I-P Controller Design for Quadruple-Tank System

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Abstract: A control system design with Coefficient Diagram Method (CDM) is proven that effective for SISO control design. But the control system design for MIMO via CDM is not concrete procedure. In this paper presents the control system design method for quadruple-tank process via CDM. By using the decentralized method for both non-minimum phase and minimum phase are made. The results from Integral-Proportional (I-P) controller's design via CDM and standard Proportional-Integral (PI) controls are also shown to compare the merits of the proposed controllers.

Keywords: Quadruple-Tank, CDM

1. INTRODUCTION

One of the main problems with mathematical models of physical systems is that the parameters used in the models cannot be determined with absolute accuracy. Inaccurate parameters can arise from many different factors. The values of parameters may change with time or various effects. These differences existing between the actual system and system model is called uncertainty. However, the actual system parameters may change during operation or the input signal takes too large. In these cases, the linear model is no longer representing the actual system and causes practical problems. Therefore, a robust controller is needed to stabilize these types of systems for the entire range of expected variations in the plant parameters.

This paper presents a quadruple-tank control system using I-P controller design by CDM [4]. By using the Structure of the I-P Controller in the controlled system, it is not necessary to use the prefilter. The parameter of I-P controller is designed based on the stability and the speed of controlled system, which are defined in the term of the standard stability index and the equivalent time constant. When the settling time of the controlled system has been selected, the equivalent time constant is obtained. The stability index and the equivalent time constant specify the coefficients of the characteristic polynomial. These coefficients are related to the controller parameters algebraically in explicit form. Hence, the transient and the steady state performances can be obtained as desired.

The step responses of the controlled system using I-P Controller compared to the step responses of the controlled system using PI Controller tuned by relay feedback method [1] are shown by various MATLAB's numerical examples. The step responses of the controlled system using I-P controller has little overshoot and reaches the desired settling time without adjustment as mentioned [4], it is better than the step response of the controlled system using PI controller designed by relay feedback method. Furthermore, the I-P controller can reduce the effect of interaction between two-loops of outputs better than PI controller. The numerical results of step responses for both minimum and non-minimum phase system are also show a good robustness when the plant parameters are varied.

2. QUADRUPLE-TANK PROCESS

Consider the quadruple-tank process shown in Fig. 1. This laboratory process has been used to illustrate many issues in multivariable control [1]. The target is to control the level in the lower two tanks with two pumps. The process inputs are u_1 and u_2 (input voltages to the pumps) and the outputs are y_1 and y_2 (voltage from level measurement devices). The linear

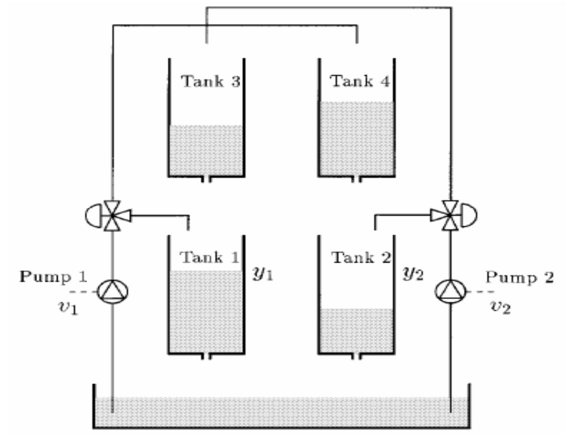


Fig. 1 Schematic diagram of the quadruple-tank process

-ised dynamics for the process is given as

$$G(s)=\begin{bmatrix}\dfrac{\alpha_1 c_1}{1+sT_1} & \dfrac{(1-\alpha_2)c_1}{(1+sT_3)(1+sT_1)}\\ \dfrac{(1-\alpha_1)c_2}{(1+sT_4)(1+sT_2)} & \dfrac{\alpha_2 c_2}{1+sT_2}\end{bmatrix}, \qquad (1)$$

where

$$T_i=\frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}, \quad i=1,\cdots,4 \qquad (2)$$

and $c_1=T_1k_1k_c/A_1$, $c_2=T_2k_2k_c/A_2$. Here A_i is the cross-sectional area of tank i , a_i is the cross-sectional area of the outlet hole, h_i^0 is the steady-state water level, k_i is the gain of the pump i , k_c is the measurement gain, and g is the acceleration of gravity. The parameters $\alpha_1,\alpha_2\in(0,1)$ are determined from how the valves are prior set to an experiment; the flow to tank 1 is proportional to α_1 and the flow to tank 4 is proportional to $(1-\alpha_1)$, and similarly for α_2 with respect to tank 2 and tank 3. Since

