A Distributed Power Optimization Method for CDMA Cellular Mobile Systems Using an Adaptive Search Scheme

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Abstract: Future cellular networks will mainly be driven by, high quality channels, high band with utilization, low power consumption and efficient network management. For a given channel allocation, the capacity and quality of communication of cellular radio systems using CDMA(Code Division Multiple Access) can be increased by using a transmitter power control scheme to combat the near-far problem. Centralized power control schemes or distributed ones to maximize the minimum signal-to-interference in each user of CDMA wireless network have been investigated. This paper has proposed a distributed power control algorithm, which employs an adaptive search scheme, in order to solve quickly the linear systems of equations for power update in CDMA cellular radio systems. The simulation results show that the proposed scheme has faster convergence rate than the typical bang-bang type of distributed power control algorithm, which has been much used as a reference algorithm in IS-95A and W-CDMA communication network.

Keywords: **distributed power control, cellular mobile network, convergence rate, acceleration**

1. INTRODUCTION

DS/CDMA(Direct Sequence CDMA) communication systems achieve multiple-access capability by assigning a distinct signature waveform to each user from a set of waveforms with low mutual cross correlation. Because of the lack of orthogonality between signals, performance is affected by the so-called near-far problem.

In fact, adjacent channel interference or near-far problem, is the single most restraining factor on the system capacity of current mobile radio networks using DS/CDMA. To alleviate the severe reduction of multiple access capability caused by the near-far problem, transmitter power control has been a target of researches in the area for several years and the only remedies implemented in practice.

In a cellular wireless network, certain QoS(Quality of Service) should be maintained for all active users in the network. A quantity that measures the user's provided QoS is the SIR (Signal to Interference Ratio) or CIR (Carrier to Interference Ratio). A general idea to achieve the desired SIR for all the active users is to allocate the network resources in the most efficient way. Resource allocation in a cellular wireless network includes channel allocation, power control, etc. The power control is the most valuable resource in the network. The central idea in power control schemes is to maximize the minimum SIR(Signal to Interference Ratio) in each user of wireless network system. By proper power control of each transmitter power, the interference can be minimized.

Power control algorithms can be centralized or distributed [1][2][3][4]. A lot of researches have been concentrated more on DPC(Distributed Power Control) schemes than on the CPC(Centralized Power Control) schemes because the CPC suffers from the large-scaled data management and incurs network vulnerability. The DPC schemes are performed at each cell by using its current transmitter power. It is natural that DPC is simpler and requires less information than CPC. The DPC schemes require only the desired-path interference measurement, made at each base, and relayed to the corresponding mobile. However, a DPC algorithm takes more time for achieving minimum SIR than a CPC algorithm due to the limited amount of information. In practical systems, it is desirable to reduce transmitting powers as much as possible while maintaining the required quality of communication, especially for mobile terminals where transmitting power is provided by battery.

This paper has proposed an adaptive search for DPC od CDMA cellular systems to speed up the convergence rate in comparison with the typical DPC algorithm.

2. SYSTEM MODEL

Consider an uplink of a cellular CDMA system, in which ***N*** mobiles are active in the system. We consider that the stationary link gain between every base station *i* and every mobile *j* is stationary, and is given by g_{ij}. Without loss of generality, we will assume that mobile *i* is communicating with base station *i.* In a CDMA system, many mobiles will communicate with the same base station through the same frequency channel. Thus, in our notation, base station *i* and *j* may denote the same physical one if the mobile *i* and *j* are assigned to the same base station.

