An Improved Method to Construct T-S Fuzzy Model

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Abstract: This paper presents an improved method that constructs an equivalent T-S fuzzy model for nonlinear systems expressed by nonlinear differential equations including terms of power series. The method in this paper has fewer numbers of the rules than the previous methods as well as exactly expresses nonlinear systems. Moreover, this method can get wider feasible area satisfying the stability conditions than the previous methods. We show the improvement of modeling by comparing the proposed method with two previous methods through an inverted pendulum on a cart.

Keywords: T-S Fuzzy, Equivalent modeling

1. INTRODUCTION

There are two major methods to construct T-S fuzzy model. One is the identification of T-S fuzzy model from input-output data[1], [2], [4], [8], the other is the construction of T-S fuzzy model from a given nonlinear dynamics[3], [7].

In latter case, Taniguchi *et al.*[5] presented a systematic method of T-S fuzzy modeling to represent a given nonlinear dynamics exactly. However their method may construct so many rules to represent it because each rule is constructed from every element with nonlinear functions.

Later, Bae *et al.*[3] presented another method that expresses a nonlinear system as a form of the sum of product of linearly independent functions. And an equivalent T-S fuzzy model is obtained in this form.

But if nonlinear systems are expressed by nonlinear differential equations including terms of power series, there are room for improvement to reduce the number of rules.

This paper presents an improved method that constructs an equivalent T-S fuzzy model for nonlinear systems expressed by nonlinear differential equations including terms of power series. The method in this paper has fewer numbers of the rules than the previous methods as well as exactly expresses nonlinear systems. Moreover, this method has wider feasible area satisfying the stability conditions than the previous methods.

This paper is organized as follows: Section 2 reviews T-S fuzzy model and the method of T-S fuzzy modeling proposed by Taniguchi *et al.*[5] and Bae *et al.*[3] Section 3 presents the improved method for construction of T-S fuzzy model. Section 4 shows the improvement by comparing the proposed method with two previous T-S fuzzy modeling methods through the T-S fuzzy modeling of an inverted pendulum on a cart. And we conclude in section 5.

2. THE PREVIOUS T-S FUZZY MODELING METHODS

2.1 T-S Fuzzy model

The T-S fuzzy model is represented as follows:

Rule i IF $z_1(t)$ is M_{i1} & $\cdots$ & $z_p(t)$ is M_{ip}

$$\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t) \tag{1}$$

where $i = 1,2,\cdots,r$. M_{ij} is the fuzzy set and r is the numbers of rules. $u(t) \in R^m$ is the input vector, and A_i and B_i are constant matrices with appropriate dimensions. $z(t) = [z_1(t) \quad z_2(t) \quad \ldots \quad z_p(t)]$ are known premise variable, which may be function of the states, external disturbances, and/or time which are possible to be measured. Given a pair of $[x(t), u(t), z(t)]$, by using the center of gravity for defuzzification, the final state of the T-S fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t)] \tag{2}$$

where

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$$

$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)).$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij} and it is assumed that

$$w_i(z(t)) \geq 0, \quad i = 1,2,\cdots,r$$

$$\sum_{i=1}^{r} w_i(z(t)) > 0$$

for all t. Therefore

$$h_i(z(t)) \geq 0, \quad i = 1,2,\cdots,r$$

$$\sum_{i=1}^{r} h_i(z(t)) = 1$$

for all t.

2.2 Taniguchi's method

Taniguchi considered the following nonlinear system:

$$\dot{x}(t) = F(z(t))h(t) \tag{3}$$

where the elements of matrix function $F(z(t))$ are functions of $z(t)$ and $h^T(t) = [x^T(t) \quad u^T(t)]^T$. In oreder to exactly represent the nonlinear system (3), they proposed the following form:

$$\dot{x}(t) = \sum_{i=1}^{r} \sum_{j=1}^{n+m} \sum_{k=0}^{1} h_{ijk}(z(t)) a_{ijk} U_{ij}^F h(t) \tag{4}$$

where

$$h_{ij0}(z(t)) = \begin{cases} \dfrac{a_{ij1} - f_{ij}(z(t))}{a_{ij1} - a_{ij0}}, & a_{ij0} \neq a_{ij1} \\ 1/2 & a_{ij0} = a_{ij1} \end{cases}$$

$$h_{ij1}(z(t)) = \begin{cases} \dfrac{f_{ij}(z(t)) - a_{ij0}}{a_{ij1} - a_{ij0}}, & a_{ij0} \neq a_{ij1} \\ 1/2 & a_{ij0} = a_{ij1} \end{cases} \tag{5}$$

$$a_{ij0} = \min_z \{ f_{ij}(z) \}$$

$$a_{ij1} = \max_z \{ f_{ij}(z) \}$$

$f_{ij}(z(t))$ denotes the *(i,j)-th* element of $F(z(t))$ and U_{ij}^F denotes a matrix with same dimension to $F(z(t))$, whose *(i,j)-th* element is one and the others are zeros.

