Guided Modes along Dispersive Double Negative (DNG) Metamaterial Columns

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Abstract

Modal properties of guided waves along circular dispersive double negative (DNG) index metamaterial rod waveguides are numerically investigated. Identical forms of dispersive dielectric and magnetic material constants are used for simplicity. For degenerated azimuthally symmetric mode, a multimode region, a single mode region, a band gap region and a forbidden region are found which cannot be observed in the case of the conventional dielectric rod waveguide. As the normalized frequency goes down, discrete guided modes are continuously generated, which is a reverse property of conventional dielectric rod waveguide. Also, there are high-frequency cutoffs, which have been generally examined in dispersive circular geometries such as a plasma column or a plasma Goubau line. In the single mode region, both the low- and high-frequency cutoffs are existed where the propagation constants are continued between the guided oscillating and surface modes.

Key words: guided mode, double negative (DNG) index metamaterial column, plasma column, plasma frequency.

I. Introduction

Double negative (DNG) index metamaterials are materials that encompass simultaneously negative permittivity and permeability [1]. Compared with conventional double positive index materials such as dielectric or magnetic materials, the DNG materials have unique properties such as negative refraction, backward wave traveling, reversals of Snell's law, Doppler shift and Čerenkov radiation, extraordinary Goos-Hänchen shift and so on. Even though, nowadays, they seem to possess lots of potential applications in microwave, millimeter wave, or even higher optical frequency ranges, they have not been seriously considered until UC San Diego Group [2, 3] verify that these artificial materials can be experimentally realizable, which has been theoretically predicted since 1968 [4]. Afterward, worldwide researches on the DNG metamaterials have been explosively accelerated in viewpoints of exploring novel applications of these exotic materials as well as discovering novel physical phenomena around them, see recently published special issues [5, 6], for instances. Among wide variety of approaches to characterize or make use of the DNG metamaterials, there are some of published papers dealing with propagations along or with DNG metamaterial slab [7-10] and channel [11] waveguides. Considering recent developments and extensions of their applications to novel electromagnetic devices such as lenses, resonators, Bragg reflectors, leaky wave antennas, couplers, high- or band pass filters, T-junctions, and so on, modal properties other than slab or channel waveguide should be investigated in advance for future potential applications. For a circular open constant DNG metamaterial rod structure, the guided mode analysis has been firstly made by us [12] using the concept of effective medium approach. However, practical DNG metamaterials can be generally considered as a dispersive media. Thus, in this work, we consider the DNG metamaterials as lossless dispersive magnetodielectric media whose dielectric and magnetic constants take identical forms of dielectric constant of plasma medium. From properly chosen expressions of axial electric and magnetic fields, we derived the characteristic equations which reveal dispersion relation of the DNG metamaterial cloumn. Guided mode solutions are determined and unique normalized frequency range(s) are found that cannot be observed in the case of conventional circular dielectric rods, plasma columns or even in DNG metamaterial columns with constant negative indexes.

II. Characteristic Equation of Guided Modes

A. Double Negative Material Constants

Simultaneous negative material constants of the DNG metamaterials can be realized by metallic rods and split ring resonators (SRRs) [2, 3]. Generally, their dispersive properties of dielectric and magnetic material constants can be respectively expressed as follows, which is employed by others [13, 14].

$$\varepsilon_r(\omega) = 1 + \frac{\omega_{pe}^2}{\omega_{0e}^2 - \omega^2 - j\omega\gamma_e} \quad (1)$$

$$\mu_r(\omega) = 1 + \frac{\omega_{pm}^2}{\omega_{0m}^2 - \omega^2 - j\omega\gamma_m} \quad (2)$$

where ω_{pe} and ω_{pm} are coupling strengths, ω_{0e} and ω_{0m} are resonance frequencies, and γ_e and γ_m are absorption parameters, all are dependent on the dimensions and substances of artificially composite electromagnetic structures, i.e., metallic rods and SRRs.

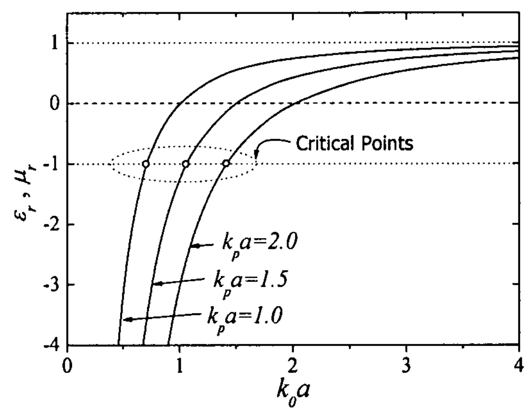

Fig. 1. Dielectric and magnetic constants in eq. (3).