Fig. 1 illustrates gains of the cellular CDMA links between mobile terminals and base stations

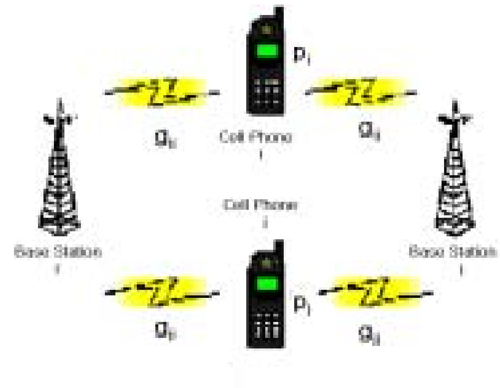


Fig. 1 Gain of cellular CDMA links

The current SIR $\gamma_i(k)$ at the base *i* is given by

$$\gamma_i(k) = \frac{q_i(k)}{I_i(k)} = \frac{g_{ii} p_i(k)}{\sum_{j=1, j\neq i}^{N} g_{ij} p_j(k) + \sigma_i^2}, \qquad i = 1,2,\cdots,N \tag{1}$$

In the above, k denotes an instant time, $q_i(k)$ in the numerator part is the power received from transmitter i at receiver i and $I_i(k)$ in the denominator part is the received interference plus noise power at receiver i. The quantity σ_i^2 is the thermal noise at receiver i. For the instant time k, let us assume that each mobile should achieve the target SIR γ_i^t as follows.

$$\gamma_i(k) \ge \gamma_i^t, \qquad i = 1,2,\cdots,N \tag{2}$$

Let us define a N by N matrix $H = [h_{ij}]$ such that

$$h_{ij} = \begin{cases} \dfrac{\gamma^t g_{ij}}{g_{ij}}, & for \quad i \ne j \\ 0, & for \quad i = j \end{cases} \tag{3}$$

Additionally, let us define a vector $b = [b_i]$ such that

$$b_i = \frac{\gamma^t \sigma_i^2}{g_{ii}} \tag{4}$$

Then, converting Eq.(1) into a matrix form, we have the following linear algebraic equations of power control problem.

$$Ap = b \tag{5}$$

Where, $A = I - H$ and $p = [p_i]$ denotes the power vector.

In [5], a necessary condition is derived to achieve feasible power for link gain matrix $G = [g_{ij}]$.

$$\gamma_i^t \gamma_j^t \le \frac{g_{ii} g_{jj}}{g_{ij} g_{ji}} \tag{6}$$

Similarly, maximum achievable SIR is derived in [4]. Let $\bar{\lambda}$ be the largest real eigenvalue of the following matrix

$$Z = \begin{cases} \dfrac{g_{ii}}{g_{ij}} & i \ne j \\ 0 & i = j \end{cases} \tag{7}$$

Then, the maximal achievable eigenvalue of matrix Z is given by

$$\bar{\gamma} = \frac{1}{\bar{\lambda} - 1} \tag{8}$$

and the power vector p^* becomes the corresponding eigenvector.

In this paper, we focus on the case in which the above power control problem has a unique solution of a positive power vector p^*.

2. TYPICAL DPC SCHEME

It was suggested that Eq.(5) be solved using the Jacobi fixed-point iterations [6]

$$p_i(k+1) = (I - A) p_i(k) + b \tag{9}$$

, which leads to the following algorithm.

$$p_i(k+1) = \frac{\gamma^d}{\gamma_i(k)} p_i(k) \tag{10}$$

where, SIR $\gamma_i(k)$ is given like in Eq.(1) at the base i. A nice feature of the algorithm in Eq.(10), which is called DPC, is that only information about the current mobile power and the current SIR is sufficient to update the mobile power. Assuming that the transmitted powers are constrained to

$$0 < p_i^L \le p_i(k) \le p_i^U \tag{11}$$

where, p_i^L and p_i^U describes the lower bound and upper bound of mobile terminal power, respectively. Then, algorithm Eq.(10) appropriately modified into the following DCPC(Distributed Constrained Power Control) algorithm [7].

$$p_i(k+1) = \min\left(\frac{\gamma^t}{\gamma_i(k)} p_i(k),\ p_i^U \right) \tag{12}$$

The DCPC provides a theoretical background of IS-95 and W-CDMA power control. It has also been much employed for simulators of cellular wireless system.

Convergence properties of this algorithm were studied in [7][8]]. Since this method is based upon a fixed-point algorithm, it usually has slow convergence to the sought solution. Note that the fixed-point algorithms, in general have the linear rate of convergence.

