Distributed Constrained Power Control with Fast Outage Convegence in CDMA systems

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Abstract: This paper proposes a fast distributed constrained power control (FDCPC) with non-stationary relaxation factor to next power update in CDMA cellular power control system. We review unconstrained control algorithms, the distributed power control (DPC), unconstrained second order power control (USOPC) and DPC with stationary relaxation factor (DPCSRF). Under the unconstrained condition, the convergence analysis shows theoretically that the convergence rate of DPC is the fastest one. However, under the constrained control algorithms, DCPC is not the fastest one any more because of transmission power constraint. To improve the convergence speed, the DCPC with non-stationary relaxation factor (FDCPC) are proposed. Under the constrained condition, the convergence rate of FDCPC outperforms that of DCPC and CSOPC.

Keywords: CDMA, SIR, relaxation factor, DCPC, CSOPC

1. INTRODUCTION

The power control of CDMA cellular system is to obtain an acceptable SIR for all users and to maximize the system capacity. And also the near-far problem has to be solved through the calculation of optimum transmission power according to the user locations. Various distributed power control (DPC) algorithms have been studied by many researchers in recent decades. The merit of DPC is distributiveness that is the ability of adjustment to the power levels of each mobile to make connection with only local measurements. And one of the most important performance measure of DPC is convergence speed of outage probability through the each DPC algorithms. The goal of this paper is to present the method that has faster convergence speed than conventional algorithms.

[1] presents a framework for uplink power control in cellular radio systems, while [2] introduces a distributed power control algorithm based on a more general model, where the algorithm calculates the transmission power required for each mobile that can accommodate all users with an acceptable SIR and produces a fixed point convergence. Grandhi *et al*. [3] propose distributed constrained power control (DCPC), which has become one of the most frequently referenced algorithms in later studies. Meanwhile, [4] describes a second-order constrained power control (CSOPC) algorithm that updates the transmission power using the current and past power. CSOPC has been shown to be more effective, including the ability to converge within a lower iteration number, than the DPC algorithm in [2]. El-Osery *et al.* [5] review various power control methods, including earlier referenced studies, then present a state-space equation that applies modern control theory and design the controller using linear quadratic control (LQ). LQ power control has a faster convergence time and higher CDMA channel capacity than CSOPC [5].

The current paper proposes a fast distributed constrained power control (FDCPC) with uses non-stationary relaxation factor to next power update in CDMA cellular power control system. We will review unconstrained control algorithms, the DPC, USOPC and DPCSRF. Under the unconstrained condition, the convergence analysis is given theoretically and it is verified that the convergence rate of DPC is the fastest.

After that, constrained control algorithms are considered. Here, the DCPC is not the fastest one any more because of transmission power constraint. To improve the convergence rate, the DCPC with non-stationary relaxation factor (FDCPC) are proposed. Under the constrained condition, the convergence rate of FDCPC outperforms that of DCPC and CSOPC.

In the following sections, we will review unconstrained control algorithms, DPC, USOPC and DPCSRF. Successively we will analyze the convergence to each algorithm using characteristics of iterative linear matrices. Next, we will briefly review transmission power constrained case and propose the fast distributed constrained power control (FDCPC). In section 3, we present the simulation results compared with other power control algorithms and discuss the performance of FDCPC.

2. SYSTEM DESCRIPTION OF DPC AND CONVERGENCE ANALYSIS

2.1 Distributed Power Control (DPC)

In our consideration, the CDMA cellular system is assumed that Q mobiles in a cell share the same channel at a given instance and the received SIR for each mobiles in the cell is unaffected by the received signal power from mobiles in other adjacent cells. Here, only the uplink power control case is considered, and it is assumed that the signal of mobile i is received correctly if SIR at base i is not less than a given target SIR value $\gamma*$. In order to make a connection with the minimal transmission power in the distributed power control, the SIR for mobile i should satisfy the following SIR constraint (1).

$$s_i = \frac{g_{ki} p_i}{\sum_{j=0, j \neq i}^{Q} g_{kj} p_j + n_i} \geq \gamma* \qquad (1)$$

where

p_i : transmission power of mobile i

g_{kj} : link gain from mobile j to base k

n_i : receiver noise at base i

s_i : SIR of mobile i

γ^* : desired SIR value.

