MILP model for short-term scheduling of multi-purpose batch plants with batch distillation process

Jin-Juk Ha*, and Euy Soo Lee**

* Department of Chemical & Biochemical Engineering, Dongguk University, Seoul, Korea
(Tel : +82-2-2260-3706; E-mail: hasung@dgu.ac.kr)
** Department of Chemical & Biochemical Engineering, Dongguk University, Seoul, Korea
(Tel : +82-2-2260-3706; E-mail: eslee@dgu.edu)

Abstract: Fine chemical production must assure high-standard product quality as well as characterized as multi-product production in small volumes. Installing high-precision batch distillation is one of the common elements in the successful manufacturing of fine chemicals, and the importance of the process operation strategy with quality assurance cannot be overemphasized. In this study, we investigate the optimal operation strategy and production planning of a sequential multi-purpose plants consisting of batch processes and batch distillation with unlimited intermediate storage. We formulated this problem as an MILP model. A mixed-integer linear programming model is developed based on the time slot, which is used to determine the production sequence and the production path of each batch. Illustrative examples show the effectiveness of the approach.

Keywords: short-term scheduling, batch distillation, optimization, multipurpose

1. INTRODUCTION

Manufacturing technology for the production of high value-added fine chemical products is emphasized and getting more attention as the diversified interests of customers and the demand of high quality products are getting bigger and bigger everyday. Thus, the development of advanced batch processes, which is the preferred and most appropriate way of producing these types of products, and the related technologies are becoming more important. Fine chemical production must assure high-standard product quality, and multi-product production in small volumes is one of its characteristics. Especially, establishing raw material requirements satisfying high quality standards and advanced quality control throughout the whole production process is exceptionally required. Therefore, high-precision batch distillation is one of the important elements in the successful manufacturing of fine chemicals, and the importance of the process operation strategy with quality assurance cannot be overemphasized. In the last decade, many theories were proposed for the design and operation of batch processes and many papers for the production planning and scheduling of batch processes have been published [1, 2, 3]. But, optimal operation strategy and production planning of the processes consisting of batch processes and batch distillation has not been reported. Therefore, proposing a process structure explanation and operation strategy of such processes including batch processes and batch distillation would be of great value.

Batch processes consist of a system of processing equipment where batches of the various products are produced by executing a set of processing tasks or operations. And, scheduling is one of the core areas of batch process operation. In general, scheduling deals with the allocation of available resources over time to perform a collection of tasks. Many scheduling and planning problems can be posed as mixed integer linear programming (MILP) problems since the corresponding mathematical optimization models involve both discrete and continuous variables that satisfy a set of linear equality and inequality constraints. And, based on the nature of the recipes and allowable equipment assignment, batch operations can be roughly classified into multi-product and multipurpose. The classical multiproduct plant is employed for a set of products whose recipe structure is the same, the production line employs fixed many-to-one equipment assignments, the line is operated cyclically, and multiple products are accommodated through serial campaigns. The multipurpose plant is appropriate for products with dissimilar recipe structures, allows many-to-many equipment assignments, and employs multiple campaign involving one or more production lines, each operated cyclically.

In this study, we investigate optimal operation strategy and production planning of multipurpose plants consisting of batch processes and batch distillation for the manufacturing of fine chemical products. Especially, the multipurpose batch plants are divided in sequential and non-sequential plants [4, 5]. In a sequential multipurpose plant, the production paths of all of the products follow the same order in the sequence of units in the recipe but do not necessarily include all of the same units. Non-sequential plant, the production paths of all of the products don't follow the same order in the sequence of units in the recipe. Among these, sequential multi-purpose plants are mostly common in industry.

Therefore, we propose a mathematical model for the short-term scheduling of a sequential multi-purpose batch plant consisting of batch distillation in a mixed product campaign. Also, we consider that the waste product of being produced on batch distillation is recycled to the batch distillation unit for the saving of raw materials.

2. MODEL ANALYSIS OF BATCH PROCESS WITH BATCH DISTILLATION UNIT

In the scheduling of batch process, the fundamental issue in assignment and sequencing decision concerns the time domain representation. A common feature among scheduling models is the definition of the time slots, i.e., the time intervals for unit allocation. In a continuous time domain representation, time slots have variable length. Moreover, they can be either associated with units or defined globally, in which case they are often denoted as time events. Time slot is determined based on how to define the production sequence and to select the production path for each batch of product. Every batch of the production is assigned to a time slot. Namely, there are an equal number of time slots and required production batches.

We propose a mathematical model for the short-term scheduling of a sequential multipurpose batch plant consisting of batch distillation in a mixed product campaign under unlimited intermediate storage (UIS) policy.

