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PN 부호의 위상오프셋을 이용한 동기 방법

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Synchronization Scheme Using Phase Offsets of PN Sequences

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요약

대역확산 통신에서의 동기는 수신 PN (Pseudo Noise) 부호의 위상오프셋과 수신기의 PN 부호 발생기의 확산부호와의 위상오프셋을 동일하게 하는 것과 동일하므로, PN 부호의 위상오프셋을 안다는 것은 매우 중요하다. 본 논문은 PN 부호의 위상오프셋 계산, 오류검출 방법과 이를 이용한 동기 방법을 제안한다. 한 주기의 PN 부호가 수신되면, 수신 부호의 위상오프셋과 오류검출은 계산되며, 계산된 위상오프셋 값을 이용하여 수신기의 PN 부호 발생기의 쉬프트 레지스터 상태를 초기화함으로써 동기를 쉽게 이룰 수 있다. 제안된 방법의 평균 초기동기 시간은 해석적으로 구해지며, 비교적 높은 SNR에서는 매우 빠른 동기를 이룸을 알 수 있다.

Abstract

It is important to know phase offsets of PN (Pseudo Noise) sequences in spread spectrum communications since the acquisition is equivalent to make a phase offset between a receiving PN sequence and a PN sequence of local PN generator be identical. In this paper, a phase offset enumeration method for PN sequences with error detection, and its application to the synchronization are proposed. The phase offset enumeration for an n-tuple PN sequence and its error detection are performed when one period of the sequence is received. Once the phase offset of the receiving sequence is calculated, we can easily accomplish the synchronization by initializing shift registers of a local PN generator according to the phase offset value. The mean acquisition time of the proposed synchronization method is derived analytically, and we see that the method acquires very fast acquisition in the high SNR (Signal-to-Noise Ratio) environment.

Key words: PN code, spread spectrum, phase offset, synchronization, error detection

1. Introduction

In this paper, a phase offset enumeration method for PN sequences with error detection, and its application to the synchronization are proposed. When the period of the sequence is not very long, the relative phase offset between the sequence and its shifted replica can be found by counting the number of bits delayed from the sequence of the same bit streams. But as the period of the sequence increases, it becomes difficult to find the phase offset.

The phase offset enumeration for an n-tuple PN sequence and its error detection are performed when one period of the sequence is received. Once the phase offset of the receiving sequence is calculated, we can easily accomplish the synchronization by initializing shift registers of a local PN generator according to the phase offset value. The mean acquisition time of the proposed synchronization scheme is derived analytically, and we see that the method acquires very fast acquisition in the high SNR environment.

2. Definitions

Let **S** be the set of all n-tuple binary sequences. We define a cyclic shift to the right operator T: $\mathbf{S} \to \mathbf{S}$ by

$$\mathbf{C} = (C_0, C_1, \mathrm{K}, C_{n-2}, C_{n-1})$$
$$\mathrm{M}$$
$$T^i\mathbf{C} = (C_{n-i}, C_{n-i+1}, \mathrm{K}, C_{n-i-2}, C_{n-i-1}) \qquad (1)$$
$$\mathrm{M}$$
$$T^{n-1}\mathbf{C} = (C_1, C_2, \mathrm{K}, C_{n-1}, C_0)$$

for every sequence $\mathbf{C} = (C_0, C_1, \Lambda, C_{n-1}) \in \mathbf{S}$. Here we define $T^0\mathbf{C} = \mathbf{C}$. We construct the following polynomial $C(x)$ corresponding to a sequence $T^j\mathbf{C} \in \mathbf{S}$.

$$C(x) = C_{n-j} + C_{n-j+1}x + \Lambda + C_{n-j-1}x^{n-1} \qquad (2)$$

We define a phase offset evaluation function A^l : $\mathbf{S} \to \mathbf{Z}$ by

$$\mathrm{A}^l(T^j\mathbf{C}) = \frac{d}{dx}x^l C(x)\bigg|_{x=1} \qquad (3)$$

for all $T^j\mathbf{C} \in \mathbf{S}$, where $\mathbf{Z}$ is the set of all integer numbers and l is an integer. Here we call the integer l the weight of the phase offset evaluation function. The sequence $T^i\mathbf{C}$ is said to be a reference sequence of the phase offset evaluation function with weight l if it satisfies $\mathrm{A}^l(T^i\mathbf{C}) \equiv 0 \pmod{n}$. The reference sequence has a value of $0 \pmod{n}$ of $\mathrm{A}^l(T^i\mathbf{C})$.

