A Novel Range Estimator for Surface to Air Missile with Closing Velocity Measurements

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Abstract: A practical range estimator based on the robust Kalman filter is proposed to solve the range estimation problem for surface to air missile(SAM) homing guidance. Apart from the previous works based on the extended Kalman filter(EKF) with bearing only measurement, the proposed scheme makes use of line-of-sight(LOS) rate to ensure the fast convergency at long-range. In this reason, the robust Kalman filter is considered to deal with LOS rate measurement error. The recursive linear structure of proposed filter is easy to implement and make it possible to reduce computational burdens. Moreover, it shows good estimation performance without specific guidance law such as oscillation proportional navigation guidance(OPNG).

Keywords: range estimation, bearing only measurement, passive ranging, robust Kalman filter

1. Introduction

The objective of range estimation is to extract the range between the missile and the moving target from available information such as line-of-sight(LOS) angle. This kind of problem is often called passive ranging. During the last decades, there are many attempts to solve the passive ranging problem based on the extended Kalman filter(EKF) or modified gain EKF(MGEKF) [1][2][3]. However, nowadays, it is very well known that there exist bias errors in range estimates caused by a correlation between the EKF gain and its innovation sequence [4]. As well, it should be pointed out that the bearing only passive ranging filter cannot avoid lack of observability with conventional proportional navigation guidance(PNG) law. Making an alternative solution, most studies have been concentrated on the development of unbiased EKF and design of irregular homing guidance law to generate a homing trajectory which guarantees high observability condition [5]. But, the passive ranging algorithm based on bearing only measurements cannot be free from the slow convergency problem in long range situation.

Figure 1 gives an insight to us about the essence of passive ranging. As shown in the figure, more accurate range information, that is, less range uncertainty, Δr, is obtained as the missile takes farther angular position, $\Delta\lambda$. It means that, in order to get a good range information at long range, missile has to move longer arc range. This property makes a bearing only passive ranging filter has bad convergency characteristics such as long setting time or even divergence of the filter. In general, the air defense missile has quite short engagement time within several minutes and follows optimal trajectory to maximize final velocity. Therefore, there may be no opportunity gathering the good range information by using the above manner. Also, using the specific oscillation proportional guidance causes serious aerodynamic drag force and results loss of energy at homing phase. In this reasons, the long setting time is a critical disadvantage for air defense missile. And it is necessary to design the range estimator having fast conversance characteristics.

To solve the inherent problem of conventional passive ranging filter, we propose a new range estimator which exploits

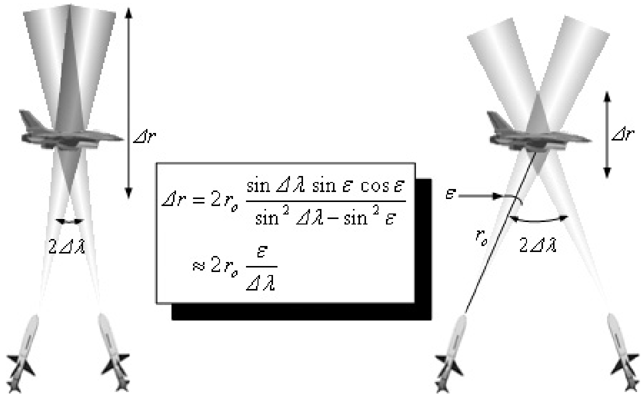

Fig. 1. Observability in Passive Ranging Problem

the fact that the closing velocity mismatch can be expressed by the multiplication of the range and LOS rate. Since the LOS rate can be easily magnified by increasing the missile velocity perpendicular to LOS vector, it guarantees fast convergency at long-range. At this point, it should be noted that the LOS rate measurements are contaminated by various error sources. To tackle the LOS rate measurement errors, the linear robust Kalman filter is applied. The conventional EKF not only adopt higher order kinematic model to describe full relative geometry but also requires approximations to handle the nonlinearity between polar coordinates and cartesian coordinates [6]. On the other hand, the proposed filter can be constructed with only two states and take benefit of the linearity of measurement equation. The proposed filter requires minimal computational efforts for its simple structure. In addition, it is possible to ensure the reliability for real application.

The arrangement of the paper is as follows : Section 2 introduces a simple kinematics in polar coordinates. In Section 3, it is shown that the range estimation problem can be formulated as the robust filter design problem for uncertain system. Also, by using the Krein space Kalman filter, a new range estimator is proposed. In addition, an adaptation algorithm is also proposed to eliminate bias in LOS rate measurements. Section 4 is devoted to certification of the range estimation performance and robustness against LOS rate measurement uncertainties of the proposed filter. To do this, the typical homing scenario is considered. Finally, the conclusions are stated in Section 5.

