-Whyte, "A New Method for the Nonlinear Transformation of Means and Covariance in Filters and Estimators," *IEEE Transactions on Automatic Control*, Vol. AC-45, No. 3., pp. 477-482, March 2000.

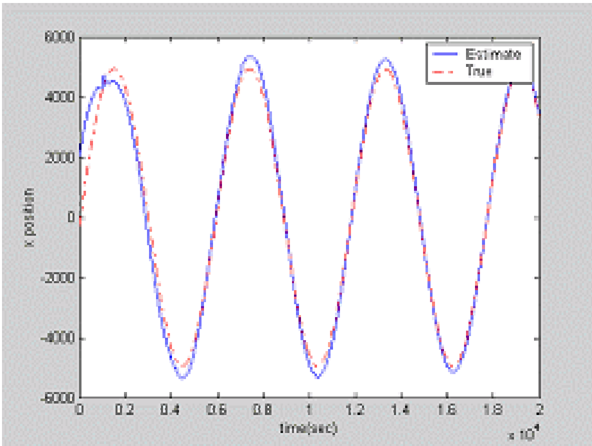

Fig. 1. x position time trajectories(initial RSS error=3300km).

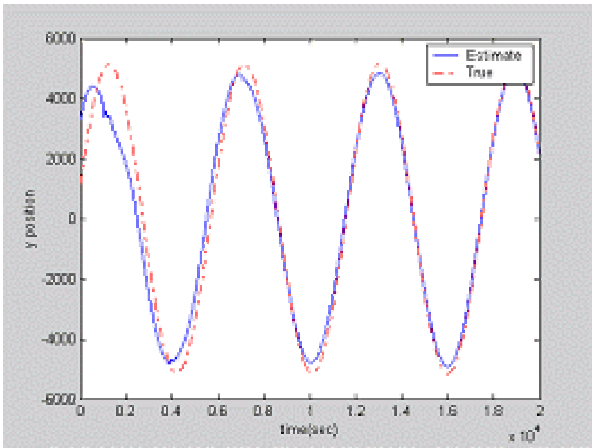

Fig. 2. y position time trajectories (initial RSS error=3300km).

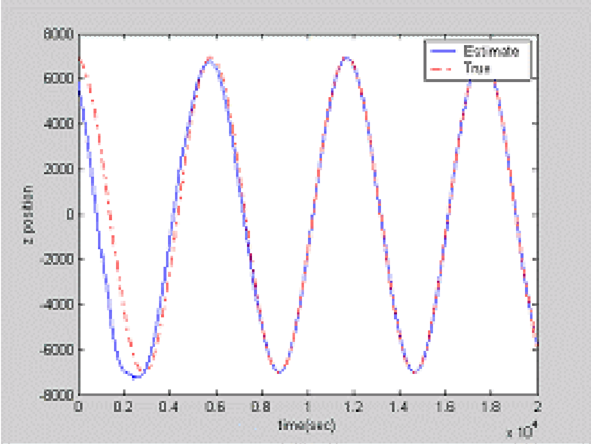

Fig. 3. z position time trajectories (initial RSS error=3300km).

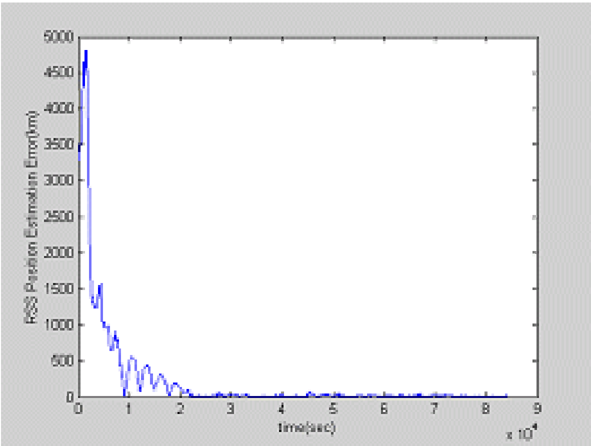

Fig. 4. RSS of position estimation error trajectory (initial RSS error=3300km).