3. Error detection and phase offset calculation

Let $f(x) = f_0 + f_1 x + \Lambda + f_m x^m$ be a primitive polynomial of degree m over $\mathrm{GF}(q)$. Then we see that [1]-[2]

$$\frac{1}{f(x)} = (C_0 + C_1 x + \Lambda + C_{n-1}x^{n-1}) \cdot (1 + x^n + x^{2n} + \Lambda) = C(x)\frac{1}{1-x^n} \qquad (4)$$

and

$$C(x)f(x) = (x^n - 1) \equiv 0 \ (\mathrm{mod}\ x^n - 1) \qquad (5)$$

where $n = q^m - 1$. Note that $C(x)f(x)$ is zero cyclic code polynomial in $\mathrm{GF}(q)[x]/(x^n - 1)$. Thus by the definition of a cyclic code [3]-[4], for $C(x)f(x)$ to be a cyclic code polynomial, $C(x)$ or $f(x)$ should also be a cyclic code polynomial.

Now let $g(x)$ be a generator polynomial of cyclic code in $\mathrm{GF}(2)[x]/(x^n - 1)$ whose degree is less than n. If $f(x)$ is not a code polynomial of the cyclic code, then $C(x)$ is a cyclic code of length n generated by $g(x)$. If $f(x)$ is a code polynomial of the cyclic code, then $C(x)$ can be a cyclic code polynomial or not.

It is necessary to note that if the Hamming weight of $\mathbf{f} = (f_0, f_1, \Lambda, f_m)$, $w(\mathbf{f})$, is less than the minimum distance of the cyclic code of length n generated by $g(x)$, then $C(x)$ is a code polynomial of the cyclic code. Thus choosing such a generator polynomial always make an n-tuple PN sequence be a cyclic code of code length n. Thus we can detect errors occurred in the PN sequence by simply checking the syndrome with no redundancy since the n-tuple PN sequence itself is the cyclic code generated by $g(x)$.

Example 1 **:** Consider the PN sequence generated by $f(x) = x^5 + x^2 + 1$. Then the n-tuple PN sequence $\mathbf{C}$ generated by $f(x)$ with initial shift register states all ones becomes

$$\mathbf{C} = (1,1,1,0,0,0,1,1,0,1,1,1,0,1,0,1,0,0,0,0,1,0,0,1,0,1,1,0,0,1,1)$$

where $n = 2^5 - 1 = 31$. Now consider the cyclic code generated by the following generator polynomial.

$$g(x) = x^{25} + x^{24} + x^{21} + x^{19} + x^{18} + x^{16} + x^{15} + x^{14} + x^{13} + x^{11} + x^9 + x^5 + x^2 + x + 1 \qquad (6)$$

Then, the minimum distance of the code is $d_{\min} = 15$, and $A_0 = 1$, $A_{15} = 31$, $A_{16} = 31$, $A_{31} = 1$, where $A_i, 0 \le i \le n$, are called the weight distribution of the code. Since $w(\mathbf{f}) = 3$, we know that $\mathbf{f}$ is not a code word and $\mathbf{C}$ should be a cyclic code word for $f(x)C(x)$ to be zero code polynomial. By the definition of the cyclic code, we see that $T^i\mathbf{C}, 0 \le i \le 30$, are also code words of the cyclic code. ■

If the PN sequence is used for BSC (Binary Symmetric Channel), the probability of undetectable error becomes [3]

$$P_u(E) = \sum_{i=d_{\min}}^{n} A_i p^i (1-p)^{n-i} \qquad (7)$$

where p is the transition probability of the BSC.

The phase offset of the PN sequence is given by [5]

$$2 \cdot [\mathrm{A}^l(T^{i+j}\mathbf{C}) - \mathrm{A}^l(T^i\mathbf{C})] \equiv j \pmod{n}. \qquad (8)$$

Since the phase offset evaluation function of (3) can be implemented by the following circuit [5], we can easily obtain the phase offset of the n-tuple PN sequence by using (8), where T_c is a chip time duration of the PN sequence.

4. Synchronization scheme using phase offset

From section 3, we know that the PN sequence generated by the characteristic polynomial $f(x)$ becomes a cyclic code of length n generated by $g(x)$ whose degree is less than n if the Hamming weight of $\mathbf{f} = (f_0, f_1, \Lambda, f_m)$ is less than the minimum distance of the cyclic code. Thus after choosing the proper $f(x)$ and $g(x)$, we can enumerate the phase offset of the sequence with error detection.

Once the phase offset value is given, the shift registers of the PN generator in the receiver is initialized by using the value. This process is equivalent to the code acquisition. After this process, for the exact synchronization the receiver goes to the code tracking mode. In the code tracking mode, if the phase offset value is not correct, the receiver fails synchronization and go back to the initial state. This time loss is given by penalty time by the false alarm. Fig. 2 shows the synchronization algorithm using the phase offset calculation with error detection.