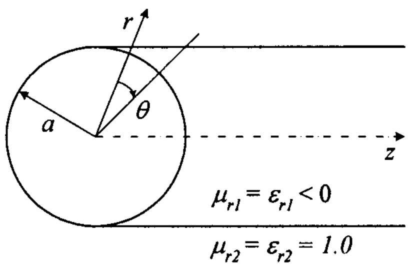

Fig. 2. DNG metamaterial column.

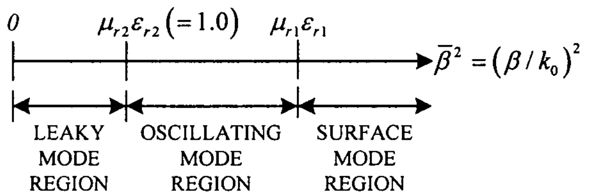

Fig. 3. Guided and leaky mode regions of DNG metamaterial column. Guided modes are composed of oscillating and surface mode regions.

Since they are controllable and we are not currently interested in resonances and losses, let us restrict to our attention to simpler form of eq. (1) and (2). Assuming that $\gamma_e = \gamma_m = 0$, $\omega_{0e} = \omega_{0m} = 0$ and $\omega_{pe} = \omega_{pe} = \omega_p$ give the following simpler form.

$$\varepsilon_r = \mu_r = 1 - \frac{\omega_p^2}{\omega^2} = 1 - \frac{\omega_p^2 a^2 / c^2}{\omega^2 a^2 / c^2} = 1 - \frac{(k_p a)^2}{(k_0 a)^2} \quad (3)$$

where c is the speed of light, $k_0 a$ is the normalized frequency related to angular frequency ω and the radius of the DNG metamaterial column a , ω_p is a electric or magnetic plasma frequency, and $k_p a$ is the normalized plasma frequency. Fig. 1 shows the dielectric and magnetic material constants of dispersive magnetodielectric materials for three different values of $k_p a$. Below the normalized frequencies of 1.0, 1.5, and 2.0, respectively, this magnetodielectric material provides simultaneous negative permittivity and permeability. In the Fig. 1, there are critical normalized frequencies corresponding to $\mu_r \varepsilon_r = 1.0$ ($\mu_r = -1.0$ and $\varepsilon_r = -1.0$), i.e., 0.707, 1.061, and 1.414 for $k_p a = 1.0$, $k_p a = 1.5$, $k_p a = 2.0$, respectively. Significances of these critical frequency points with guided mode characteristics will be discussed in Section III.

B. Derivation of Characteristic Equation

Fig. 2 shows the schematic diagram of the circular DNG metamaterial column embedded in free space region. The radius of the DNG column is a and the inner region ($r < a$) are characterized by μ_{r1} and ε_{r1} as assumed in eq. (3). In the free space region, i.e., at $r > a$, both the dielectric and magnetic material constants are assumed to be unities. Fig. 3 shows the possible ranges of solutions for discrete modes. $\bar{\beta}(=\beta/k_0)$ is normalized propagation constant, which is propagation constant normalized by the free space wavenumber k_0 . As shown in Fig. 3, guided mode solutions can be classified into two distinct modes, i.e., oscillating and surface modes, whose propagation constants are real and corresponds to the slow waves because $\bar{\beta} > \sqrt{\mu_{r2}\varepsilon_{r2}}(=1.0)$. Below the guided mode region in Fig. 3, there is a leaky mode region, whose propagation constant is complex and corresponds to the fast wave ($\bar{\beta} < 1.0$), however, this region is currently not our concern. We take on the principles of choosing proper Bessel functions of the axial field components of dielectric rod [15], but it is extended here to our DNG metamaterial column case. If the normalized propagation constant $\bar{\beta}$ is at $\mu_{r2}\varepsilon_{r2}(=1) < \bar{\beta}^2 < \mu_{r1}\varepsilon_{r1}$, that is, the oscillating mode, the axial scalar electric and magnetic field components can be expressed as follows. The propagation factor $\exp[j(\omega t - \beta z)]$ is assumed but it is omitted here.

