Application of Coefficient Diagram Method for Multivariable Control of Overhead Crane System

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Abstract: In this paper, the controller design by coefficient diagram method (CDM) for controlling the trolley position, load-swing angle and hoisting rope length of the overhead crane system simultaneously is proposed. The overhead crane system is a MIMO system consisting of two inputs and three outputs**.** Its mathematical model is nonlinear with coupling characteristics. This nonlinear model can be approximated to obtain a linear model where the first input mainly affects the trolley position and the load-swing angle while the second input mainly affects the hoisting rope length. In order to utilize the CDM concept for assigning the controllers, namely PID, PD and PI controllers separately, the model is approximated to be three transfer functions in accordance with trolley position, the load-swing angle and the hoisting rope length controls respectively. The satisfied performances of the overhead crane system controlled by the these controllers and fast rejection of the disturbance effect occurred at the trolley position are shown by simulation and experimental results.

Keywords: Coefficient diagram method, multivariable control, overhead crane system

1. INTRODUCTION

Overhead crane systems are widely used in industrial applications. It is mainly used for loading and unloading at wharf. The purpose of control is to transport loads rapidly and safely, reduce load-swing angle and hoist the loads to avoid the obstruction. It is known that the trolley position, load-swing angle and hoist motion of overhead crane system must be controlled simultaneously and properly, otherwise the load might be damaged. In recent years, many control schemes have been proposed to control the trolley position and load anti sway of the overhead crane system in order to obtain a good response. Those control schemes are anti-swing control of the container crane by fuzzy control [1], model reference adaptive control of a gantry crane scale model [2] and gantry crane control using neural network two degree of PID controller [3]. However, it is quite complicated to design and implement those controllers to meet the desired performances.

In this paper, the trolley position, load-swing angle and hoisting rope length of the overhead crane system controlled simultaneously by three controllers designed by CDM [4] is proposed. The overhead crane system is a MIMO system and its mathematical model is nonlinear with the coupling characteristics. This nonlinear model can be approximated to be a linear model of trolley position, load-swing angle and hoisting rope length. From the linearized model, it can be noticed that the first input mainly affects the trolley position and the load-swing angle while the second input mainly affects the hoisting rope length. Hence, this model can be approximated to consist of three transfer functions: the transfer function from the input duty cycle D of the trolley motor driver to the trolley position, the transfer function from the same input to the load-swing angle and transfer function from the input voltage V_i of the hoisting motor driver to the rope length. In this work, the controllers, namely PID, PD and PI controllers are employed to control the trolley position, the load-swing angle and the hoisting rope length respectively. These three controllers are designed independently by CDM. The parameters of the PID, PD and PI controllers are designed based on the stability and speed of the closed-loop system. In the CDM, stability and speed are designed from the standard stability index and the equivalent time constant respectively. The stability index and the equivalent time constant are defined based on the coefficients of the characteristic polynomial of the closed-loop transfer function. These coefficients are related to the controller parameters algebraically in an explicit form. Consequently, the parameters of the PID, PD and PI controller can be obtained.

The simulation and experimental results in controlling the trolley position, load-swing angle and hoisting rope length of the overhead crane system using the proposed controllers whose parameters designed by CDM are shown in this paper. The coefficient diagram of the trolley movement control when the values of the equivalent time constant τ varied for increasing the speed of the trolley motion and the rejection of input disturbance effect are also shown.

2. DYNAMICS MODEL OF OVERHEAD CRANE

The overhead crane system model is shown in Fig.1, where M is the mass of the trolley, m is the mass of the load, l is the rope length, θ is the swing angle, x is the position of the trolley, and F_x and F_l are respectively the control force of the trolley and the hoisting rope.

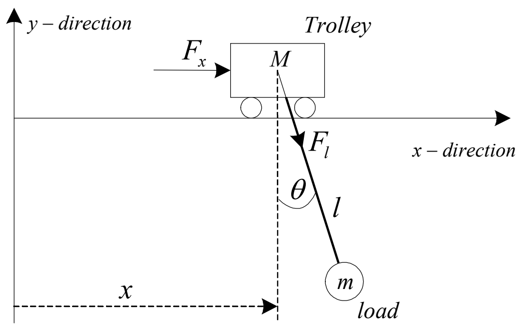

Fig. 1 Model of overhead crane system.