The expression (4) looks like giving $2^{n\times(n+m)}$ rules although some elements of $F(z(t))$ are constant real numbers. So we introduce another expression. Define the set S as

$$S = \{(i, j) \mid f_{ij}(z(t)) \text{ is not a constant real number}\},$$

and let n_s and s_l be the number of the elements of the set S and the *l-th* element of the set S respectively, then (4) becomes

$$\dot{x}(t) = [F_0 + \sum_{l=1}^{n_s} \sum_{k=0}^{1} h_{s_l k}(z(t)) a_{s_l k} U_{s_l}^F] h(t) \tag{6}$$

where

$$h_{s_l 0}(z(t)) = \frac{a_{ij1} - f_{s_l}(z(t))}{a_{ij1} - a_{ij0}}$$

$$h_{s_l 1}(z(t)) = \frac{f_{s_l}(z(t)) - a_{ij0}}{a_{ij1} - a_{ij0}} \tag{7}$$

for $l = 1,2,..., n_s$, and

$$F_0 = \sum_{(i,j)\in \bar{S}} f_{ij} U_{ij}^F$$

$$\bar{S} = \{(i, j) \mid f_{ij}(z(t)) \text{ is a constant real number}\}.$$

From (6), we easily find out how many rules are needed. Thus, the number of rules is 2^{n_s}. Since the expression (6) is same to (4), we will use the form (6) to obtain a T-S fuzzy model. The nonlinear system (6) is equivalent to the following T-S fuzzy model:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} h_i(z(t)) F_i h(t) \\ &= \sum_{i=1}^{r} h_i(z(t)) [A_i x(t) + B_i u(t)] \end{aligned} \tag{8}$$

where

$$\begin{aligned} & r = 2^{n_s}, \\ & h_i(z(t)) = \prod_{l=1}^{n_s} h_{s_l i_l}(z(t)) \\ & F_i = F_0 + \sum_{i=1}^{n_s} a_{s_l i_l} U_{s_l i_s}^F \\ & [A_i \quad B_i] = F_i \end{aligned} \tag{9}$$

for $i = 1,2,\cdots,r$, and i_l is the *l-th* bit of the binary expression of (*i-1*). Note that it is obvious that the T-S fuzzy model (8) expresses the nonlinear system (3) exactly. However, this method may construct so many rules.

2.3 The sum of products of linearly independent functions

Bae presented another method of constructing T-S fuzzy model using the sum of products of linearly independent functions from nonlinear systems. This method considered a nonlinear system such as (3):

$$\dot{x}(t) = F(z(t))h(t)$$

where $h^T(t) = [x^T(t) \quad u^T(t)]^T$, the matrix function $F(z(t))$ is

$$F(z(t)) = \begin{bmatrix} f_{11}(z(t)) & f_{12}(z(t)) & \cdots & f_{1(n+m)}(z(t)) \\ f_{21}(z(t)) & f_{22}(z(t)) & \cdots & f_{2(n+m)}(z(t)) \\ \vdots & \vdots & \ddots & \vdots \\ f_{n1}(z(t)) & f_{n2}(z(t)) & \cdots & f_{n(n+m)}(z(t)) \end{bmatrix} \tag{10}$$

where

$f_{ij}(z(t))$ is the (i, j) element of the $F(z(t))$ matrix.