Based on constraint (1), distributed power control applies on iterative method to adjust the power levels of each transmitted signal, using only local measurements. For the ideal situation, it is assumed that constraint (1) is the same as (2).

$$s_i = \frac{g_{ki} p_i}{\sum_{j=0, j\neq i}^{Q} g_{kj} p_j + n_i} = \gamma^* \tag{2}$$

Here, the $Q \times Q$ matrix $H = [h_{ij}]$ is defined such that $h_{ij} = \gamma^* g_{kj} / g_{ki}$ for $i \neq j$ and $h_{ij} = 0$ for $i = j$, and the vector $\eta = (\gamma^* n_i / g_{ki})$ has the length Q. Using the predefined matrices, (2) can be converted into a matrix form as follows:

$$AP = \eta \tag{3}$$

where $A = I - H$ and $P = (p_i)$ is the power vector of length Q.

To solve (3), the general iterative method can be considered as follows:

$$P(n+1) = M^{-1} N P(n) + M^{-1} \eta, \quad n = 0, 1, \cdots \tag{4}$$

where M and N are matrices of an appropriate size such that $P^* = M^{-1} N P^* + M^{-1} \eta$; vector $P(n) = (p_i(n))$ power level at iteration n. Based on the appropriate selection of M and N, the above iterative method can converge, i.e.,

$$\lim_{n \to \infty} P(n) = P^*. \tag{5}$$

When applying (4) to (2), (4) can be written as a set of linear equations that can be iteratively solved for P [3]. Through some manipulations, for each mobile i, (4) becomes

$$p_i(n+1) = \frac{\gamma^*}{s_i(n)} p_i(n), \quad n = 0, 1, \cdots \tag{6}$$

where $s_i(n)$ denotes the received SIR of mobile i at iteration n. Thus, as seen above, applying the DPC algorithm enables the next transmission power of mobile i to be iteratively calculated using the current power and received SIR. This is the distributed power control (DPC) algorithm and a control block diagram of DPC is shown in fig. 1.

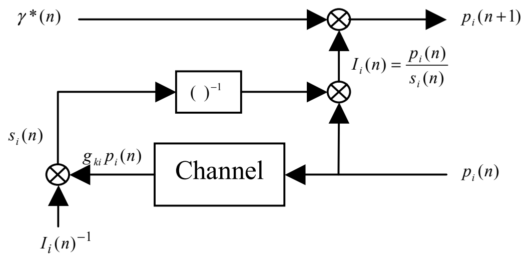

Fig. 1 Control block diagram of DPC for mobile i

2.2 DPC with stationary relaxation factor (DPCSRF)

As seen in (6), DPC algorithm uses the ratio between the desired SIR (γ^*) and measured SIR ($s_i(n)$) obtain next optimum transmission power. The proposed method in this paper computes the next transmission power using the error between γ^* and $s_i(n)$. Thus we can write the power update equation of DPC with stationary relaxation factor (DPCSRF) as follows:

$$p_i(n+1) = p_i(n) + u_i(n), \quad n = 0, 1, \cdots \tag{7}$$

The control input $u_i(n)$ of (7) is defined as

$$u_i(n) = k_i(n) e_i(n) \tag{8}$$

where $k_i(n)$ is the state feedback control gain. And $e_i(n)$ can be described as follows:

$$\begin{aligned} e_i(n) &= (\gamma^* - s_i(n)) I_i(n) / g_{ki}(n) \\ &= (\gamma^* - s_i(n)) p_i(n) / s_i(n) \end{aligned} \tag{9}$$

where $s_i(n)$ is the measured SIR. Now, (8) can be rewritten as (10).

$$u_i(n) = k_i(n) \{ \frac{\gamma^*}{s_i(n)} p_i(n) - p_i(n) \} \tag{10}$$

Thus the calculation of the next transmission power is performed as follows:

$$p_i(n+1) = p_i(n) + k_i(n) \left(\frac{\gamma^*}{s_i(n)} p_i(n) - p_i(n) \right) \tag{11}$$

Notice that, if $k_i(n) = 1$ in (11), (11) is the same as the next power update equation (6) of DPC.