In order to obtain the mathematical model of the overhead crane system, the kinetic energy KE and the potential energy PE are found first. The kinetic energy of the trolley and the load which equals to the kinetic energy of the model is represented by

$$KE = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + 2\dot{x}\dot{\theta}l\cos\theta + 2\dot{x}\dot{l}\sin\theta + \dot{\theta}^2 l^2 + \dot{l}^2), \quad (1)$$

And the potential energy measured relative to the starting position of the load is also represented by

$$PE = -mgl\cos\theta, \quad (2)$$

where g is the gravitational acceleration. From the Lagrangian, $L = KE - PE$ is obtained as

$$L = \frac{1}{2}M\dot{x}^2 + \frac{1}{2}m(\dot{x}^2 + \dot{l}^2 + \dot{\theta}^2 l^2 + 2\dot{x}\dot{\theta}l\cos\theta + 2\dot{x}\dot{l}\sin\theta) + mgl\cos\theta. \quad (3)$$

By using the Euler-Lagrange equations of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q, \quad (4)$$

where Q is generalized force of system. The equations of motion associated with the generalized coordinates $q = [x, \theta, l]^T$ can be derived as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F_x - b_m\dot{x}, \quad (5)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0, \quad (6)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{l}}\right) - \frac{\partial L}{\partial l} = F_l - b_l\dot{l}. \quad (7)$$

where b_m and b_l are the viscous damping coefficient of trolley motion and hoisting rope respectively. Consequently, the equations of motion can be written, respectively, as

$$x \;:\; F_x - b_m\dot{x} = (M+m)\ddot{x} + ml\left(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta\right) + m\ddot{l}\sin\theta + 2m\dot{l}\dot{\theta}\cos\theta, \quad (8)$$

$$\theta \;:\; 0 = l\ddot{\theta} + 2\dot{\theta}\dot{l} + \ddot{x}\cos\theta + g\sin\theta, \quad (9)$$

$$l \;:\; F_l - b_l\dot{l} = m\ddot{l} + m\ddot{x}\sin\theta - ml\dot{\theta}^2 - mg\cos\theta. \quad (10)$$

Since the electrical time constant of motors is much smaller than the mechanical time constant, then the electrical dynamics is neglected. That is, the control force of the trolley F_x in (8) is approximated to be

$$F_x = K_m T_m \cong K_r D \quad (11)$$

where K_m and T_m are the trolley motor constant and torque, respectively, D (duty cycle) is the input to current driver of the trolley motor and K_r is a force constant of motion which depends on the trolley motor constant K_m and the power transmission (toothed-belt drive pulley ratio) of the trolley system. The control force of the hoisting F_l in (10) is also approximated to be

$$F_l = K_{ml} T_l \cong K_l V_i, \quad (12)$$

where K_{ml} and T_l are the hoisting motor constant and torque, respectively, V_i is the input voltage to current driver of the hoisting motor and K_l is a force constant of hoist which depends on the hoisting motor constant K_{ml} and the power transmission (gear ratio) of the hoist system. By using Lyapunov's linearization method, the linear equations of motion of (8) to (10) after substituting F_x and F_l can be given by

$$\ddot{x} = \frac{-b_m}{M}\dot{x} + \frac{K_r}{M}D \quad (13)$$

$$\ddot{\theta} = \frac{b_m}{Ml}\dot{x} - \frac{g}{l}\theta - \frac{K_r}{Ml}D \quad (14)$$

$$\ddot{l} = -\frac{b_l}{m}\dot{l} + \frac{K_l}{m}V_i. \quad (15)$$

Taking Laplace transformation to (13) – (15), the three transfer functions are obtained as

$$G_{px}(s) = \frac{x(s)}{D(s)} = \frac{K_r}{s(Ms + b_m)} \quad (16)$$

$$G_{p\theta}(s) = \frac{\theta(s)}{D(s)} = -\frac{K_r s}{(Mls^3 + b_m ls^2 + Mgs + b_m g)} \quad (17)$$

$$G_{pl}(s) = \frac{l(s)}{V_i(s)} = \frac{K_l}{s(ms + b_l)}. \quad (18)$$

These transfer functions are employed to design the controllers for the overhead crane system by CDM. Note that the values of K_r, K_l, b_m and b_l are not known and their values can be found from the experiment.