$$\det G(s)=\frac{c_1c_2}{\alpha_1\alpha_2\prod_{i=1}^{4}(1+sT_i)}\times\left[(1+sT_3)(1+T_4)-\frac{(1-\alpha_1)(1-\alpha_2)}{\alpha_1\alpha_2}\right] \qquad (3)$$

and transfer matrix G has two finite zeros for $\alpha_1,\alpha_2\in(0,1)$.

The system is non-minimum phase for

$$0 < \alpha_1 + \alpha_2 < 1,$$

and minimum phase for

$$1 < \alpha_1 + \alpha_2 < 2 .$$

Hence, by changing a single valve we can make the multivariable level control problem more or less difficult.

The relative gain (RGA) was introduced by Bristol [2] as a measure of interaction in multivariable control systems. The RGA Λ is defined as $\Lambda = G(0) * G^{-T}(0)$, where the asterisk denotes the schur product (element-by-element matrix multiplication) and $-T$ inverse transpose. It is possible to show that the elements of each row and column of Λ sum up to one, so for a 2×2 system the RGA is determined by the scalar $\lambda = \Lambda_{11}$. The RGA is used as a tool mainly in the process industry to decide on control structure issues such as input-output pairing for decentralized controllers [3]. The RGA of the Quadruple-Tank Process is given by the simple expression

$$\lambda = \frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2 - 1},$$

where

$$\Lambda = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix}.$$

The physical modeling and RGA of minimum phase system and non-minimum phase system give the two transfer matrices as follows [1]:

2.1 Case of minimum phase system

$$G_{-}(s) = \begin{bmatrix} \dfrac{2.6}{1+62s} & \dfrac{1.5}{(1+23s)(1+62s)} \\ \dfrac{1.4}{(1+30s)(1+90s)} & \dfrac{2.8}{1+90s} \end{bmatrix} \quad (4)$$

$$\Lambda_{-} = \begin{bmatrix} 1.4 & 0.4 \\ 0.4 & 1.4 \end{bmatrix}$$

2.2 Case of non-minimum phase system

$$G_{+}(s) = \begin{bmatrix} \dfrac{1.5}{1+63s} & \dfrac{2.5}{(1+39s)(1+63s)} \\ \dfrac{2.5}{(1+56s)(1+91s)} & \dfrac{1.6}{1+91s} \end{bmatrix} \quad (5)$$

$$\Lambda_{-} = \begin{bmatrix} -0.635 & 1.635 \\ 1.635 & -0.635 \end{bmatrix}$$

3. STRUCTURE OF THE CONTROL SYSTEM WITH I-P CONTROLLER

From RGA analysis suggests that input-output pairing for decentralized control be chosen. In case of minimum phase system, transfer function G_{11} and G_{22} are used for design the controller, but the case of non-minimum phase system the transfer function G_{12} and G_{21} are instead used.

The structure of MIMO control system using I-P controller for minimum phase and non-minimum phase are shown in Fig.