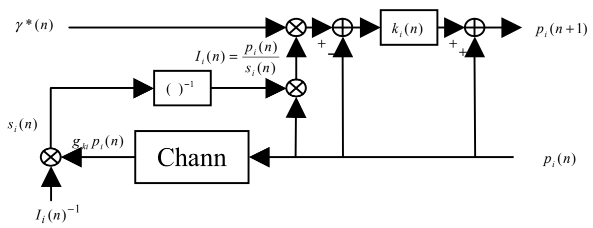

Fig. 2 Control block diagram of the DPC-SRF for mobile i

2.3 Convergence Analysis

Note that (11) can be rearranged like as (12), which is the similar form with the next transmission power update equation of USOPC in [4].

$$p_i(n+1) = k_i(n) \frac{\gamma^*}{s_i(n)} p_i(n) + (1 - k_i(n)) p_i(n) \tag{12}$$

Therefore, the convergence of the DPC with relaxation factor can be verified with the successive overrelaxation iterative method (SOR) as the same manner in [3]-[4]. Here, let's define the matrices M and N for the proposed algorithm are as follows:

$$M^{-1} = \omega I, \; N = \frac{1}{\omega}(1 - \omega I + \omega H) \tag{13}$$

where ω is relaxation factor. Applying (13) to (4), we can rewrite (12) to (14).

$$p_i(n+1) = \omega_i(n) \frac{\gamma^*}{s_i(n)} p_i(n) + (1 - \omega_i(n)) p_i(n) \tag{14}$$

For convergence, let's find ω satisfying the condition $\rho(M^{-1}N)_\omega < \rho(M^{-1}N)_{\omega=1} < 1$. Here,

$\rho(M^{-1}N)_\omega = \max\left| eigen\ values\ of\ (M^{-1}N)_\omega \right|$.

Proposition 1: In DPC system (4), when $M^{-1} = \omega I, N = \frac{1}{\omega}(1 - \omega I - \omega H)$ and $0 \le \omega \le 2$, ω satisfying the condition $\rho(M^{-1}N)_\omega < \rho(M^{-1}N)_{\omega=1} = \alpha\gamma^* < 1$ does not exist. Here, $\alpha = \sqrt{\frac{g_{12}g_{21}}{g_{11}g_{22}}}$.

Proof: For simplicity, let us find the spectral radius of the DPC system, $\rho(M^{-1}N)$, assume that it has two mobiles only like as in [3]. The eigenvalues of the iteration matrix, $M^{-1}N$, are

$$\lambda = (1-\omega) \pm \omega\alpha\gamma^*. \tag{15}$$

When $0 \le \omega \le 1$,

$$\left|(1-\omega) - \omega\alpha\gamma^*\right| \le \left|(1-\omega) + \omega\alpha\gamma^*\right| \tag{16}$$

is always satisfied.

Therefore, ω satisfying following equation does not exist.

$$\rho(M^{-1}N)_\omega = \left|(1-\omega) + \omega\alpha\gamma^*\right| < \rho(M^{-1}N)_{\omega=1} = \alpha\gamma^* < 1 \tag{17}$$

Thus $\omega = 1$ is the largest universal relaxation constant and this is verified in [3].

When $1 < \omega \le 2$,

$$\left|(1-\omega) - \omega\alpha\gamma^*\right| \ge \left|(1-\omega) + \omega\alpha\gamma^*\right| \tag{18}$$

is always satisfied

Therefore, it is satisfied with $\rho(M^{-1}N)_\omega = \left|(1-\omega) - \omega\alpha\gamma^*\right|$. To find ω satisfying the condition $\rho(M^{-1}N)_\omega < \rho(M^{-1}N)_{\omega=1} = \alpha\gamma^*$, the following equation is established.

$$\left|(1-\omega) - \omega\alpha\gamma^*\right| < \alpha\gamma^* \tag{19}$$

The condition $\omega < 1$ is obtained from (19) but this result is contract with the condition $1 < \omega \le 2$ mentioned before. Therefore, when $0 \le \omega \le 2$, ω satisfying the condition $\rho(M^{-1}N)_\omega < \rho(M^{-1}N)_{\omega=1} = \alpha\gamma^* < 1$ does not exists. ■