3. PROPOSED ADAPTIVE SCHEMES

3.1 Background of Scheme 1

The preliminary step for the proposed Scheme 1 is to make the transmission power proportional to the error between the actual SIR and the desired SIR. Define the transmission power change from time step k to $k+1$ as

$$\Delta p_i(k+1) = p_i(k+1) - p_i(k) \tag{13}$$

Let the error between the actual SIR $\gamma_i(k)$ and the desired SIR γ_i^t be noted by $e_i(k)$ for each mobile i at the k-th time instant,

$$e_i(k) = \gamma_i^t - \gamma_i(k) \tag{14}$$

We consider the following control algorithm

$$\Delta p_i(k+1) = \alpha_i(k) e_i(k)$$

or $p_i(k+1) = p_i(k) + \alpha_i(k)(\gamma_i^t - \gamma_i(k))$ (15)

where, $\alpha_i(k)$ is the control gain to be determined through the optimization procedure, in which the square norm of the error is minimized. The first stage of our scheme is to find the optimal control gain $\alpha_i(k)$ such that minimize the cost

$$J_i(k) = e_i(k+1)^2 \ge 0 \tag{16}$$

To solve this problem, let us denote the channel variation

$$\beta_i(k) = \frac{g_{ii}}{I_i(k)} = \frac{\gamma_i^t(k)}{p_i(k)} \tag{17}$$

, and the control gain as

$$\alpha_i(k) = \begin{cases} \dfrac{1}{\beta_i(k)}\left(1 - \dfrac{\gamma_i^t}{e_i(k)}\right) + \dfrac{1}{\beta_i(k+1)} \dfrac{\gamma_i^t}{e_i(k)} & if\ e_i(k) \ne 0 \\ 0 & if\ e_i(k) = 0 \end{cases} \tag{18}$$

This is the optimal, because the partial derivative of the cost $J_i(k)$ over the gain $\alpha_i(k)$ is zero.

$$\frac{\partial J_i(k)}{\partial \alpha_i(k)} = \frac{\partial (e_i(k+1)^2)}{\partial \alpha_i(k)} = 0 \tag{19}$$

Furthermore, the quadratic cost function of error is semi-definitely positive as it can be seen in Eq.(16). Because a mobile terminal has limitations in instantaneous power transmission, it is needed to find the optimal gain considering the constraints in Eq.(11), which minimizes the cost $J_i(k)$. The solution of this nonlinear programming problem is given by

$$p_i(k+1) = \begin{cases} p_i^L & \text{if } \beta_i(k+1) \succ \dfrac{\gamma_i^t}{p_i^L} \\ p_i^U & \text{if } \beta_i(k+1) \prec \dfrac{\gamma_i^t}{p_i^U} \\ p_i(k) + \alpha_i(k) e_i(k) & \text{otherwise} \end{cases} \quad (20)$$

The detailed derivation of the results in Eq.(20) can be obtained by using the well-known Kuhn-Tucker conditions [9], which can be found in [10].

3.2 Procedure of Scheme 1

It is to be reminded that an estimator or a predictor is needed in order to compute $\alpha_i(k)$. In this paper, we simply employed the typical DPC in Eq.(10) to make one-step prediction of the channel variation. The overall procedure of the proposed Scheme 1 can be described as follows.

[*Scheme 1*]

Step 1: Start with some initial guess of none zero mobile power $p_i(0)$ and with the achievable target SIR γ_i^t.

Step 2: With the initial values defined in step 1, calculate the current instant SIR $\gamma_i(k)$ with Eq.(1), the current channel variation $\beta_i(k)$ with Eq.(17) and the SIR error $e_i(k)$ with Eq.(14), respectively.

Step 3: Predict the next instant mobile power $p_i(k+1)$ by Eq.(10), the next instant SIR $\gamma_i(k+1)$ by Eq.(1), the next instant channel variation $\beta_i(k+1)$ with Eq.(17), respectively.

Step 4: Compute the control gain $\alpha_i(k)$ using Eq.(18).

Step 5: Modify the next instant power $p_i(k+1)$ using Eq.(15) and Eq.(20).