One of the most important parameters to evaluate the performance of synchronization methods is mean acquisition time. Assume that the PN sequence is transmitted through the BSC, and let p be the transition probability. Then the probability of receiving one period of the PN sequence with no error is $(1-p)^n$. Now let t be the number of receiving n-tuple PN sequences. Then the probability of receiving an n-tuple PN sequence with no error is given by

$$P(t) = (1-p)^n (1-(1-p)^n)^{t-1} \quad (9)$$

and the average of t becomes

$$\begin{aligned} \bar{t} &= \sum_{t=1}^{\infty} t \cdot P(t) \\ &= \sum_{t=1}^{\infty} t \cdot (1-p)^n \cdot (1-(1-p)^n)^{t-1} \cdot P(t). \quad (10) \\ &= \frac{1}{(1-p)^n} \end{aligned}$$

The mean time to determine whether there are error in the received n-tuple PN sequence is expressed as

$$\overline{T_e} = P_u(E) \cdot KnT_c \quad (11)$$

where K is the penalty due to undetectable error pattern by the cyclic decoder. Therefore, the mean acquisition time can be expressed as

$$\begin{aligned} \overline{T_{acq}} &= \bar{t} \cdot (nT_c + \overline{T_e}) \\ &= \bar{t} \cdot nT_c \cdot (1 + KP_u(E)) \end{aligned} \quad (12)$$

When the generator polynomial (6) is used for error detection of the PN code by $f(x) = x^5 + x^2 + 1$, the mean acquisition time performance over AWGN and Rayleigh fading channel are depicted in Fig. 3, where $n = 31$, $K = 10$, and BPSK with coherent detection are assumed. From Fig. 3, we see that it acquires very rapid acquisition in high SNR environment.

5. Conclusion

In this paper, we proposed a synchronization scheme using phase offsets of PN sequences with error detection. An error detection of phase offsets for PN sequences using the property of cyclic code, and its application to synchronization were discussed. Once the phase offset of a PN sequence is calculated, we can easily accomplish the synchronization by initializing shift registers of a local PN generator according to the phase offset value. The mean acquisition time performance of the proposed scheme was derived analytically.

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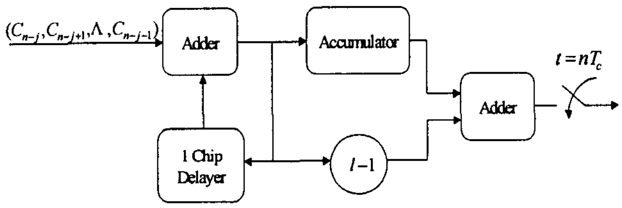

Fig. 1. Circuit to calculate offsets of PN sequences using the phase offset evaluation function with weight l.

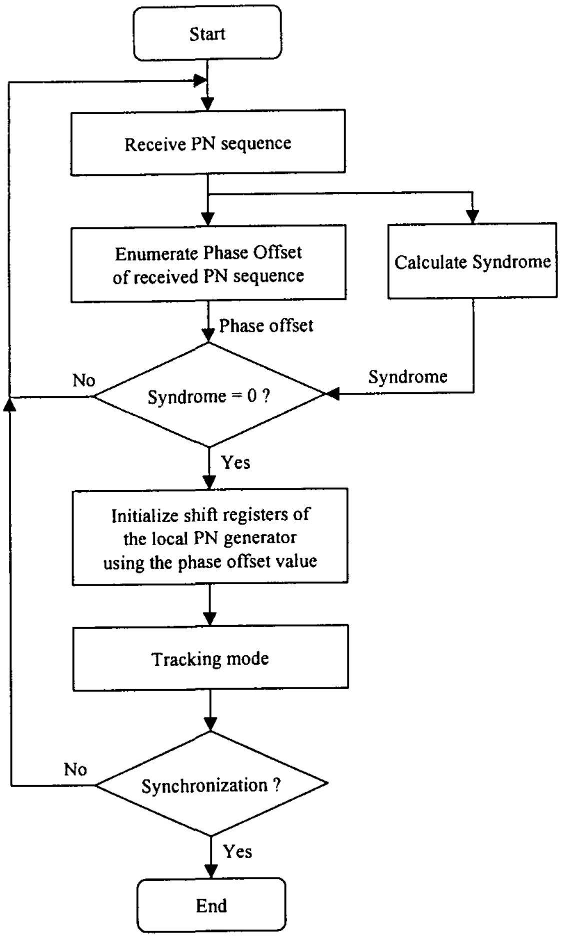

Fig. 2. Synchronization algorithm using phase offset with error detection.

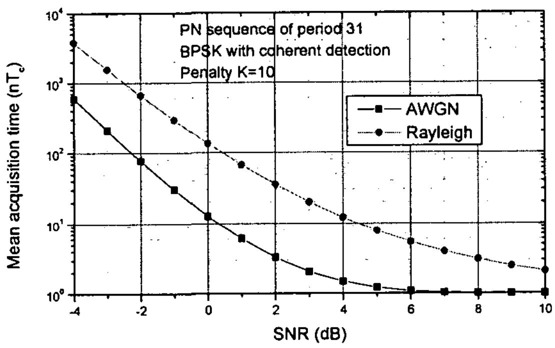

Fig. 3. Mean acquisition time for PN sequence of period 31 over AWGN and Rayleigh fading channel with the coherent BPSK when the generator polynomial by (6) is used for error detection.