$$E_{z1} = A_{mn} J_m(k_1 r) \tag{4}$$

$$H_{z1} = B_{mn} J_m(k_1 r) \tag{5}$$

$$E_{z2} = C_{mn} K_m(k_2 r) \tag{6}$$

$$H_{z2} = D_{mn} K_m(k_2 r) \tag{7}$$

where J_m and K_m are the Bessel function and modified Bessel function of the second kind, respectively. At $r < a$, waveguide region, the electric and magnetic fields are governed by the Bessel functions, which means fields are oscillated transversely. Thus, we call this mode as a "*oscillating mode*". m is azimuthal eigenvalue. A_{mn} to D_{mn} are real constants to be determined. k_1 and k_2 are the transverse propagation constants and are related with the axial propagation constant β as follows.

$$k_1^2 = k_0^2 \mu_{r1}\varepsilon_{r1} - \beta^2 = k_0^2\left(\mu_{r1}\varepsilon_{r1} - \bar{\beta}^2\right) \tag{8}$$

$$k_2^2 = \beta^2 - k_0^2 \mu_{r2}\varepsilon_{r2} = k_0^2\left(\bar{\beta}^2 - \mu_{r2}\varepsilon_{r2}\right) \tag{9}$$

From eq. (4)-(7), azimuthal field components are obtained and matching the tangential fields at the boundary $r = a$ resulted in characteristic equation through standard steps of arithmetic manipulations as follows.

$$\left(\frac{\beta m}{k_0 a}\right)^2 \left(\frac{1}{k_1^2} + \frac{1}{k_2^2}\right)^2 = \left\{\frac{\varepsilon_{r1}}{k_1}\frac{J_m'(k_1 a)}{J_m(k_1 a)} + \frac{\varepsilon_{r2}}{k_2}\frac{K_m'(k_2 a)}{K_m(k_2 a)}\right\} \times \times \left\{\frac{\varepsilon_{r1}}{k_1}\frac{J_m'(k_1 a)}{J_m(k_1 a)} + \frac{\varepsilon_{r2}}{k_2}\frac{K_m'(k_2 a)}{K_m(k_2 a)}\right\} \tag{10}$$

If azimuthal eigenvalue m is zero, the characteristic equation (10) can be reduced as following form.

$$\frac{\varepsilon_{r1}}{k_1}\frac{J_1(k_1 a)}{J_0(k_1 a)} + \frac{1}{k_2}\frac{K_1(k_2 a)}{K_0(k_2 a)} = 0 \tag{11}$$

Since we assumed that the dielectric and magnetic constants in region 1 ($r < a$) and region 2 ($r > a$) to be identical in eq. (3) and to be unities, respectively, the azimuthally symmetric modes, i.e., TM_{0n} and TE_{0n} modes are degenerated together. Thus, here, we call this degenerated symmetric mode as $m = 0$ mode, which is our main concern in this work. On the other hand, if the normalized propagation constant $\bar{\beta}$ is at $\bar{\beta}^2 > \mu_{r1}\varepsilon_{r1} > \mu_{r2}\varepsilon_{r2}(=1)$, that is, the surface mode, the axial field components in region 1 are expressed as follows keeping the field expression in region 2 to be identical.

$$E_{z1} = A_{mn} I_m(k_1 r) \tag{12}$$

$$H_{z1} = B_{mn} I_m(k_1 r) \tag{13}$$

where I_m is the modified Bessel function of the first kind. As r goes to origin at $r < a$ and r goes to infinity at $r > a$, the amplitudes of the fields decay monotonically. Thereby, the fields are strongly confined near interface $r = a$, so it can be called as a "*surface mode*". Of course, the separation relation in eq. (8) should also be changed as follows.

$$k_1^2 = \beta^2 - k_0^2 \mu_{r1}\varepsilon_{r1} = k_0^2\left(\bar{\beta}^2 - \mu_{r1}\varepsilon_{r1}\right) \tag{14}$$

Following same procedures as the previous oscillating mode case, the characteristic equation for surface $m = 0$ mode is as follows.

$$\frac{\varepsilon_{r1}}{k_1}\frac{I_1(k_1 a)}{I_0(k_1 a)} - \frac{1}{k_2}\frac{K_1(k_2 a)}{K_0(k_2 a)} = 0 \tag{15}$$

In spite of positive values of magnetic material constants of the plasma column [16], i.e., $\varepsilon_r = 1 - \omega_p^2/\omega^2$ and $\mu_r = 1.0$ for plasma medium, the characteristic equation of the plasma column is same as eq. (15), because $\bar{\beta}^2 (> 0)$ is always greater than $\mu_{r1}\varepsilon_{r1} (< 0)$. The effects of same characteristic equation but different magnetic constants to each dispersion characteristics will be noticed in Section III. Characteristic equations (11) and (15) are numerically solved and the dispersion characteristics of DNG metamaterial columns will be shown in the next section.