3. CONCEPT OF CDM

In this section, the concept of CDM used to design the parameters of a controllers so that the step response of the control system satisfies stability, fast response and robustness requirements is described [4].

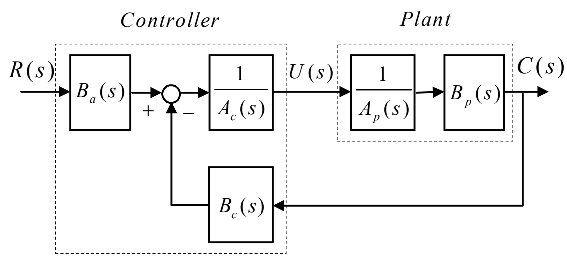

Fig. 2 Standard block diagram for CDM.

From the standard block diagram for CDM illustrated in Fig. 2, the transfer function of the plant in the polynomial form in each block is

$$A_p(s) = p_k s^k + p_{k-1}s^{k-1} + \ldots + p_0 \quad (19)$$

$$B_p(s) = q_m s^m + q_{m-1}s^{m-1} + \ldots + q_0 \quad (20)$$

and the controller polynomials are

$$A_c(s) = l_\lambda s^\lambda + l_{\lambda-1}s^{\lambda-1} + \ldots + l_0 \quad (21)$$

$$B_c(s) = k_\lambda s^\lambda + k_{\lambda-1}s^{\lambda-1} + \ldots + k_0 \quad (22)$$

$$B_a(s) = k_0 \tag{23}$$

where $\lambda < k$ and $m <$ k. $B_a(s)$ is called as a pre-filter and must be assigned to be k_0 so that the step response with zero steady-state error is obtained. The characteristic polynomial of the closed-loop system is then given as

$$\begin{aligned} P(s) &= A_c(s)A_p(s) + B_c(s)B_p(s) \\ &= a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \\ &= \sum_{i=0}^{n} a_i s^i \end{aligned} \tag{24}$$

where $a_0, a_1 \ldots, a_n$ are the coefficients of the characteristic polynomial. The stability index γ_i, the equivalent time constant τ and stability limit γ_i^* are defined as follows

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}} \tag{25}$$

$$\tau = \frac{a_1}{a_0} \tag{26}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0\ , \gamma_n = \infty \tag{27}$$

where $i = 1,\ldots,n-1$. In order to meet the specifications,

$$t_s = 2.5 \sim 3\tau \tag{28}$$

and

$$\gamma_i > 1.5\gamma_i^* . \tag{29}$$

are chosen. In general, the equivalent time constant τ can be obtained from $\tau = t_s / 2.5$ according to (28) and the standard stability index γ_i is recommended as

$$\gamma_{n-1} = \ldots = \gamma_3 = \gamma_2 = 2, \gamma_1 = 2.5 . \tag{30}$$

The standard values stated in (30) can be used to design the controller if the following condition is satisfied

$$p_k / p_{k-1} > \tau / \left(\gamma_{n-1}\gamma_{n-2} \cdots \gamma_1\right) \tag{31}$$

where p_k and p_{k-1} are the coefficients of the the plant at k^{th} and $(k-1)^{th}$. If the above condition is not satisfied, γ_{n-1} is first increased and then γ_{n-2} and so on, until (31) is satisfied. From (25)-(27), the coefficient a_i can be written by

$$a_i = a_0 \tau^i \frac{1}{\gamma_{i-1} \cdots \gamma_2^{i-2} \gamma_1^{i-1}} = a_0 \tau^i \prod_{j=1}^{i-1} \frac{1}{\left(\gamma_{i-j}\right)^j} . \tag{32}$$

Then the characteristic polynomial used to design the parameters of a controller is expressed as

$$P(s) \quad = a_0 \left[\left\{ \sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} 1 \Big/ \gamma_{i-j}^{j} \right) (\tau s)^i \right\} + \tau s + 1 \right] \tag{33}$$

By equating the characteristic polynomial (24) with a controller included to the characteristic polynomial (33), the parameters of the controller can be assigned.

4. CONTROLLER DESIGN

The controller design by CDM for the overhead crane system is described in this section. The controllers, namely PID, PD and PI controllers are designed independently and employed to control the trolley position, the load-swing angle and the hoisting rope length simultaneously. The block diagram of the multivariable control system is shown in Fig. 3.