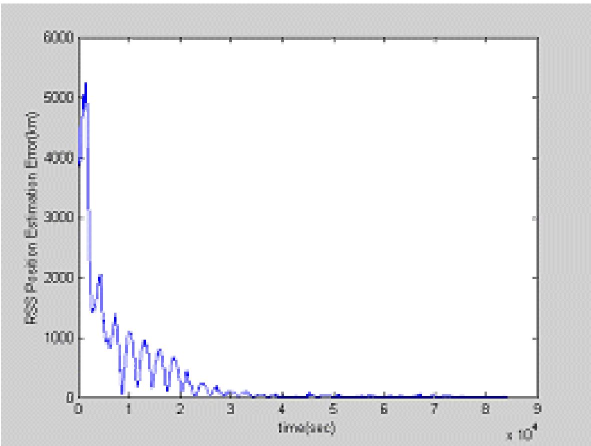

Fig. 5. RSS of position estimation error trajectory (initial RSS error=4500km).

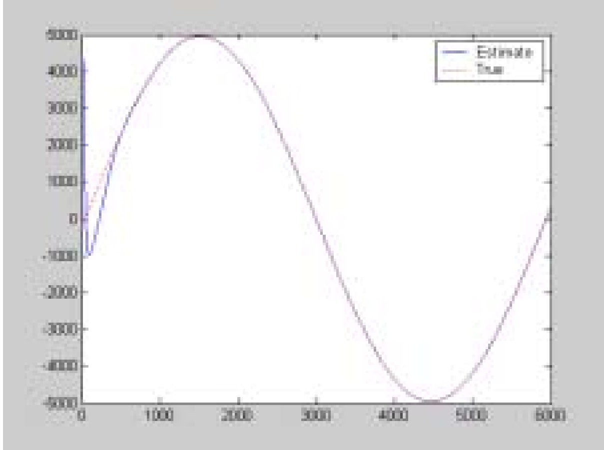

Fig. 6. x position time trajectories(initial RSS error=7200km).

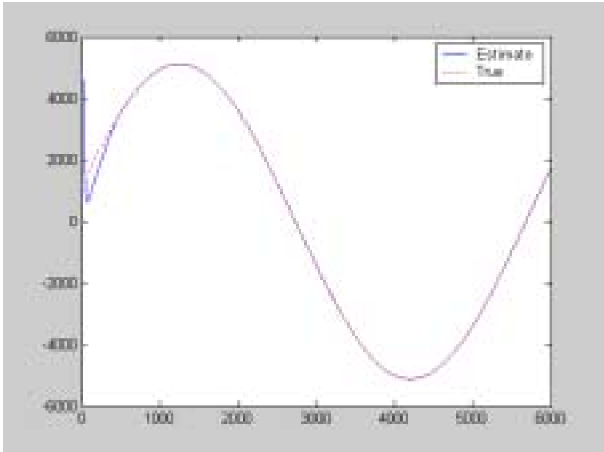

Fig. 7. y position time trajectories(initial RSS error=7200km).

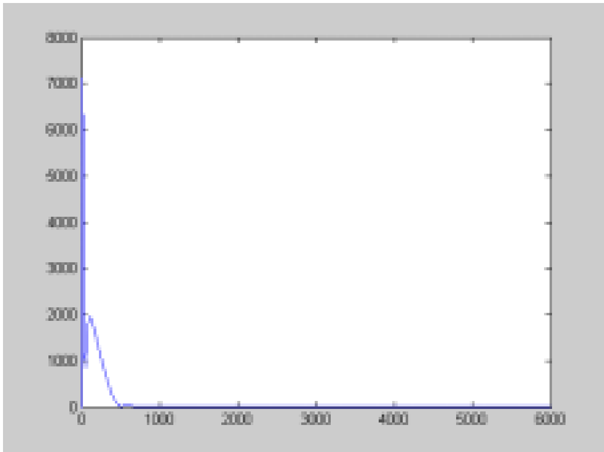

Fig. 8. RSS of position estimation error trajectory (initial RSS error=4500km).