The convergence analysis of USOPC in [4] can be performed as the same method of proposition 1 and the iterative matrices are $M^{-1} = \omega\left(I - \omega\begin{bmatrix} 0 & 0 \\ H & 0 \end{bmatrix}\right)^{-1}$ and $N = \frac{1}{\omega}\left((1-\omega)I - \omega\begin{bmatrix} 0 & H \\ 0 & 0 \end{bmatrix}\right)$. Thus the eigenvalues of USOPC are

$$\lambda = (1-w) + 0.5 \cdot w\alpha\gamma^* \left\{ w\alpha\gamma^* \pm \sqrt{w^2\alpha^2\gamma^{*2} - 4(1-\omega)} \right\}$$

and

$$\lambda = (1-w) + 0.5 \cdot w\alpha\gamma^* \left\{ w\alpha\gamma^* \pm \sqrt{w^2\alpha^2\gamma^{*2} + 4(1-\omega)} \right\}. \tag{20}$$

And the spectral radius when $\omega = 1$ is $\rho(M^{-1}N)_{\omega=1} = (\alpha\gamma^*)^2$. When $0 \le \omega \le 2$, the theoretical proof of existence of ω satisfying the condition $\rho(M^{-1}N)_\omega < \rho(M^{-1}N)_{\omega=1} = (\alpha\gamma^*)^2 < 1$ is very difficult because of nonlinearity of eigenvalues in (20). So, we will verified through the numerical example that all eigenvalues of USOPC in (20) are calculated as a function of the relaxation factor when $\alpha\gamma^* = 0.5$. Fig. 3a shows the comparison of the DPC and DPCSRF and fig. 3b shows the comparison of spectral radius of the DPC and USOPC as a function of relaxation factor (ω). Note that the DPC is used as a reference in both numerical examples, the reference spectral radius of DPC is $\rho(M^{-1}N)_{\omega=1} = (\alpha\gamma^*)^2 = 0.25$ in USOPC case and the reference spectral radius of DPC is $\rho(M^{-1}N)_{\omega=1} = \alpha\gamma^* = 0.5$ in DPC with stationary relaxation factor. The comparison of spectral radius should be performed based on $\rho(M^{-1}N)_{\omega=1} = (\alpha\gamma^*)^2 = 0.25$ which is reference spectral radius of DPC in USOPC case.

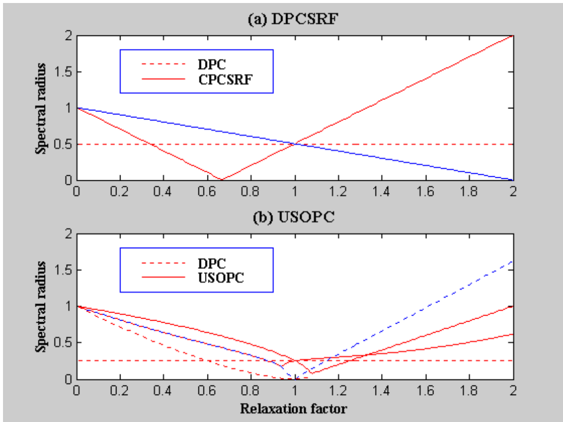

Fig. 3 Comparison of spectral radius, $\rho(M^{-1}N)$

In fig. 3a, when $0 \le \omega \le 1$, at least one of eigenvalues is bigger than 0 in the case of DPCSRF. Because of the biggest absolute value among eigenvalues is the spectral radius, we can see that the spectral radius of DPCSRF is bigger than 0.5 in fig. 3a. The result in fig. 3a is consistent with proposition 1 very well. In fig. 3b, the biggest eigenvalues of USOPC is always bigger than 0.25, the spectral radius of DPC. And for any other values except for $\alpha\gamma^* \ne 0.5$ in $0 \le \alpha\gamma^* \le 1$, similar result was seen through simulations. Therefore, the convergence rate of DPC is the fastest among them under the unconstrained condition.