The nonlinear system (3) can be rewritten as follows:

$$\dot{x}(t) = [F_0 + \sum_{i=1}^{w} f_i(z(t)) F_i] h(t) \tag{11}$$

where

$$f_i(z(t)) = \prod_{j=1}^{v} g_j^{l_{ij}}(z(t)), \quad \forall i = 1,2,\cdots,w \tag{12}$$

where v is the least number of linearly independent functions $g_j(z(t))$ when we express $f_i(z(t))$ to the form of equation (12). l_{ij} is '1' if $f_i(z(t))$ has the term of $g_j(z(t))$, otherwise l_{ij} is '0'. Also it is always possible to rewrite the nonlinear system (3) to the equation (11). Substituting (12) into (11) gives

$$\dot{x}(t) = [F_0 + \sum_{i=1}^{w} \prod_{j=1}^{v} g_j^{l_{ij}}(z(t)) F_i] h(t) \tag{13}$$

and equation (13) is equivalent to

$$\dot{x}(t) = [F_0 + \sum_{i=1}^{w} \prod_{j=1}^{v} \sum_{k=0}^{1} h_{jk}(z(t)) g_{jk}^{l_{ij}} F_i] h(t) \tag{14}$$

where

$$h_{j0}(z(t)) = \frac{g_{j1} - g_j(z(t))}{g_{j1} - g_{j0}}$$

$$h_{j1}(z(t)) = \frac{g_j(z(t)) - g_{j0}}{g_{j1} - g_{j0}} \tag{15}$$

$$g_{j0} = \min_z\{g_j(z)\}$$

$$g_{j1} = \max_z\{g_j(z)\}$$

for all $j = 1,2,\cdots,v$. To verify that (13) and (14) are equivalent, we need to show the following are true:

$$g_j^{l_{ij}}(z(t)) = \sum_{k=0}^{1} h_{jk}(z(t))\, g_{jk}^{l_{ij}} \tag{16}$$

$$\sum_{k=0}^{1} h_{jk}(z(t)) = 1 \tag{17}$$

for all $j = 1,2,\cdots,v$. Equation (17) is shown from (14), and (15) is derived from

$$\begin{aligned} g_j^{l_{ij}} &= [h_{j0}(z(t))g_{j0} + h_{j1}(z(t))g_{j1}]^{l_{ij}} \\ &= \begin{cases} h_{j0}(z(t))g_{j0} + h_{j1}(z(t))g_{j1}, & l_{ij} = 1 \\ 1 (= h_{j0}(z(t)) + h_{j1}(z(t))), & l_{ij} = 0 \end{cases} \\ &= h_{j0}(z(t))g_{j0}^{l_{ij}} + h_{j1}(z(t))g_{j1}^{l_{ij}} \end{aligned}$$

Using the T-S fuzzy model representation, (14) is rewritten as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t)] \tag{18}$$

where

$$r = 2^v$$

$$h_i(z(t)) = \prod_{k=1}^{v} h_{ki_k}(z(t)) \tag{19}$$

$$[A_i \quad B_i] = F_0 + \sum_{j=1}^{q}\prod_{k=1}^{v} g_{ki_k}^{l_{jk}} F_j$$

for all $i = 1,2,\cdots,r$ and i_k is a *k-th* bit of binary number of *(i-1)* which is represented to binary number having v bits. From (14) and (19), we have

$$h_i(z(t)) \geq 0, \qquad i = 1,2,\cdots,r$$

$$\sum_{i=1}^{r} h_i(z(t)) = 1 \tag{20}$$

for all *t*.

3. AN IMPROVED METHOD

We consider a nonlinear system expressed by nonlinear differential equations including terms of power series as follows:

$$\dot{x}(t) = [F_0 + \sum_{i=1}^{a} g^i(z(t))F_i]h(t) \tag{21}$$

The nonlinear system (21) is represented in terms of $g^i(z(t))$ $(i = 1,2,\cdots,a)$. So if we express $g(z(t))$ as follows:

$$g(z(t)) = h_{10}(z(t))g_{10} + h_{11}(z(t))g_{11} \tag{22}$$

where

$$h_{10}(z(t)) = \frac{g_{11} - g(z(t))}{g_{11} - g_{10}},$$

$$h_{11}(z(t)) = \frac{g(z(t)) - g_{10}}{g_{11} - g_{10}} \tag{23}$$

$$g_{10} = \min_z\{g(z)\}$$

$$g_{11} = \max_z\{g(z)\}$$

then

$$\begin{aligned} g^l(z(t)) &= [h_{10}(z(t))g_{10} + h_{11}(z(t))g_{11}]^i \\ &= \sum_{j=0}^{i} h_{10}^{i-j}(z(t))h_{11}^{j}(z(t))g_{10}^{i-j}g_{11}^{j} \end{aligned} \tag{24}$$

where

$$\begin{pmatrix} i \\ j \end{pmatrix} = \frac{i!}{(i-j)!\,j!}$$

$$j! = \begin{cases} i\times(i-1)\times\cdots\times 2\times 1, (i \neq 0) \\ 1, (i = 0) \end{cases}$$