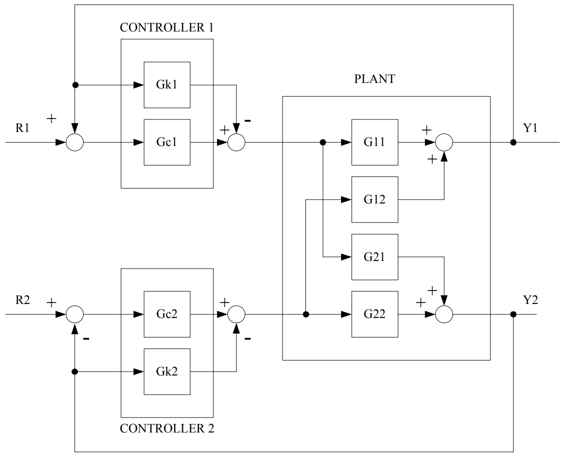


Fig. 2 Structure of the MIMO minimum phase control system

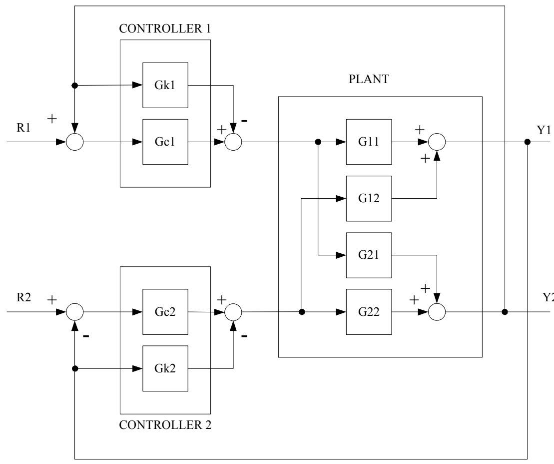


Fig. 3 Structure of the MIMO non-minimum phase control system.

2 and Fig. 3 respectively, where

$$G_{c1}(s) = \frac{K_{i1}}{s}, G_{k1}(s) = K_{p1},$$

$$G_{c2}(s) = \frac{K_{i2}}{s}, G_{k2}(s) = K_{p2},$$

and transfer matrix of system is

$$G(s) = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{K_{11}}{a_1 s + a_0} & \dfrac{K_{12}}{b_2 s^2 + b_1 s + b_0} \\ \dfrac{K_{21}}{c_2 s^2 + c_1 s + c_0} & \dfrac{K_{22}}{d_1 s + d_0} \end{bmatrix}.$$

The transfer functions used for design are:

3.1 Case of minimum phase system

Loop 1 (Y1-R1),

$$\frac{Y_1(s)}{R_1(s)} = \frac{G_{c1}(s)G_{11}(s)}{1 + G_{c1}(s)G_{11}(s) + G_{k1}(s)G_{11}(s)}.$$

Characteristic equation is

$$\begin{aligned} P_{-11} &= 1 + G_{c1}(s)G_{11}(s) + G_{k1}(s)G_{11}(s) \\ &= a_1 s^2 + (a_0 + K_{p1}K_{11})s + K_{i1}K_{11}. \end{aligned} \tag{6}$$

Loop 2 (Y2-R2),

$$\frac{Y_2(s)}{R_2(s)} = \frac{G_{c2}(s)G_{22}(s)}{1 + G_{c2}(s)G_{22}(s) + G_{k2}(s)G_{22}(s)}.$$

Characteristic equation is

$$\begin{aligned} P_{-22} &= 1 + G_{c2}(s)G_{22}(s) + G_{k2}(s)G_{22}(s) \\ &= d_1 s^2 + (d_0 + K_{p2}K_{22})s + K_{i2}K_{22}. \end{aligned} \tag{7}$$

3.2 Case of non-minimum phase system

Loop 1 (Y1-R2),

$$\frac{Y_1(s)}{R_2(s)} = \frac{G_{c2}(s)G_{12}(s)}{1 + G_{c2}(s)G_{12}(s) + G_{k2}(s)G_{12}(s)}.$$

Characteristic equation is

$$\begin{aligned} P_{+12} &= 1 + G_{c2}(s)G_{12}(s) + G_{k2}(s)G_{12}(s) \\ &= b_2 s^3 + (b_1 + K_{12})s^2 + (b_0 + K_{p2}K_{12})s + K_{i2}K_{12}. \end{aligned} \tag{8}$$

Loop 2 (Y2-R1),

$$\frac{Y_2(s)}{R_1(s)} = \frac{G_{c1}(s)G_{21}(s)}{1 + G_{c1}(s)G_{21}(s) + G_{k1}(s)G_{21}(s)}.$$

Characteristic equation is

$$\begin{aligned} P_{+21} &= 1 + G_{c1}(s)G_{21}(s) + G_{k1}(s)G_{21}(s) \\ &= c_2 s^3 + (c_1 + K_{21})s^2 + (c_0 + K_{p1}K_{21})s + K_{i1}K_{21}. \end{aligned} \tag{9}$$

4. COEFFICIENT DIAGRAM METHOD

The CDM is used for design the controller so that the step response of the controlled system satisfies both transient and steady state response specifications, and also satisfies the requirements of stability, faster response and robustness. Generally, the order of the controller designed by CDM is lower than the order of the plant [4]. However, when using the I-P controller for the first order plant, the order of the controller is equal to the order of the plant, and for the second order plant, the order of the controller is lower than the order of the plant equal one, but the integrator of the $G_{c1}(s)$ and $G_{c2}(s)$ virtually makes the plant to be second order and third order respectively. Thus, the CDM condition is satisfied.