2. Range Kinematics in Polar Coordinates

Here, the kinematic relation for range estimation problem is formulated. To do this, the engagement scenario depicted in figure 2 is considered. Let's define the inertial position and velocity represented in the LOS frame as follows :

$$\mathbf{R}^{LOS}=\begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix},\ \frac{d\mathbf{R}}{dt}\Big|_{I}^{LOS}=\begin{bmatrix} v_c \\ v_y \\ v_z \end{bmatrix},\ \frac{d^2\mathbf{R}}{dt^2}\Big|_{I}^{LOS}=\begin{bmatrix} a_c \\ a_y \\ a_z \end{bmatrix} \quad (1)$$

According to the Corilois' law, one can obtain the equations.

$$\frac{d\mathbf{R}}{dt}\Big|_{I}^{LOS}=\frac{d\mathbf{R}}{dt}\Big|_{LOS}^{LOS}+\omega_{ILOS}^{LOS}\times\mathbf{R}^{LOS}, \quad (2)$$

$$\frac{d\mathbf{V}}{dt}\Big|_{I}^{LOS}=\frac{d^2\mathbf{R}}{dt^2}\Big|_{LOS}^{LOS}+\dot{\omega}_{ILOS}^{LOS}\times\mathbf{R}^{LOS} \quad (3)$$
$$+\ 2\omega_{ILOS}^{LOS}\times\frac{d\mathbf{R}}{dt}\Big|_{LOS}^{LOS}+\omega_{ILOS}^{LOS}\times\left(\omega_{ILOS}^{LOS}\times\mathbf{R}^{LOS}\right).$$

From eq. (2) and (3), the relative velocity and acceleration can be written by

$$\begin{bmatrix} v_c \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} \dot{r} \\ r\omega_z \\ -r\omega_y \end{bmatrix}, \quad (4)$$

$$\begin{bmatrix} a_c \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \ddot{r}-r(\omega_y^2+\omega_z^2) \\ 2\dot{r}\omega_z+r\dot{\omega}_z+r\omega_x\omega_y \\ -2\dot{r}\omega_y-r\dot{\omega}_y+r\omega_x\omega_z \end{bmatrix}. \quad (5)$$

Substituting the eq. (4) for eq. (5) and considering only the components for LOS direction results the following range kinematic equation.

$$\begin{bmatrix} \dot{r} \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \omega^2 & 0 \end{bmatrix}\begin{bmatrix} r \\ v_c \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_c^t - \begin{bmatrix} 0 \\ 1 \end{bmatrix} a_c^m \quad (6)$$

where it is defined that $\omega^2=\omega_y^2+\omega_z^2$ and $a_c=a_c^t-a_c^m$. In many cases, range estimation problem does not take the target maneuver into account. Therefore, the constant velocity target model, $a_c^t\approx 0$, is used without loss of generality.

The high pulse repetition frequency(PRF) seeker provides doppler frequency f_d, and doppler frequency mismatch Δf_d. These parameters are closely related to the closing velocity and its difference as bellow

$$v_c(k) = -\frac{\lambda}{2}f_d(k), \quad (7)$$

$$\frac{v_c(k)-v_c(k-1)}{T} = -\frac{\lambda}{2T}\Delta f_d(k) \quad (8)$$

where λ is the wavelength of transmitted signal from seeker and T means sampling time.

Now, rearranging the eq. (6), one gets discrete time state space realization for range kinematics in LOS frame.

$$\begin{cases} x_{k+1} = F_k x_k + G_k w_k + u_k \\ y_k = H_k x_k + v_k \end{cases} \quad (9)$$

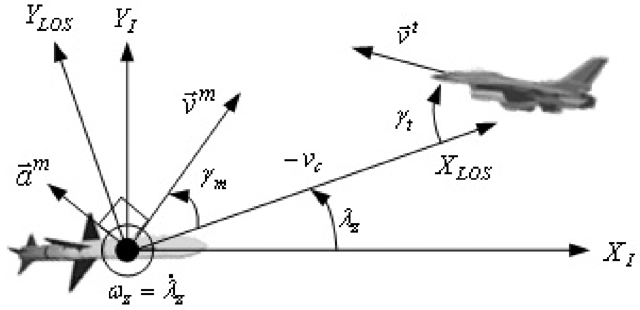

Fig. 2. Engagement Geometry

where we have defined that

$$F=\begin{bmatrix}1\end{bmatrix},\quad G=\begin{bmatrix}T\end{bmatrix},\quad H=\begin{bmatrix}\omega^2\end{bmatrix}$$
$$x=r,\quad y=-\tfrac{\lambda\Delta f_d}{2T}+a_c^m,\quad u=Tv_c.$$