Since $h_{10}(z(t)) + h_{11}(z(t)) = 1$, (21) becomes

$$\begin{aligned} & g^i(z(t)) \\ &= [h_{10}(z(t)) + h_{11}(z(t))]^{a-i}[h_{10}(z(t))g_{10} + h_{11}(z(t))g_{11}]^i \\ &= \{\sum_{j=0}^{a-i}\begin{pmatrix} a-i \\ j \end{pmatrix}[h_{10}(z(t))]^{a-i-j}[h_{11}(z(t))]^j\} \\ &\quad \times\{\sum_{k=0}^{i}\begin{pmatrix} i \\ k \end{pmatrix}[h_{10}(z(t))]^{i-k}[h_{11}(z(t))]^k g_{10}^{j-k}g_{11}^{k}\} \\ &= \sum_{j=0}^{a-i}\sum_{k=0}^{i}\begin{pmatrix} a-i \\ j \end{pmatrix}\begin{pmatrix} i \\ k \end{pmatrix}[h_{10}(z(t))]^{a-j-k}[h_{11}(z(t))]^{j+k}g_{10}^{j-k}g_{11}^{k} \end{aligned} \tag{25}$$

Let $b = j + k$, then (24) is equivalent to

$$\begin{aligned} & g^i(z(t)) \\ &= \sum_{b=0}^{a}\sum_{k=k_1}^{k_2}\begin{pmatrix} a-i \\ b-k \end{pmatrix}\begin{pmatrix} i \\ k \end{pmatrix}[h_{10}(z(t))]^{a-b}[h_{11}(z(t))]^b g_{10}^{j-k}g_{11}^{k} \\ &= \sum_{b=0}^{a}\begin{pmatrix} a \\ b \end{pmatrix}[h_{10}(z(t))]^{a-b}[h_{11}(z(t))]^b \\ &\qquad \times\left\{\sum_{k=k_1}^{k_2}\frac{\begin{pmatrix} a-i \\ b-k \end{pmatrix}\begin{pmatrix} i \\ k \end{pmatrix}}{\begin{pmatrix} a \\ b \end{pmatrix}}g_{10}^{j-k}g_{11}^{k}\right\} \end{aligned} \tag{26}$$

where

$$k_1 = \max\{0, i + b - a\}$$

$$k_2 = \min\{i, b\}$$

Substituting (26) into (21) gives

$$\begin{aligned} \dot{x}(t) &= \{F_0 + \sum_{i=1}^{a}\sum_{b=0}^{a}\begin{pmatrix} a \\ b \end{pmatrix}[h_{10}(z(t))]^{a-b}[h_{11}(z(t))]^b \\ &\qquad \times\left\{\sum_{k=k_1}^{k_2}\frac{\begin{pmatrix} a-i \\ b-k \end{pmatrix}\begin{pmatrix} i \\ k \end{pmatrix}}{\begin{pmatrix} a \\ b \end{pmatrix}}g_{10}^{j-k}g_{11}^{k}\right\}F_i\}h(t) \end{aligned}$$

$$\dot{x}(t) = \{F_0 + \sum_{b=0}^{a}\begin{pmatrix} a \\ b \end{pmatrix}[h_{10}(z(t))]^{a-b}[h_{11}(z(t))]^{b} \times \sum_{i=1}^{a}\sum_{k=k_1}^{k_2}\frac{\begin{pmatrix} a-i \\ b-k \end{pmatrix}\begin{pmatrix} i \\ k \end{pmatrix}}{\begin{pmatrix} a \\ b \end{pmatrix}} g_{10}^{j-k} g_{11}^{k} F_i\} h(t)$$