Step 6: If $\sum_{i=1}^{N} |e_i(k)| / N \le \varepsilon$ for $k \ge 1$, where ε is a desired tolerance, then stop since the result is obtained. Otherwise, increase k by 1 and go to Step 2.

3.3 Background of Scheme 2

In numerical analysis, a variety of acceleration techniques exist to speed up the convergence of fixed-point algorithms. The acceleration technique called Aitkens Δ^2 process applied to a linearly convergent fixed-point iterations leads to the procedure known as the Steffensen method [11]. This method produces the quadratic convergent rate for the corresponding fixed-point algorithm. In the following, under the assumption that the current SIRs are known ($\gamma_i(k)$ for each *k*-th instant), we derive the accelerated version of the Scheme 1 using the Steffensen method. In that direction, we present the Steffensen method and then apply it to our scheme 1 for DPC.

Assume that we have a linearly convergent fixed-point sequence defined by $x(k+1) = f(x(k))$, which is accelerated by using the Steffensen method as follows.

Step 1: Start with some initial guess $x(0)$.

Step 2: With the initial values defined in Step 1, perform two iterations on the original fixed-point sequence, that is, find $x(1)$ and $x(2)$, respectively.

Step 3: Evaluate a Steffensen iteration using the following formula

$$x^s(k) = x(0) - \frac{(x(1)-x(0))^2}{x(2)-2x(1)+x(0)}, \qquad k = 1,2,\cdots \quad (21)$$

Step 4: If $|x^s(k) - x^s(0)| \le \varepsilon$, where ε is a desired tolerance, stop and the result is obtained. Otherwise,

Step 5: Replace $x(0)$ with $x^s(k)$, increase *k* by 1, and go to step 2.

3.4 Procedure of Scheme 2

It is obvious that Steffensen method accelerates the original sequence by evaluating or inserting the Steffensen iteration after two iterations performed on the original fixed-point recursion. Such an obtained accelerated sequence is known to have the quadratic convergent rate [11]. Therefore, we applied Sffenson method to the Scheme 1 for speed up of the search. Consequently, it produces the following Scheme 2, which is named as an adaptive scheme.

[*Scheme 2*]

Modify the step 6 in Scheme 1 as

Step 6:* If k equals to 2, and mobile powers $p_i(0)$, $p_i(1)$ and $p_i(2)$ are obtained, then escape from the loop procedure.

That is, two initial iterations are performed using Scheme 1. After then, perform the following Steffensen iterations.

Step 1: Set $p_i^s(0) = p_i^s(0)$, $p_i^s(1) = p_i^s(1)$ and $p_i^s(2) = p_i^s(2)$

Step 2: Calculate

$$p_i^s(k) = p_i^s(0) - \frac{(p_i^s(1) - p_i^s(0))^2}{p_i^s(2) - 2p_i^s(1) + p_i^s(0)}, \qquad k = 1,2,\cdots \quad (22)$$

Step 3: Evaluate the current instant SIR $\gamma_i(k)$ and the error $e_i(k)$ employing $p_i^s(k)$ with the current instant power $p_i(k)$.

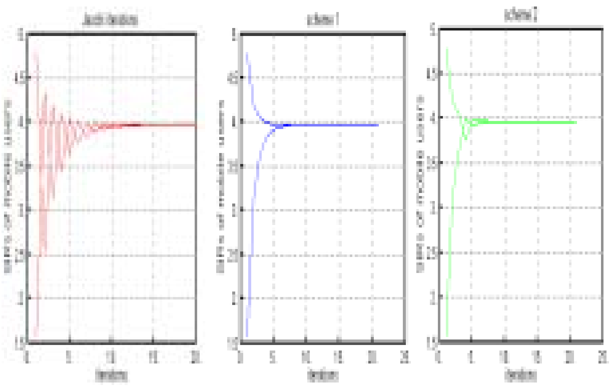
Step 4: If $\sum_{i=1}^{N} |e_i(k)| / N \le \varepsilon$ for $k \ge 1$, where ε is a desired tolerance, then stop since the result is obtained. Otherwise, increase *k* by 1 and go to step 2.