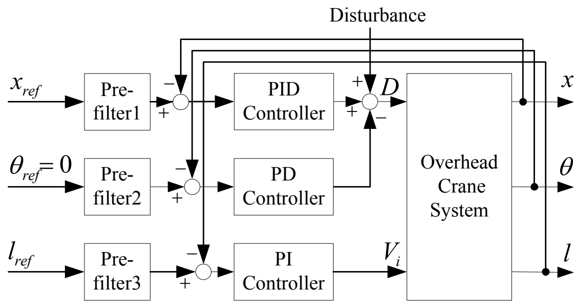

Fig. 3 The structure of multivariable control system.

4.1 PID controller design

The polynomial forms of the conventional PID controller for trolley motion control of (16) which has the standard transfer function

$$G_{cx}(s) = K_{cx}\left(1 + \frac{1}{T_{ix}s} + T_{dx}s\right) \tag{34}$$

can be expressed as

$$B_{cx}(s) = k_{dx}s^2 + k_{px}s + k_{ix}$$

and

$$A_{cx}(s) = s ,$$

where $k_{dx} = K_{cx}T_{dx}$, $k_{px} = K_{cx}$ and $k_{ix} = K_{cx}/T_{ix}$, and where K_{cx} is the proportional gain , T_{ix} is the integral time and T_{dx} is the derivative time. The values of k_{dx}, k_{px} and k_{ix} are then designed by the following procedures:

1) Define the stability index γ_1, γ_2 and the equivalent time constant τ.
2) Derive the characteristic polynomial (24), which includes the PID controller parameters, and equate its polynomial to the characteristic polynomial obtained from (33). Then the values of k_{dx}, k_{px} and k_{ix} are obtained.
3) Set the pre-filter $B_{ax}(s) = k_{ix}$

4.2 PD controller design

The standard transfer function of the conventional PD controller for controlling the swing angle of the load (17) is given by

$$G_{c\theta}(s) = K_{c\theta}\left(1 + T_{d\theta}s\right) \tag{35}$$

and its polynomial forms are

$$B_{c\theta}(s) = k_{d\theta}s + k_{p\theta}$$

and

$$A_{c\theta}(s) = 1 ,$$

where $k_{p\theta} = K_{c\theta}$ and $k_{d\theta} = K_{c\theta} T_{d\theta}$, and where $K_{c\theta}$ is the proportional gain and $T_{d\theta}$ is the derivative time. In this case, the equivalent time constant τ cannot be achieved by specifying the settling time t_s. Therefore, the values of $k_{p\theta}$ and $k_{d\theta}$ are designed by the following procedures:

1) Define the equivalent time constant τ as a variable after choosing the proper values of the stability index γ_2 and γ_1.

2) Derive the characteristic polynomial (24), which includes PD controller parameters, and equate its polynomial to the characteristic polynomial obtained from (33). Thus the values of $k_{p\theta}$, $k_{d\theta}$ and τ are obtained.

3) Set the pre-filter $B_{a\theta}(s) = k_{d\theta}$.

4.3 PI controller design

The standard transfer function of the conventional PI controller for hoisting rope length control of(18)

$$G_{cl}(s) = K_{cl}\left(1 + \frac{1}{T_{il}s}\right) \tag{36}$$

can be rewritten in the polynomial forms as

$$B_{cl}(s) = k_{pl}s + k_{il}$$

and

$$A_{cl}(s) = s,$$

where $k_{pl} = K_{cl}$ and $k_{il} = K_{cl}/T_{il}$, and where K_{cl} is the proportional gain and T_{il} is the integral time. As the equivalent time constant τ cannot be achieved by specifying the settling time t_s, the values of k_{pl} and k_{il} are therefore designed by the following procedures:

1) Define the equivalent time constant τ as a variable after choosing the proper values of the stability index γ_2 and γ_1.

2) Derive the characteristic polynomial (24), which includes PI controller parameters, and equate its polynomial to the characteristic polynomial obtained from (33). Thus the values of k_{pl}, k_{il} and τ are obtained.

3) Set the pre-filter $B_{al}(s) = k_{il}$

5. SIMULATION RESULTS

The parameters of PID, PD and PI controllers for the overhead crane system are found first. Then, simulations have been carried out with MATLAB for verifying the performance of the proposed multivariable control system. The parameters used in the simulation are shown in Table 1.