The polynomials form of the controller and the plant are generally be written respectively in the form [4]

$$\begin{aligned} A_c(s) &= l_\lambda s^\lambda + l_{\lambda-1}s^{\lambda-1} + \ldots + l_0 \\ B_c(s) &= k_\lambda s^\lambda + k_{\lambda-1}s^{\lambda-1} + \ldots + k_0 \end{aligned} \tag{10}$$

and

$$\begin{aligned} A_p(s) &= p_k s^k + p_{k-1}s^{k-1} + \ldots + p_0 \\ B_p(s) &= q_m s^m + q_{m-1}s^{m-1} + \ldots + q_0 , \end{aligned} \tag{11}$$

where $\lambda < k$ and $m < k$.

The polynomials characteristic equation from Fig. 2 and Fig. 3 can be written in the form

$$\begin{aligned} P(s) &= a_n s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \\ &= \sum_{i=0}^{n} a_i s^i , \end{aligned} \tag{12}$$

where $a_0, a_1, \cdots, a_n$ are the real coefficients.

The stability index γ_i, the equivalent time constant τ and the stability limit γ_i^* are defined as follows:

$$\gamma_i = \frac{a_i^2}{a_{i+1}a_{i-1}} , \tag{13}$$

$$\tau = \frac{a_1}{a_0} , \tag{14}$$

$$\gamma_i^* = \frac{1}{\gamma_{i+1}} + \frac{1}{\gamma_{i-1}} \quad ; \quad \gamma_0 , \gamma_n = \infty . \tag{15}$$

From equation (13) - (15), the coefficients a_i and the characteristic equation $P(s)$ is

$$a_i = a_0 \tau^i \frac{1}{\gamma_{i-1} \cdots \gamma_2^{i-2} \gamma_1^{i-1}} = a_0 \tau^i \prod_{j=1}^{i-1} \frac{1}{(\gamma_{i-j})^j} , \tag{16}$$

$$P_{\mp}(s) = a_0 \left\{ \left[\sum_{i=2}^{n} \left(\prod_{j=1}^{i-1} \frac{1}{\gamma_{i-j}^j} \right) (\tau s)^i \right] + \tau s + 1 \right\} . \tag{17}$$

4.1 Case of minimum phase system

From equation (12) and (17), the characteristic equation can be expressed as

$$P_{-}(s) = a_2 s^2 + a_1 s + a_0 . \tag{18}$$

4.2 Case of non-minimum phase system

In the same way, the characteristic equation for this case is

$$P_{+}(s) = a_3 s^3 + a_2 s^2 + a_1 s + a_0 . \tag{19}$$

The coefficients in equation (12) came from the chosen stability index γ_i and equivalent time constant τ. By equating the $P(s)$ of equation (12) with the $P_{-}(s)$ of equation (18) or the $P_{+}(s)$ of equation (19), such that the I-P controller for each case are obtained.

5. NUMERICAL EXAMPLES

In this section, the design procedures of I-P Controller via CDM for both cases are verified through the MATLAB. The step responses of the control systems that employ the proposed I-P controller and the PI controller tuned by relay feedback method [1] are also be compared.

5.1 Case of minimum phase system

Loop 1 (Y1-R1)

$$G_{-11}(s) = \frac{2.6}{62s+1}$$

Here, the desired settling time $t_s = 20$ secs, then the equivalent time constant $\tau = 8$. The stability index $\gamma_1 = 3$ is chosen, hence the $P_-(s)$ can be expressed as

$$P_-(s) = 62s^2 + 23.427392s + 2.927938 .$$

Equated the above $P_-(s)$ to the $P_{-11}(s)$ of the plant with the I-P controller in the equation (6), then the I-P controller is

$$K_{p1} = 8.62592, \quad K_{i1} = 1.12613 .$$

Loop 2 (Y2-R2)