It should be emphasized that in Scheme 2, two iterations of Scheme 1 are used only to initiate Steffensen iteration procedure. Then, Steffensen iterations run until the completion.

4. NUMERICAL RESULTS

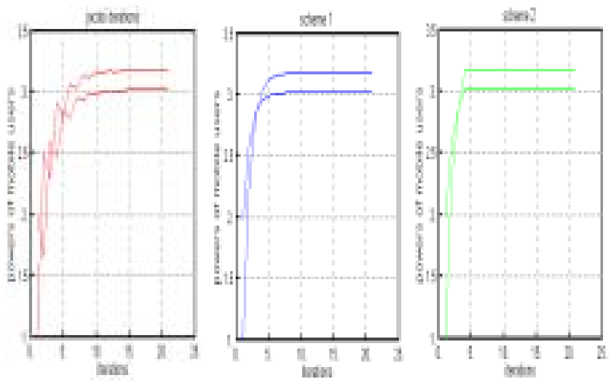
In this section, we solve a simple numerical example for $N = 2$ using data taken from Jantti and Kim [12]. The link gains are given by $g_{11} = 0.3268$, $g_{12} = 0.0534$, $g_{21} = 0.0602$, $g_{22} = 0.3836$ and the receiver noise is set to $\sigma^2 = 0.1$. The target SIR is assumed to be identical for both user data and equal to 6 *dB*(3.981 in absolute values). The initial mobile powers are given by $p_1(0) = 1$ and $p_2(0) = 2$.

It can be seen in Fig.1, Fig.2 and Fig. 3 that our proposed schemes - Scheme 1 and Scheme 2 - have rapid convergence over the typical DPC scheme using Jacobi iterations. Furthermore, it is observed in Fig.3 (b) and (c) that Scheme 2 is faster than Scheme 1.



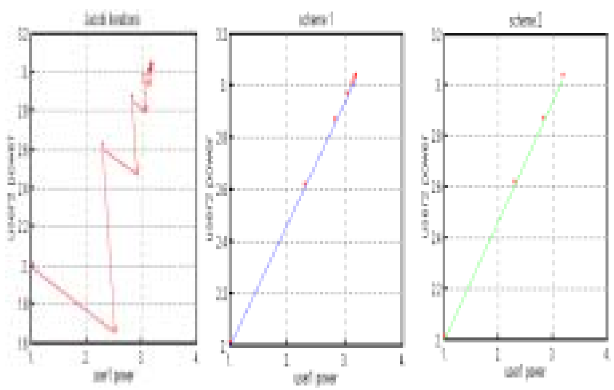
(a) Typical DPC (b) Scheme 1 (c) Scheme 2

Fig.1 Mobile user's power per iterations



(a) Typical DPC (b) Scheme 1 (c) Scheme 2

Fig.2 Mobile user's SIRs per iterations



(a) Typical DPC (b) Scheme 1 (c) Scheme 2

Fig.3 Comparison of search patterns

It is notable that the proposed schemes show almost straight line of search pattern unlike zigzag search pattern in the typical DPC scheme using Jacobi method, which can be observed in Fig.3.

Thus, the proposed schemes have an advantage over typical Jacobi based DPC scheme in convergence rate and smoothness, which confirm validity and effectiveness of this work.

5. CONCLUSION

This paper has proposed adaptive schemes that aim at shortening iteration time of solution search for distributed power control in CDMA wireless network. The features of the proposed adaptive schemes are as follows.

- Rapid convergence: The theoretical background of Scheme 1 is described in section 3.1, which is based on optimal control theory.
- Scheme 1 needs one-step predictor on channel variation, which is provided simply by the typical DPC in this paper.
- Scheme 2 employs the advantage of Steffensen iteration to accelerate Scheme 1. Furthermore, scheme 2 lessens the computational burden of Scheme 1. It begins only after two initial iterations of Scheme 1 and the next running Steffensen iterations require less computational overhead than scheme 1.

Consequently, our proposed schemes have a high potential advantage for decreasing the mobile terminal power consumption and for increasing the CDMA cellular radio network capacity.

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