The specification for designing the PID controller to control the trolley position is that the settling time t_s of the closed-loop system is 12 seconds. In this simulation, the stability indices $\gamma_1 = 4$ and $\gamma_2 = 4$ are utilized and the equivalent time constant τ is assigned from $t_s/3$. Therefore, the equivalent time constant τ becomes 4 seconds. The values of the standard stability indices $\gamma_2 = 2$ and $\gamma_1 = 2.5$, and $\gamma_2 = 2$ and $\gamma_1 = 4$ are chosen for designing the parameters of the PD and PI controllers, respectively. From the design steps described in section 4, the parameters of PID, PD and PI controllers are summarized in Table 2.

Table 1 Parameters of the overhead crane system.

Mass of trolley (M)	6.12 kg
Mass of load (m)	0.3 kg
Rope length (l)	0.9 m
Gravitation acceleration (g)	9.8 $m/\sec^2$
Viscous damping coefficient of motion (b_m)	12.32 $N\sec/m$
Viscous damping coefficient of hoist (b_l)	3 $N\sec/m$
Force constant of motion (K_r)	0.57024
Force constant of hoist (K_l)	2.135

Table 2 Parameters of the controllers.

Controller	γ_1	γ_2	τ (sec)	K_c	T_i	T_d
PID	4.0	4.0	4.00	43.21	4.0	0.50
PD	2.5	2.0	0.80	70.10	-	0.56
PI	4.0	2.0	0.83	7.03	0.8	-

The specification for controlling the overhead crane system is that to control the trolley moving from 0 m to 1 m with small load-swing angle while the hoisting rope length moving from initial length of 1 m to 0.5 m and then return back to 1 m again. The simulation results are shown in Fig. 4.

It is seen from Fig. 4(a) and 4(c) that the PID and PI controllers respectively control the trolley position and hoisting rope length as desired. There are no overshoot P_o and steady-state error. It is also seen from Fig. 4(b) that the PD controller can reduce the swing angle of the load and the angle reaches to zero $(\theta = 0)$ rapidly.

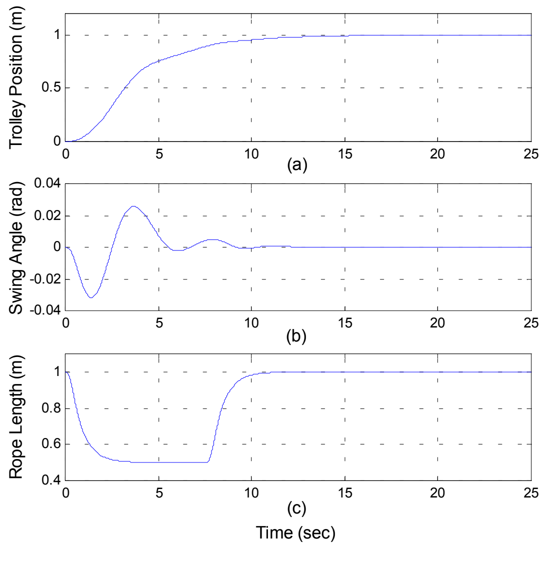

Fig. 4 Simulation results of closed-loop system (load 0.3kg); (a) trolley position, (b) load-swing angle, (c) hoisting lope length.

6. EXPERIMENTAL RESULTS

The experiments have been performed with a miniature of a overhead crane system shown in Fig. 5 in order to demonstrate the effectiveness of the proposed scheme. A DC servomotor and a DC geared-motor are used to turn a drive toothed-belt to move the crane trolley and to lift the load respectively. The load angle is detected by a rotary encoder attached to a small stainless rod. The position of trolley and the rope length are also detected by rotary encoders. The controllers are implemented digitally using C language on PC unit equipped with suitable I/O interface. The trolley has a maximum travel range of 1.7 m and a working length of the rope is 1 m.

The three controllers designed are implemented to control the overhead crane system shown in Fig. 6 by using the same control specifications stated in section 5. The similar results of Fig. 4 can be obtained and are shown in Fig. 6. It is noticed that the small load-swing angle can be achieved by experiment.

Fig. 5 Structure of overhead crane system.