Therefore, a T-S fuzzy model for the nonlinear system (21) is as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))[A_i x(t) + B_i u(t)] \tag{27}$$

where

$$r = a + 1$$

$$h_i(z(t)) = \begin{pmatrix} r-1 \\ i-1 \end{pmatrix}[h_{10}(z(t))]^{r-i}[h_{11}(z(t))]^{i-1}$$

$$\begin{bmatrix} A_i & B_i \end{bmatrix} = F_0 + \sum_{j=1}^{a}\sum_{k=k_1}^{k_2}\frac{\begin{pmatrix} r-1-j \\ i-1-k \end{pmatrix}\begin{pmatrix} i \\ k \end{pmatrix}}{\begin{pmatrix} r-1 \\ i-1 \end{pmatrix}} g_{10}^{j-k} g_{11}^{k} F_j \tag{28}$$

where $k_1 = \max\{0, j+b-a\}, k_2 = \min\{j, b\}$

for $i = 1, 2, \cdots, r$

Remark) This example shows that the nonlinear system (21) can be exactly represented as the T-S fuzzy model (27) with $a+1$ rules. For nonlinear systems such as (21), the method explained in this section constructs fewer rules than the previous methods.

Example) We consider a nonlinear system such as

$$\dot{x}(t) = a\sin^2 x(t)\cdot x(t) + b\sin x(t)\cdot u(t)$$

for $-\frac{p}{6} \le x(t) \le \frac{p}{6}$.

If we choose $f_1(z(t)) = \sin x(t)$ and $f_2(z(t)) = \sin^2 x(t)$, the method proposed by II.3 constructs 4 rules.

We can get $\begin{bmatrix} A_i & B_i \end{bmatrix}$ as follows:

$$\begin{bmatrix} A_1 & B_1 \end{bmatrix} = \begin{bmatrix} ag_{10} & bg_{20} \end{bmatrix}$$
$$\begin{bmatrix} A_2 & B_2 \end{bmatrix} = \begin{bmatrix} ag_{10} & bg_{21} \end{bmatrix}$$
$$\begin{bmatrix} A_3 & B_3 \end{bmatrix} = \begin{bmatrix} ag_{11} & bg_{20} \end{bmatrix}$$
$$\begin{bmatrix} A_4 & B_4 \end{bmatrix} = \begin{bmatrix} ag_{11} & bg_{21} \end{bmatrix}$$

where

$$g_{10} = \min_x(\sin^2 x)$$
$$g_{11} = \max_x(\sin^2 x)$$
$$g_{20} = \min_x(\sin x)$$
$$g_{21} = \max_x(\sin x) \quad .$$

And the feasible area satisfying the stability conditions is shown Fig. 1. marked by '*'.

If we choose $g = \sin x$, the method explained in this section constructs 3 rules. We can get $\begin{bmatrix} A_i & B_i \end{bmatrix}$ as follows:

$$\begin{bmatrix} A_1 & B_1 \end{bmatrix} = \begin{bmatrix} ag_{10}^2 & bg_{10} \end{bmatrix}$$
$$\begin{bmatrix} A_2 & B_2 \end{bmatrix} = \begin{bmatrix} ag_{11}^2 & bg_{11} \end{bmatrix}$$
$$\begin{bmatrix} A_3 & B_3 \end{bmatrix} = \begin{bmatrix} 2a(g_{11}\cdot g_{10}) & b(g_{10}+g_{11}) \end{bmatrix} \quad .$$

And the feasible area satisfying the stability conditions is shown Fig. 2. marked by '*'.

In this example, method of this paper has more possibility to get wider feasible area satisfying the stability conditions than the previous methods.

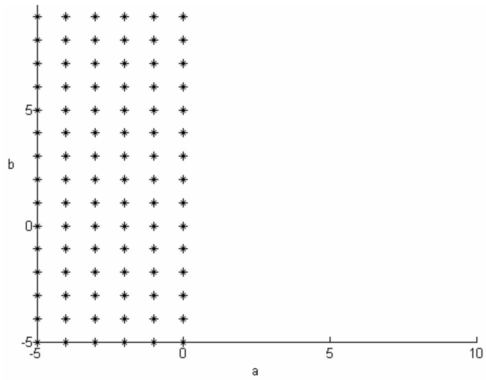

Fig. 1

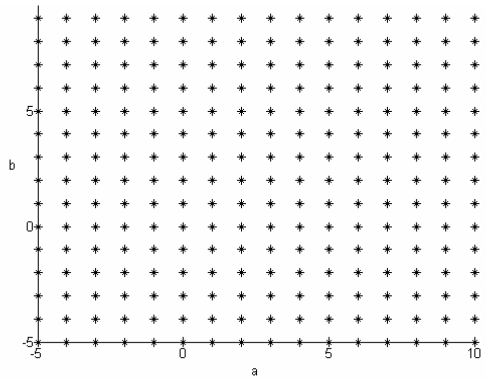

Fig. 2

4. AN EXAMPLE : INVERTED PENDULUM ON A CART

In this section, we consider the nonlinear dynamics of an inverted pendulum on a cart to construct T-S fuzzy models. The equations of motion for inverted pendulum system are as follows:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = x_5(t)$$
$$\dot{x}_3(t) = x_4(t)$$
$$\dot{x}_4(t) = x_6(t)$$