III. Numerical Results

Fig. 4 show the dispersion characteristics of DNG metamaterial column embedded in the free space. At a first glance, there are finite numbers of guided modes at a given frequency and there exists high-frequency cutoff while in the conventional circular dielectric waveguide case, there are only low-frequency cutoffs. The high-frequency cutoff, which is due to the dispersive nature of material constants, can also be observed in plasma columns [16, 17] and Goubau lines with plasma layers instead of dielectrics [18]. As the normalized frequency decreases, the number of modes increases. Arrows headed to zero frequency in Fig. 4 denote the continuations of the mode generation with respect to the decreases of the normalized frequency. The number of guided modes along the general conventional circular dielectric rod increases as the normalized frequency increases. As the normalized frequency decreases, the value of $\mu_{r1}\varepsilon_{r1}$ are increased as shown in

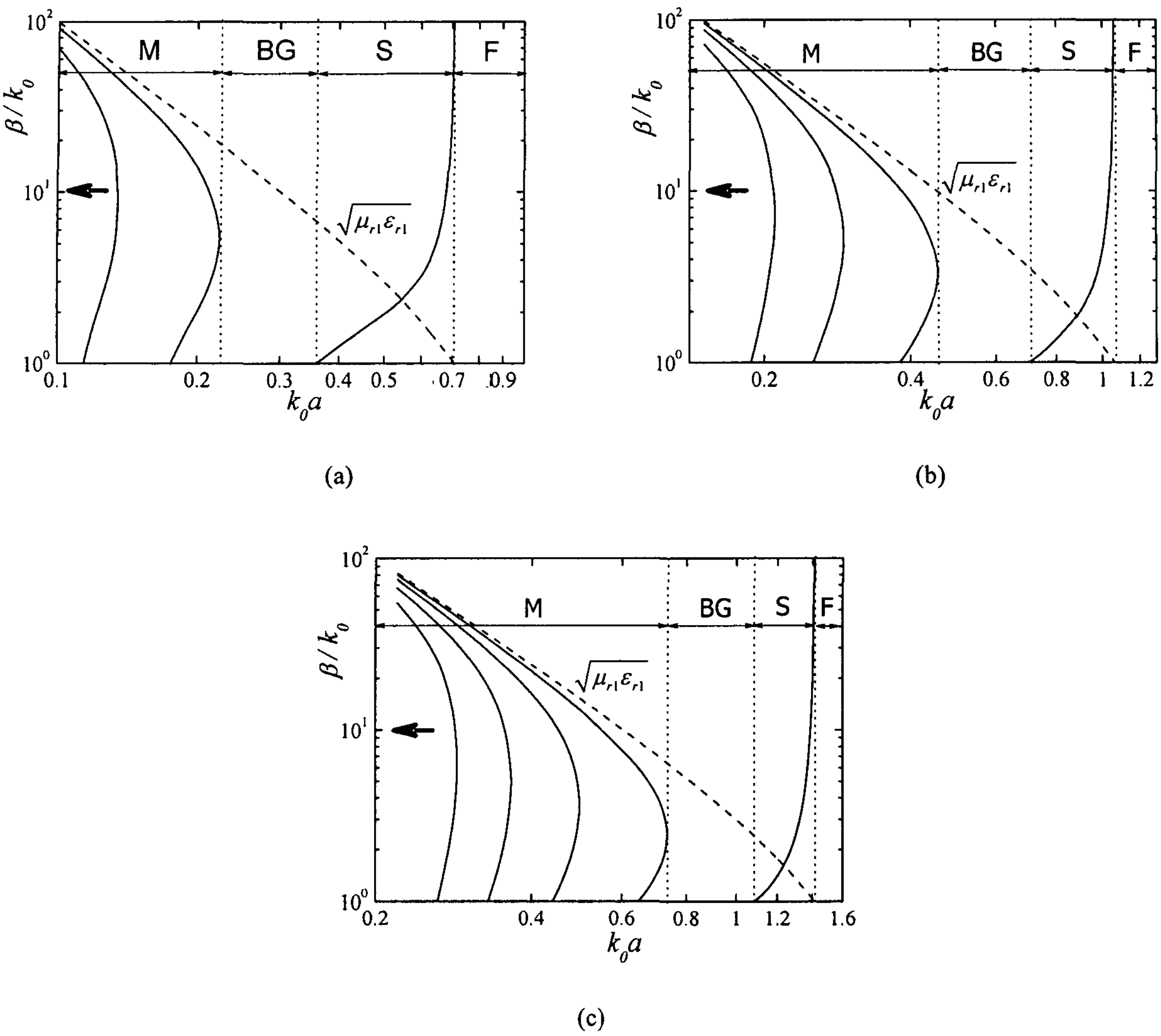

Fig. 4. Dispersion curves of LHM column. (a) $k_p a = 1.0$, (b) $k_p a = 1.5$, and (c) $k_p a = 2.0$. Dashed lines denote the boundary between the oscillating and surface modes. M, BG, S, and F represent multimode, band gap, single mode, forbidden regions, respectively.

Fig. 1, thus, allowable number of modes can be increased. In Fig. 4, there are multimode (M), band gap (BG), single mode (S), forbidden (F) regions. In a multi mode region, there are two possible solutions at a given normalized frequency, which can be usually observed in DNG media slab [7-10] and channel waveguides [11]. Two branches from a high-frequency cutoff reveal different power confinement characteristics [12]. At higher frequency, there is a single mode region where the normalized propagation constant has both low- and high-frequency cutoffs and it also laid both in the oscillating ($1 < \bar{\beta} < \sqrt{\mu_{r1}\varepsilon_{r1}}$) and surface ($\bar{\beta} > \sqrt{\mu_{r1}\varepsilon_{r1}}$) mode regions. The high-frequency cutoffs in the single mode region are $k_p a / \sqrt{2}$, which is identical in the case of the plasma column [16, 17] and the plasma Goubau line [18]. This high-frequency