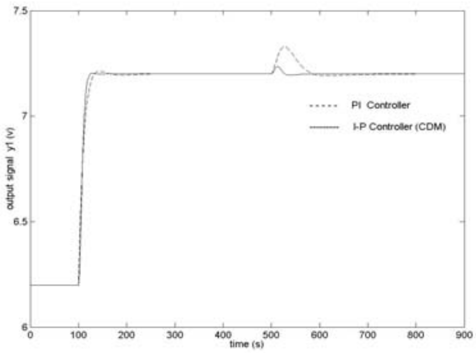
$$G_{-22}(s) = \frac{2.8}{90s+1}$$

In the same manner, the $P_-(s)$ is

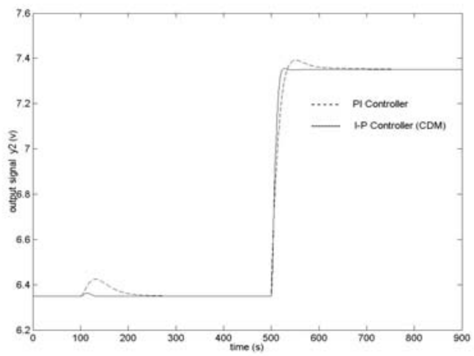
$$P_-(s) = 90s^2 + 33.445812s + 4.179084 .$$

Equated the above $P_-(s)$ to the $P_{-22}(s)$ of the plant with the I-P controller in equation (7), the I-P controller is

$$K_{p2} = 11.58779, \quad K_{i2} = 1.49253 .$$



(a)



(b)

Fig. 4. Step responses of minimum phase system

The step response in Fig. 4(a) is for the tank 1 and Fig. 4(b) is for tank 2. In both figures, it is evidence that the settling time from the proposed I-P controller is satisfied for both tanks, while the responses obtained from PI controller tuned by relay feedback method have more than effect from the interaction among the tanks.

5.2 Case of non-minimum phase system

Since the I-P controller cannot be achieved by specifying the settling time t_s, the equivalent time constant τ is obtained by specifying the values of the stability index γ_2, γ_1. Then the equivalent time constant τ and parameter K_i and K_p of I-P controller are obtained by equating the characteristic polynomial (12) to the characteristic polynomial (19).

Loop 1 (Y1-R2)

$$G_{+12}(s) = \frac{2.5}{(63s+1)(39s+1)} = \frac{2.5}{2457s^2 + 102s + 1}$$

Selected the stability index $\gamma_1 = 3$ and $\gamma_2 = 2$, then $\tau =$ 146.5 secs, and settling time t_s is approximate 366.25 secs. The $P_+(s)$ is then

$$P_+(s) = 2457s^3 + 104.5s^2 + 2.1312925s + 0.014684 .$$

Equated the above $P_+(s)$ to the $P_{+12}(s)$ of the plant with the I-P controller in equation (8), the I-P controller is

$$K_{p2} = 0.452517, \quad K_{i2} = 0.0058736 .$$

Loop 2 (Y2-R1)

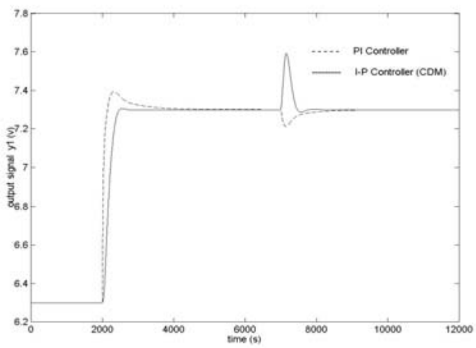
$$G_{+21}(s) = \frac{2.5}{(91s+1)(56s+1)} = \frac{2.5}{5096s^2 + 147s + 1}$$

In the same manner, from the selected $\gamma_1 = 3$ and $\gamma_2 = 2$, then $\tau = 206.145$ secs and settling time t_s is approximate 515.3625 secs. The $P_+(s)$ is then

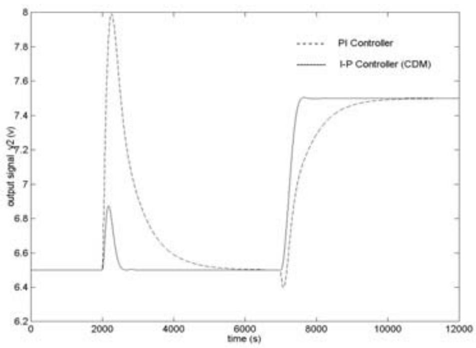
$$P_+(s) = 5096s^3 + 149.5s^2 + 2.1065075s + 0.01011 .$$