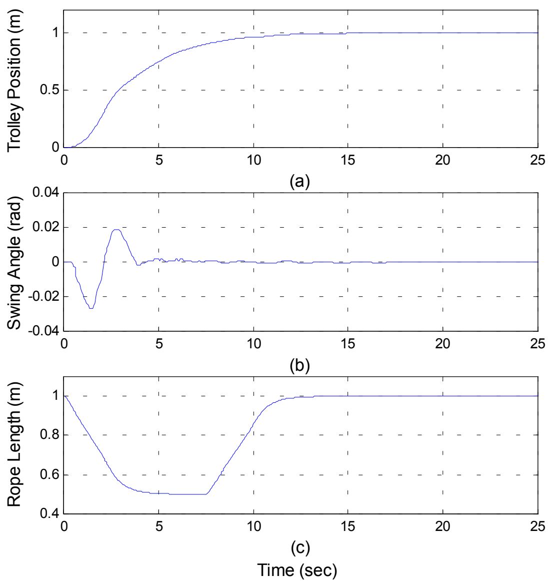

Fig. 6 Experimental results of closed-loop system (load 0.3kg); (a) trolley position, (b) load-swing angle, (c) hoisting lope length.

The speed of trolley movement can be increased by decreasing the equivalent time constant τ from 4 seconds to 3 seconds and to 2 seconds while keeping the previous values of stability indices γ_1 and γ_2. The experimental results of closed-loop system are shown in Fig. 7.

It is seen from Fig. 7 that smaller value of the equivalent time constant τ is the faster speed of trolley movement becomes as shown in Fig. 7(a). However, it is seen from Fig. 7 (b) that the small overshoot occurs and amplitude of the load-swing angle is larger when the speed of the response increases. The parameters of PID controller and the performance of trolley motion control system according to the variation of τ can be summarized in Table 3.

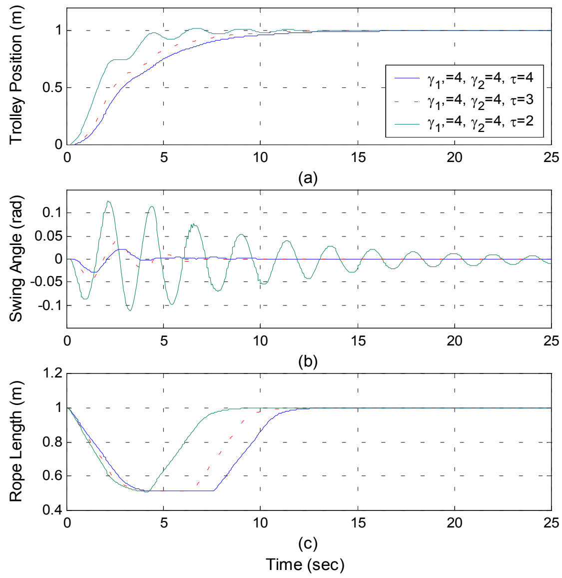

Fig. 7 Experiment results of closed-loop system (load 0.3kg); (a) trolley position, (b) load-swing angle, (c) hoisting lope length.

Table 3 PID controller parameters and system performances.

τ (sec)	K_c	T_i (sec)	T_d (sec)	t_r (sec)	t_s (sec)	P_o (%)
4	43.21	4	0.500	7.04	12.86	0.00
3	76.82	3	0.469	5.33	9.42	0.00
2	172.84	2	0.375	6.58	9.40	1.02

The coefficient diagrams of closed-loop system for trolley movement when the values of the equivalent time constant τ varied are also shown in Fig. 8. When the curve is more left-end down, the system becomes fast response [4] but the amplitude of load-swing angle is increased accordingly.

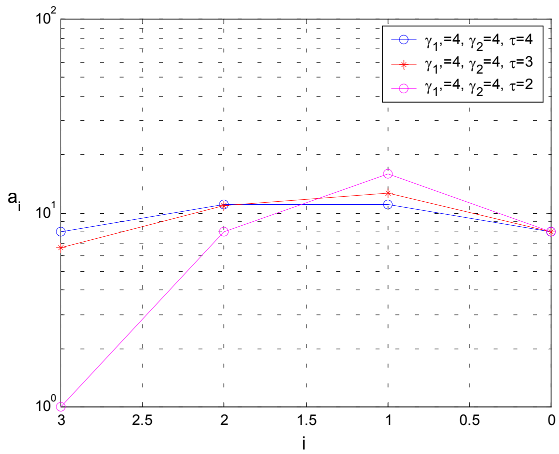

Fig. 8 Coefficient diagram of the trolley movement control.