$$\begin{aligned} \dot{x}_5(t) = &[m^2l^2g_3^2(z(t)) - ab]x_5(t) \\ &+ g_2(z(t))[amgl \\ &- m^2l^2g_3(z(t))g_4(z(t))]x_1(t) \\ &- f_p a x_2(t) + f_c ml\ g_3(z(t))x_4(t) \\ &- ml\ g_3(z(t))u(t) \end{aligned} \tag{29}$$

$$\begin{aligned} \dot{x}_6(t) = &[m^2l^2g_3^2(z(t)) - ab]x_6(t) \\ &+ g_2(z(t))[bml\ g_4(z(t)) \end{aligned}$$

$$- m^2 l^2 g \ g_3(z(t))]x_1(t)$$
$$+ f_p ml \ g_3(z(t)) x_2(t)$$
$$- f_c b x_4(t) + bu(t)$$

where

$$a = M + m, \quad b = J + ml^2.$$
$$g_1(z(t)) = [ab - m^2 l^2 \cos^2 x_1(t)]^{-1}$$
$$g_2(z(t)) = \mathrm{sinc} x_1(t)$$
$$g_3(z(t)) = \cos x_1(t)$$
$$g_4(z(t)) = x_2^2(t)$$

$x_1(t)$ is the angle(rad) of the pendulum from the vertical, $x_2(t)$ is the displacement(m) of the cart, $x_4(t)$ is the velocity(m/s) of the cart, $u(t)$ is the force(N) applied to the cart. $g = 9.8[m/s^2]$ is the gravity constant, M is the mass of the cart, m is the mass of the pendulum, l is the length from the center of mass of the pendulum to the shaft axis, J is the moment of inertia of the pendulum round its center of mass, f_c is the friction constant of the cart, and f_p is the friction constant of the pendulum.

We represent (29) as

$$E\dot{z}(t) = \left[F_0 + \sum_{i=1}^{6} f_i(z(t))F_i\right]\begin{bmatrix} z(t) \\ u(t) \end{bmatrix}$$

$$z(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_5(t) \\ x_3(t) \\ x_4(t) \\ x_6(t) \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$f_1(z(t)) = g_2(z(t))$$
$$f_2(z(t)) = g_3(z(t))$$
$$f_3(z(t)) = g_3^2(z(t))$$
$$f_4(z(t)) = g_2(z(t)) g_3(z(t))$$
$$f_5(z(t)) = g_2(z(t)) g_4(z(t))$$
$$f_6(z(t)) = g_2(z(t)) g_3(z(t)) g_4(z(t))$$

$$F_0 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -f_p a & -ab & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -f_c b & -ab & b \end{bmatrix}$$

$$F_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ amgl & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & f_c ml & 0 & -ml \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & f_p ml & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m^2 l^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m^2 l^2 & 0 \end{bmatrix}$$

$$F_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -m^2 g l^2 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ bml & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$F_6 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -m^2 l^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since $f_i(z(t))$ $(i = 1, 2, \cdots, 6)$ are represented by $g_2(z(t))$, $g_3(z(t))$, $g_4(z(t))$, and $g_3^2(z(t))$,
(2^2) [from $g_2(z(t))$, $g_4(z(t))$]
$\times(2+1)$ [from $g_3(z(t))$, $g_3^2(z(t))$]
=12 rules will be constructed.

Although the numbers of rules made by methods of II.3 are 16, the method presented in this paper only make 12 rules.

5. CONCLUSION

This paper has presented the improved method of constructing T-S fuzzy model to exactly represent a given nonlinear dynamics. For nonlinear systems expressed by nonlinear differential equations including terms of power series, we show that the improved method has fewer rules than previous methods. And this method has been able to get wider feasible area satisfying the stability conditions than previous methods.

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