cutoffs correspond to the normalized frequencies when $\mu_r \varepsilon_r = 1.0$. These are critical frequency points in Fig. 1. If the refractive index $n = \sqrt{\mu_{r1}\varepsilon_{r1}}$ is smaller than unity, there is no possible solution satisfying characteristic equation of the DNG metamaterial column. In other words, at higher normalized frequencies than a single mode region, guided mode cannot be supported. This is a forbidden region. Between multi- and single mode regions in Fig. 4, i.e., in the band gap region, there is no solution satisfying characteristic equations as well as in the forbidden region. Fig. 5 shows the guided dispersion

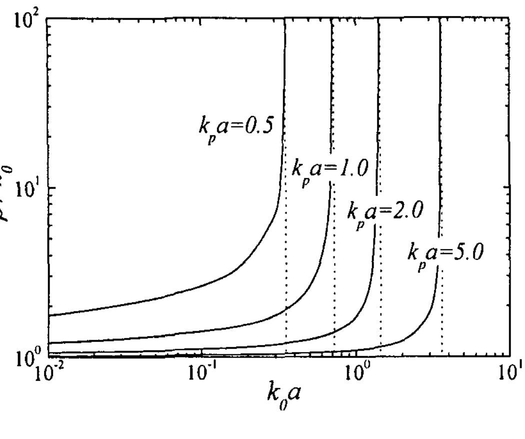

Fig. 5. Dispersion curve of the plasma column. Dotted lines are the high-frequency cutoff, i.e., $k_0a = k_pa/\sqrt{2}$.

characteristics of the plasma column of several values of normalized plasma frequency for comparisons with those of the DNG metamaterial column in Fig. 4. The high-frequency cutoffs are observable for various normalized plasma frequencies; however, there are no low-frequency cutoffs. Also, below forbidden region, only single mode exists for each azimuthal eigenvalue m [18].

IV. CONCLUSIONS

Guided modes of the dispersive DNG metamaterial column for the azimuthally symmetric case ($m = 0$) are theoretically investigated. There are two kinds of guided modes, which are the oscillating and surface modes. As the normalized frequency goes down, new guided modes are continuously generated in multi mode region, where high-frequency cutoffs are existed due to the dispersive properties of the DNG metamaterial. At higher frequency, in a single mode region, the normalized propagation constants are laid both on the oscillating and surface modes and there are both the low- and high-frequency cutoffs. The high-frequency limitation of the single mode region is $k_pa/\sqrt{2}$, which is identical in the cases of the plasma column and the plasma Goubau line. Between the single and multimode regions, there is a frequency range where the guided mode cannot be supported, that is, the band gap region as well as the forbidden region above the single mode region.

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