Equated the above $P_+(s)$ to the $P_{+21}(s)$ of the plant with the I-P controller in equation (9), the I-P controller is

$$K_{p1} = 0.442603, \quad K_{i2} = 0.004044 .$$



(a)



(b)

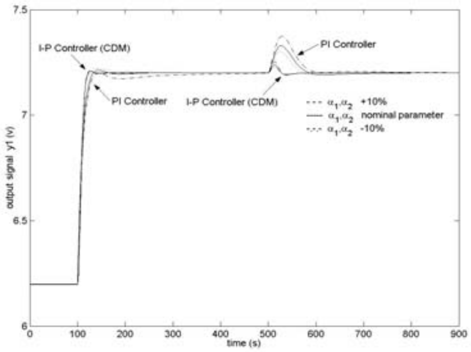
Fig. 5. Step response of non-minimum phase system

The step response in Fig. 5(a) is for the tank 1 and Fig. 5(b) is for tank 2. Although the responses from the proposed I-P controller of tank 1 compared with the PI Controller has the effect from the interaction more than PI controller, but the response of tank 2 is clearly shown that it differ from the response of the PI controller so much. Then we can conclude that even the plant is a non-minimum phase system, the proposed controller also be suitably applied. Moreover, the settling time are satisfied for both tanks with small in overshoots.

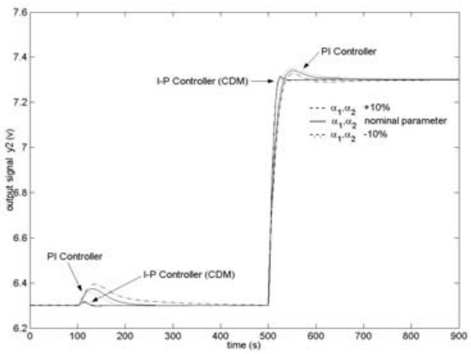
5.3 Case of the system with parameters α_1 and α_2 changed

In order to investigate the plant with variation of parameters α_1, α_2 of the quadruple-tank are changed $\pm 10\%$, we also classify the results in two cases as follows:

Case of minimum phase system



(a)



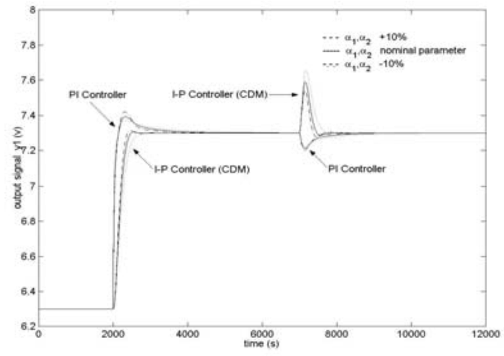
(b)

Fig. 6 Step responses of minimum phase system when α_1, α_2 varied $\pm 10\%$

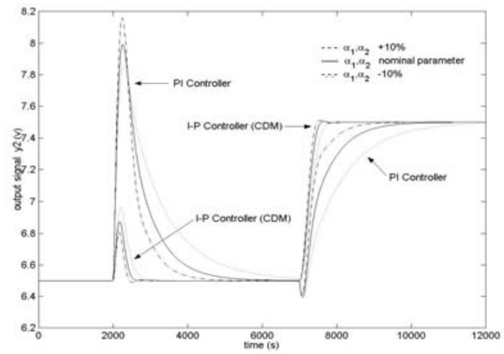
The step responses of the control system when α_1, α_2 are changed $\pm 10\%$ shown in Fig. 6(a) for tank 1 and Fig. 6(b) for tank 2 respectively. It is obviously seen that the responses from I-P controller has a little changed, while the responses from PI controller are more observable change.

Case of non-minimum phase system

This case is look like the case of nominal plant without the changing of α_1, α_2 as in section 5.2. That is the response of tank 2 from the I-P controller is still better than that of the PI controller as well.



(a)



(b)

Fig. 7 Step responses of non-minimum phase system when α_1, α_2 varied $\pm 10\%$

6. CONCLUSION

The design procedures of the I-P controller by CDM for quadruple-tank process have been proposed in this paper. The step responses between the proposed I-P controller and PI controller are compared for both minimum and non-minimum phase systems to verify that the transient and steady state response specifications are obtained with robustness in all cases.

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