6.1 Effect of load variation

In this subsection, the PID, PD and PI controllers are used to control the overhead crane system when the load is varied without redesigning these controllers. The mass of the load is varied from 0.3 kg to 1 kg. The experimental results of the overhead crane system are shown in Fig. 9. It is seen that the system is still stable. It can also be observed from the Fig. 9(b) that when the mass of the load increased, the oscillation amplitude of load-swing angle is increased.

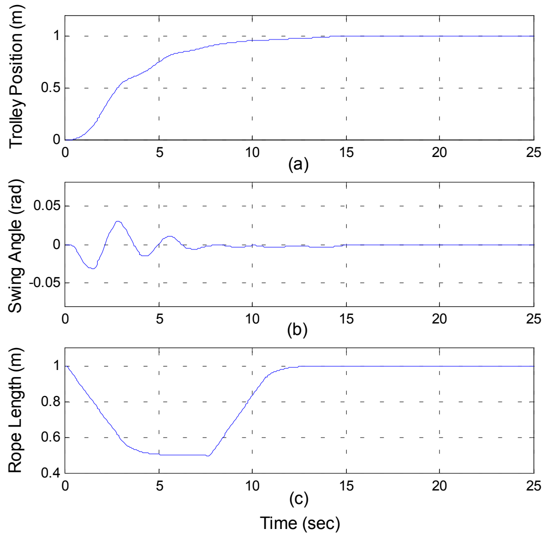

Fig. 9 Experimental results of closed-loop system (load 1kg); (a) trolley position, (b) load-swing angle, (c) hoisting lope length.

6.2 Step input disturbance

In order to demonstrate the effectiveness of the PID, PD and PI controllers, the controllers are implemented to control the overhead crane system without redesign. The step-input disturbance entering to the system at 20 seconds is employed in the experiments. The experimental results are shown in Fig. 10.

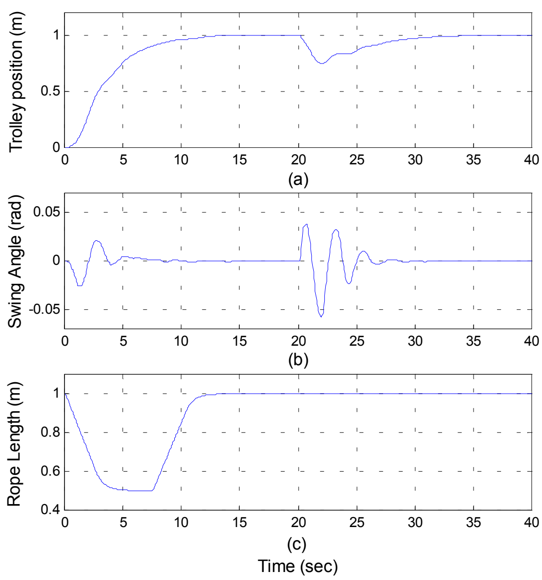

Fig. 10 Experimental results of closed-loop system with step input disturbances; (a) trolley position, (b) load-swing angle, (c) hoisting lope length.

It is seen from Fig. 10(a) and 10(c) that the PID and PI controllers can properly control the overhead crane system without overshoot P_o and steady-state error. The PD controller can suppress the load-swing angle and the effects of step input disturbance can be completely rejected as shown in Fig. 10(a) and 10(b).

7. CONCLUSIONS

The controllers namely PID, PD and PI controllers designed by CDM have been proposed to the multivariable control of the overhead crane system. These controllers have been used to control the trolley position, swing angle of the load and hoisting rope length, respectively. It has been shown that the parameters of the PID, PD and PI controllers can be properly designed. From the simulation and experimental results, the multivariable control of overhead crane system are satisfactory. Furthermore, the disturbance rejection can be achieved. It has also been demonstrated the effectiveness of the three controllers that the controllers can also be applied to control the overhead crane system when the mass of the load is varied without adjusting their controller parameters.

ACKNOWLEDGMENTS

This research work is partially supported by the Japan International Cooperation Agency (JICA).

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