In the above equation (9), the process noise w_k and measurement noise v_k are assumed that the zero mean white noise with variances Q and R. The the missile acceleration a_c^m can be measured by the inertial navigation unit mounted on the missile, and the LOS rate ω can be measured from seeker. Hence, the range estimation problem can be summarized as the Kalman filtering problem to the range kinematic equation (9) derived in polar coordinates.

3. Robust Kalman Range Estimator

As mentioned above, the range estimator can be designed by using the linear robust Kalman filter(RKF) [7]. Apart from the previous approach to the problem, in our setting, a linear measurement equations is carefully deliberated. The multiplication of squares of LOS rate ω and range r should be estimated makes up the measurement equation. Unfortunately, the measured LOS rate contains many undesirable error sources such as gyro bias, thermal noise in electronic devices in seeker hardware and target glint noise and so on. These error sources have time-varying, range dependant nature hence it cannot be predicted by off-line processing. Moreover, it affects the measurements directly and results bias errors in range estimates. To overcome this situation, a specific filtering scheme is used to handle the LOS rate measurement errors. This is the reason why one must consider the robust Kalman filter algorithm to tackle the robustness issue against measurement uncertainty. Before preceding, it should be pointed out that the LOS rate measurement errors are energy bounded so it can be modelled by using the norm bounded uncertainty.

$$\tilde{\omega}^2=(\omega+\delta\omega)^2,\quad \|\delta\omega\|\le\alpha \quad (10)$$

where $\tilde{\omega}$ and $\delta\omega$ represent measured LOS rate and LOS rate uncertainty, respectively. In addition, α is the given norm bound of LOS rate uncertainty. To replace the true LOS rate ω in the measurement matrix to the available information $\tilde{\omega}$, eq. (10) should be modified.

$$\omega^2=\tilde{\omega}^2-2\tilde{\omega}\delta\omega+\delta\omega^2 \quad (11)$$

To apply the robust Kalman filter, the upper bound of squares of the LOS rate is adopted hereafter.

$$\omega^2\le\tilde{\omega}^2+\alpha^2-2\tilde{\omega}\delta\omega \quad (12)$$

Table 1. Robust Kalman Filter

- State-space equation

$$\begin{cases} x_{k+1} &= F_k x_k + G_k w_k \\ y_k &= [H_k + E_k \Delta_k K_k] x_k + v_k \\ &= H_k x_k + \tilde{v}_k \\ s_k &= K_k x_k \end{cases}$$

- Robust Kalman filter

$$\hat{x}_k = (I + P_k K_k^T K_k)\tilde{x}_k + P_k H_k^T \tilde{R}_k^{-1}(y_k - H_k \tilde{x}_k)$$
$$\tilde{x}_{k+1} = F_k \hat{x}_k$$
$$P_{k+1}^{-1} = \tilde{P}_{k+1}^{-1} + H_k^T \tilde{R}_k^{-1} H_k - K_k^T K_k$$
$$\tilde{P}_{k+1} = F_k P_k F_k^T + G_k Q_k G_k^T$$

- Sum Quadratic Constraint

$$(x_0 - \hat{x}_0)^T \Pi_0^{-1}(x_0 - \hat{x}_0) + \sum_{i=0}^{N} w_i^T Q_i^{-1} w_i \, dt + \sum_{i=0}^{N} \tilde{v}_i^T \tilde{R}_i^{-1} \tilde{v}_i \, dt \le \varepsilon$$

- Weighting Matrix

$$\tilde{R}^{-1} = R^{-1} - \begin{bmatrix} I \\ E^T \end{bmatrix}^+ \begin{bmatrix} 0 & R^{-1}E \\ E^T R^{-1} & E^T R^{-1} E - I \end{bmatrix} \begin{bmatrix} I & E \end{bmatrix}^+$$

- Existence condition

$$P_k^{-1} = \tilde{P}_k^{-1} + H_k^T \tilde{R}_k^{-1} H_k - K_k^T K_k > 0$$

Now, the linear measurement equation in (9) can be converted as bellow by using eq. (12).

$$y_k = \begin{bmatrix} H_k + \delta H_k \end{bmatrix} x_k + v_k. \quad (13)$$

where

$$H_k = \tilde{\omega}_k^2 + \alpha^2, \quad \delta H_k = -2\tilde{\omega}_k \delta\omega_k.$$

The parametric uncertainty in measurement matrix, $\delta\omega_k$ can be rewritten by using the norm bounded uncertainty Δ_k.

$$y_k = \begin{bmatrix} H_k + E_k \Delta_k K_k \end{bmatrix} x_k + v_k, \quad \|\Delta_k\| = 1. \quad (14)$$

The time invariant matrices E and K chosen by the designer hold $E \cdot K = -2\tilde{\omega}\alpha$ from eqs (10) and (14).