2.4 DCPC with non-stationary relaxation factor (FDCPC)

Since the transmission power of a mobile is limited, we will consider the following constraint on the power:

$$0 \le p_i \le \overline{p}_i \tag{22}$$

where $\overline{p}_i$ is the maximum allowable transmission power level of each mobile. So if the constraint (22) is considered during power update, (6) is changed as follows:

$$p_i(n+1) = \min\left\{ \overline{p}_i, \frac{\gamma^*}{s_i(n)} p_i(n) \right\}, \quad n = 0, 1, \cdots. \tag{23}$$

Eq. (23) is the distributed constrained power control (DCPC), convergence rate of DCPC is not the fastest algorithm because of constraint (22), the convergence rate is not depend on the spectral radius any more. Because of the constraint is unavoidable in the practical application, the method that can be fasten the convergence rate of DCPC is needed.

Here, we consider applying non-stationary relaxation factor to state feedback gain $k_i(n)$ in (11) like as $\omega_i(n)$ in [4]. The relaxation factor of iterative method is applied as non-increasing sequence satisfying $1 < k_i(n) \le 2$, $\lim_{n\to\infty} k_i(n) = 1$ and can be described as (24).

$$k_i(n) = 1 + \frac{1}{a^n}, \quad n = 1,2\cdots \tag{24}$$

where, a is the constant that satisfying $1 < a$. Now, let's rewrite the equation of DCPC with non-stationary relaxation factor (FDCPC) to constraint form as follows:

$$p_i(n+1) = \min\left\{\bar{p}_i, p_i(n) + k_i(n)\left(\frac{\gamma^*}{s_i(n)}p_i(n) - p_i(n)\right)\right\} \tag{25}$$

The convergence of (25) has been considered in [4]. Fig. 4 shows the convergence test example of FDCPC and CSOPC with non-stationary relaxation factor and the simulation was performed based on the parameters mentioned next chapter 3. Though FDCPC needs 3 steps only to converge, DCPC needs 4-5 steps and CSOPC needs 5-6 steps to converge. We can show that convergence rate of the proposed FDCPC is the fastest one in fig. 4.

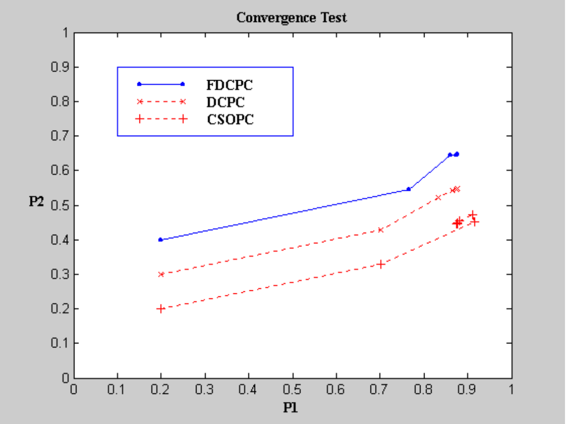

Fig. 4 Convergence test example

3. SIMULATION

3.1 Simulation Environment and parameters

The simulation of distributed power control is performed based on the IS-95 system [6], the environment and parameters are same as follows:

- Based on the 7-cell model, random choice of the number of mobiles and random allocation of the mobiles in each cell. Here the number of mobiles in each cell is determined to 10.
- Generation of the path loss based on available models. Here, it is assumed that the link gain g_{ki} is d^{-4}, where d (meter) is the distance between the mobile station and the base station. Other propagation models [6] are not considered.
  - The desired SIR is set to 8dB and the same desired SIR is applied to all mobiles in a cell
- Bit rate, R_b, is 9600 bits per second and Radio-channel bandwidth, B_c, is 1.2288 MHz
- Receiver noise, $n_i = n = 10^{-12}$
- Maximum transmission power of each mobile is 1 Watt.

3.2 Discussion

The simulation of distributed power control is performed based on the IS-95 system [6] and the seven-cell configuration and initial power of each mobile is selected to the uniformly distributed random number within the interval [0,1]. DCPC and CSOPC is used as a reference algorithms and the outage probability value was accumulated for the randomly selected 10000 instants like as [4]. The outage probability at each iteration is computed over 10000 instants by counting the portion of the number of unsupported mobiles at the iteration.