We introduce the necessary assumptions for the batch distillation processes as shown in Fig. 1.

1) Binary processes
2) After one batch processing of raw material (P1), recycle product (P4) is recycled.
3) Every product that was produced in the batch distillation is considered as an independent batch in each unit.

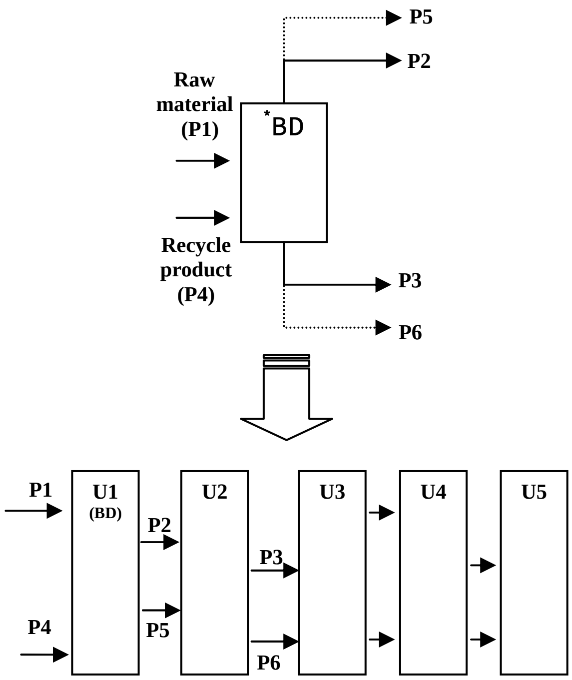
First of all, as every production batch occurs exactly once in processing sequence, the following constraints must be satisfied for the binary variable X_{ik} :

$$\sum_{k \in K} X_{ik} = n_i \qquad \forall\, i \in I \tag{1}$$

$$\sum_{i \in I} X_{ki} = 1 \qquad \forall\, k \in K \tag{2}$$

Where n_i is the required number of batches for product *i*, hence the number of required time slots must be the same as the required number of batches N, i.e., $K = N = \sum_i n_i$. And X_{ik} is represented by the following definition:

$X_{ik} =$ 1, if product *i* is manufactured in time slot *k*
0, otherwise



* BD : batch distillation

Fig. 1 A schematic diagram of conceptually converting a batch distillation process into an equivalent batch process

For batch distillation plant is applied with time slot, we introduce preordering constrains. Suppose when product P1 is processed on batch distillation, products P2, P3 and P4 (recycle product) are produced from batch distillation. Products P2 and P3 are transferred to the next unit. But product P4 is recycled to the batch distillation and another product P5 and P6 are produced from the batch distillation. Fig. 1 is a schematic diagram of conceptually converting a batch distillation processes into the equivalent batch processes. In order to start a processing on each unit, every material must have always been prepared in advance.

When product P1 finishes processing on the batch distillation and products P2 and P3 are produced, although units 2 and 3 are ready, products P2 and P3 do not start processing at the same time with product P1. Therefore, in order to assign product P1, P2 and P3 in the time slots, product P1 must have a preordering procedure than product P2, P3 and P4 in the optimal scheduling sequence. As the same way, since product P5 and P6 are the product of batch distillation of product P4, product P4 must have a preordering procedure than production P5 and P6. When such conditions are satisfied, product P2, P3, P4, P5 and P6 can start processing on each first unit. Product P2 and P3 must have a preordering procedure than production P5 and P6. Accordingly, the constraint term of the optimal scheduling sequence must satisfy the following formulation.

$$X_{i,k'} + \sum_{i' \in S_i} X_{i',k} \le 1$$
$$\forall\, i \in I_s, k, k' \in K - \{\bar{k}\}, k' < k \tag{3}$$

Where I_S, S_i : product set; *K*: time slot, $\bar{k}$: the last time slot

The product pairs in the example of fig. 1 that need the preordering procedure by using Eq. (3) are (P1,P2), (P1,P3), (P1,P4), (P2,P5), (P2,P6), (P3,P5), (P3,P6), (P4,P5) and (P4,P6). Define that I_S means the product set that is assigned to the first position of the product pairs on time slot k. I_S = {P1, P2, P3, P4}. Define that S_i means the product set of product *i* that is assigned to the second position of the product pairs on time slot k where product $i \in I_{S.}$ Namely, S_{P1}={P2,P3,P4}, S_{P2}={P5,P6}, S_{P3}={P5,P6}, S_{P4}=(P5,P6).