From the above results, we can conclude that the range estimation problem is just the robust Kalman filter design problem in table 1. Consequently, the robust range estimator can be readily obtained as in eq. (15).

$$\hat{x}_k = (I + P_k K^T K)\tilde{x}_k + P_k H_k^T \tilde{R}^{-1}(y_k - H_k \tilde{x}_k), \quad (15)$$
$$\tilde{x}_k = F \hat{x}_{k-1}$$

where a posteriori estimation error covariance matrix P_k satisfies the Riccati recursion.

$$P_{k+1}^{-1} = (F P_k F^T + G Q G^T)^{-1} + H_k^T \tilde{R}^{-1} H_k - K^T K$$

At this point, it should be noted that the proposed range estimator takes only one state and measurement. That is, we are able to implement the range estimator in real time without computational burdens. In addition, the minimal order of range filter allows us to eliminate unexpected behaviors of the filter.

Table 2. Simulation Condition

Subsystem	Assumption
Seeker	LOS meas. error : $\sigma = 0.1°$ LOS rate meas. error : $\sigma = 0.1°/sec$ f_d meas. error : $\sigma = 5KHz$ Δf_d meas. error : $\sigma = 0.5KHz$ wavelength : $\lambda = 20mm$
Missile	navigation constant : $N = 3$ OPNG : $a_c = -N v_c \omega + 10\sin(0.4 \cdot \pi \cdot t_{go})$ missile velocity : $v_m = 700m/sec$
Target	position : $\vec{r}_m = [\ 20 \ \ 0\ ]Km$ velocity : $v_t = [\ -300 \ \ 0\ ]m/sec$
RKF	$T = 0.025$ $Q = (T \cdot 0.1)^2$, $R = 0.5^2$ $\tilde{x}_0 = 30Km$, $\tilde{P}_0 = 9 \cdot 10^2 Km^2$

4. Simulation Result

In this section, the convergency and estimation performance of the proposed filter are investigated via a simple 2-D homing scenario. Table 2 shows the parameters used in the simulation. To verify the performance improvement of the proposed method, the conventional passive ranging filter based on EKF is compared.

Figure 3 shows the estimation results after 200 Monte-Carlo runs when the missile is guided by using the conventional PN guidance law with initial heading $\gamma_m = 20°$. The proposed range estimator provides good convergency characteristics but the EKF cannot ensure the convergency in such case. Moreover, the EKF may diverge in certain cases. Figure 4 gives support to mathematical outcomes in [4]. There are bias errors in EKF estimates. Also the EKF cannot give the reliability to us due to the large deviations of estimates. In the contrast, the proposed filter has negligible bias errors and very small deviations.

In the figure 5, we can see the estimation performances of the proposed filter for somewhat different observability conditions. The RMS errors of the proposed filter is in inverse proportion to the initial heading error. It means that if there are insufficient changes of LOS rate, the proposed filter might be faces with bad observability conditions. But even that case, the estimation performance of RKF is superior to that of EKF. Fortunately, the initial heading error is quite large in medium range air defense missile hence the proposed scheme is adequate for SAM applications.

It is very well known that we can obtain better estimation performance of EKF by using the OPNG. As mentioned in the figure 1, the utilization of LOS rate can easily provides good observability condition for us. In this reason, although we use the OPNG for missile homing, the range estimation performance of proposed RKF is much better than the EKF at long range as shown in the figure 6.