Fig. 4 shows the simulation results about the outage probability as a function of iteration. The outage probability of DCPC takes 15 iteration steps until approaches the fixed outage probability (5×10^{-4}), it is slower than that of the other algorithms. At the beginning, DCPC converges faster than other algorithms but DCPC becomes slower as approaching the fixed outage probability. CSOPC needs 10 iteration steps to converge, it is faster than that of DCPC. However, the outage probability increase from iteration 6 to 7 is caused by the dynamics of the second-order power control. However, FDCPC needs only 7 iteration steps to converge to the fixed outage probability and the convergence rate of FDCPC is the fastest one among them. And we can see that the outage probability increase problem from iteration 6 to 7 in CSOPC is improved greatly in FDCPC.

In fig. 6, the convergence shape of SIR example is presented to explain the difference of convergence between CSOPC and FDCPC. The number of mobiles in a cell set to 10. The convergence shape of CSOPC is shown increasing type from under the desired SIR as approaching the desired SIR (from iteration step 6 to 8). Thus at this situation, because of the probability that occur unsupported mobiles is high though the SIR of each mobiles is converging to the desired SIR, the outage probability of CSOPC near the desired SIR still maintains high. However, the convergence shape of FDCPC is shown decreasing type from over the desired SIR as approaching the desired SIR (from iteration step 5 to 8). Thus at this situation, because of the most mobiles are satisfy the desired SIR, though the convergence rate is slower than that of CSOPC, the outage probability of FDCPC near the desired SIR can be decrease to the very low value abruptly. This improvement is possible on the virtue of dynamics of non-stationary relaxation factor and the decision method of outage.

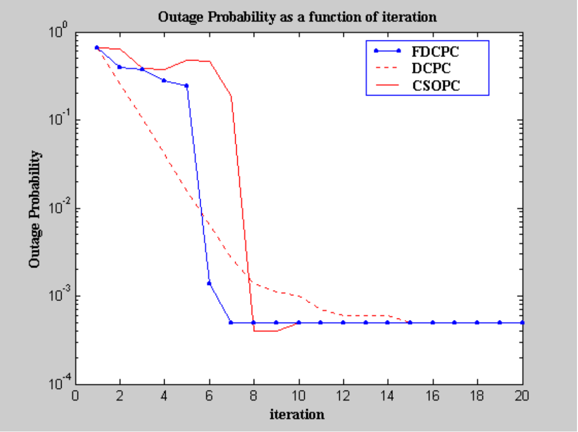

Fig. 5 Outage probability as a function of iteration

4. CONCLUSION

The current study presented a fast distributed constrained power control (FDCPC) that uses non-stationary relaxation factor to next power update in CDMA cellular power control system. Under the unconstrained condition, the convergence analysis shows theoretically that the convergence rate of DPC is the fastest one. Consideration about the constrained control algorithms shows that the DCPC is not the fastest one any more because of transmission power constraint. To improve the convergence rate, the DCPC with non-stationary relaxation factor (FDCPC) are proposed. Under the constrained condition, the simulation results are presented that the FDCPC converges the fastest one. This improvement is possible through the effect on dynamics of non-stationary relaxation factor and the decision method of outage.

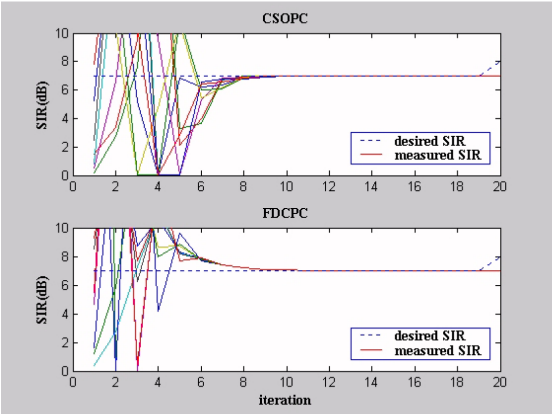

Fig. 6 Convergence shape of SIR

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