And, we define nonnegative continuous variables ST_{kj} and ET_{kj} as the start and end times of unit *j* in time slot *k*. Then, the starting, ending and processing times have the following relationship.

$$\mathrm{ET}_{kj} - \mathrm{ST}_{kj} = \sum_{i \in I} \mathrm{P}_{ij} X_{ik} \quad \forall k \in K,\, j \in J \tag{4}$$

Where P_{ij} is processing time. The timing between units, *j* and *j'*, included in a given path is expressed in equation (5).

$$\mathrm{ST}_{kj'} - \mathrm{ET}_{k'j} \ge -U(1 - \sum_{i \in I} X_{ik}) \quad \forall k \in K,\, j \in J - \{\bar{j}\} \tag{5}$$

Equ. (6) establishes the relationship between time slots, *k* and *k+1*.

$$\mathrm{ST}_{k+1,j} - \mathrm{ET}_{kj} \ge 0 \quad \forall k \in K - \{\bar{k}\},\, j \in J \tag{6}$$

Where *U* is sufficient large positive number.

For the scheduling of a sequential multi-purpose batch plant consisting of batch distillation under a mixed product campaign (MPC) and UIS policy, the objective function is set to minimize the makespan.

$$\text{Min} : \text{Makespan} \geq \text{ET}_{\bar{k}j} \quad \forall j \in J \tag{7}$$

Therefore, the optimization problem is composed of minimizing the makespan for the sequential multi-purpose batch plant consisting of batch distillation under MPC and UIS policy, as represented equ. (7), subject to constraints equs. (1)-(6). We have tested the proposed model against several examples to show its effectiveness.

3. EXAMPLE

The scheduling problem in this paper is solved on PC, Intel Pentium III 800 MHz, using the solver Hyper LINGO 6.0. The structure of example process is shown at Fig. 1. Processing time, production path and batches of each product is given in the following Table 1.

Table 1 Data for example

Product / unit	P1	P2	P3	P4	P5	P6	P7	P8
Unit 1	7			5				
Unit 2		4			4		4	5
Unit 3		5	6		5	6	3	
Unit 4		5	4		5	4	3	4
Unit 5			6			6		4
batch	1	1	1	1	1	1	2	2

The scheduling problem size of the example consisted of 80 binary variables, 181 continuous variables, and 252 constraints. The computation time was 16.0 seconds. The optimal scheduling sequence is P1-P8-P7-P4-P3-P8-P2-P7-P6-P5, as seen in Figs. 2~3, with the minimum makespan, 45h.

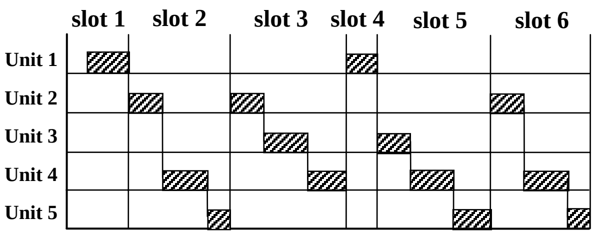


Fig. 2 Optimal schedule of example.

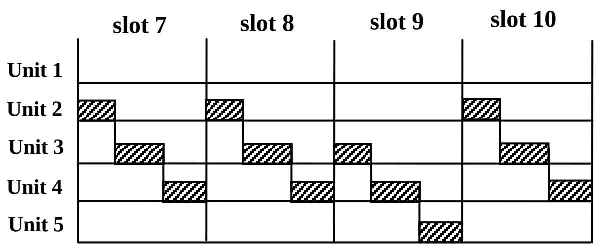


Fig. 3 Optimal schedule of example (continued).

4. CONCLUSION

We investigated a production scheduling of multi-purpose plants consisting of batch processes and batch distillation unit for the manufacturing of fine chemical products. For the short-term scheduling of a sequential multi-purpose batch plant consisting of batch distillation under MPC and UIS policy, we proposed a MILP model based on a priori time slot allocation.

The developed methodology will be especially useful for the design and optimal operations of multi-purpose and multiproduct plants that is suitable for fine chemical production.

5. SYMBOLS

Indices

i, i' products

j units

$\bar{j}$ last processing unit

k, k' time slots or time event points

$\bar{k}$ last time slot

Sets

I set of order

I_s set of tasks that receive material from state *s*

J set of units

K set of time slots

S_i set of allowed states for product *i*

Parameters

U sufficient large positive number

Variables

$X_{i,k}$ binary variable that assigns product *i* to slot *k*

P_{ij} processing time in unit *j* for product *i*

ET_{kj} completion time in unit *j* for time slot *k*

ST_{kj} starting time in unit *j* for time slot *k*

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