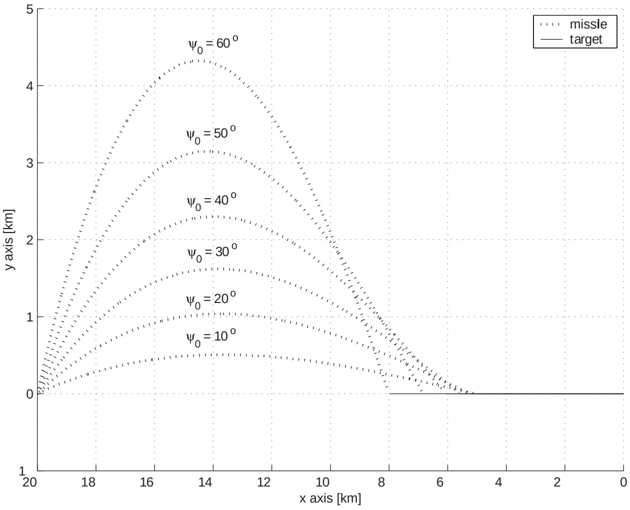

Fig. 3. Engagement Trajectory

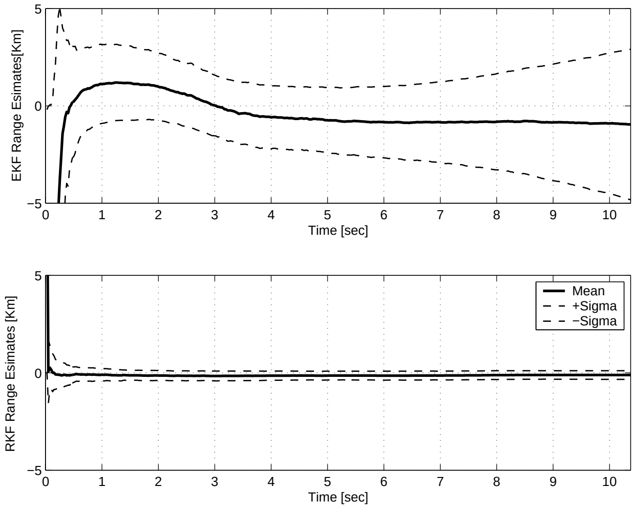

Fig. 4. Range Estimates with Conventional PNG

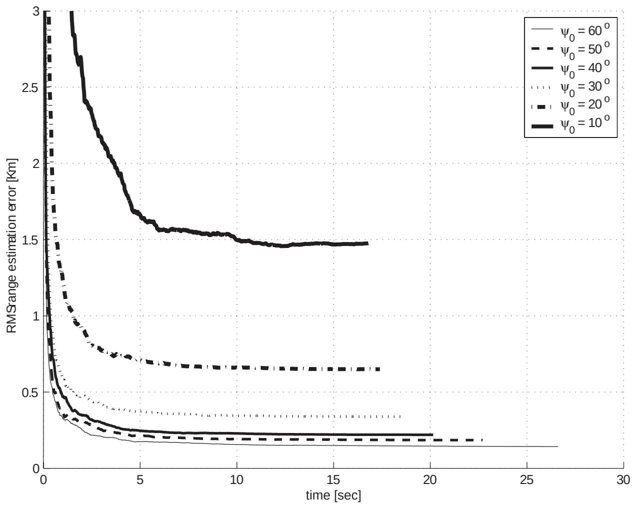

Fig. 5. RMS Range Estimation Error with PNG

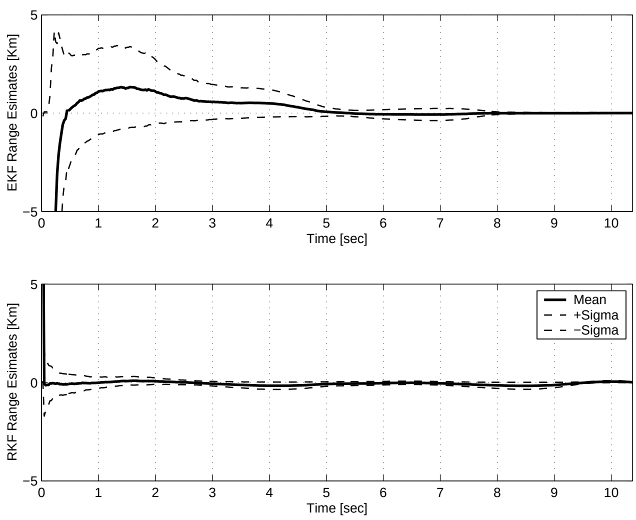

Fig. 6. Range Estimates with OPNG

5. Conclusion

In this paper, a practical range estimator based on the robust Kalman filter was proposed. It was shown that the proposed scheme can be applied to the typical SAM scenario which requires fast convergency characteristics. To do this, the LOS rate measurements is used istead of bearing measurements. Moreover, it can guarantee the operational confidency because the newly proposed range estimator can be easily implemented by using first order linear robust Kalman filter based on the simple kinematics.

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