2003 Proceedings of the Autumn Conference, Korean Statistical Society

A study for the Coverage Probability of a Confidence Interval on the Variance Component in the Unbalanced Random One-Way Model

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1 Introduction

Estimation of variance components has received a great deal of attention, particularly in the 50's and 60's. While papers on point estimation of variance components far outnumbered those dealing with interval estimation, interest in the latter has picked up in the last 15 years. The book by Burdick and Graybill (1992) is devoted in its entirety to the construction of a variety of confidence intervals on variance components and functions thereof.

In this study, we present a novel approach for the comparison of confidence intervals on variance components on the basis of their coverage probability. This approach uses generalized linear models techniques to model the coverage probability as a function of particular control variables. The proposed methodology is demonstrated using the random one-way model and four types of confidence intervals on σ_α^2, the among-group variance component. One of the main advantages of this modeling scheme is to provide a deeper insight into the combined effects of the degree of imbalance of the associated design and the true values of the variance components on the coverage probabilities of the confidence intervals. This is accomplished by examining contour plots of the coverage probability that can be easily generated from the derived model for each of the four types of confidence intervals.

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2 Confidence Intervals on σ_α^2

Consider the one-way random model

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}, \quad i = 1, 2, ..., k, \quad j = 1, 2, ..., n_i, \tag{1}$$

where the α_i and ϵ_{ij} are independently distributed as normal variates with zero means and variances σ_α^2 and σ_ϵ^2, respectively. No exact confidence intervals on σ_α^2 exist. However, there are several procedures for deriving approximate confidence intervals on σ_α^2. We consider four such procedures, a brief description of which follows.

2.1 The modified large sample procedure

This procedure is based on applying a particular modification to the balanced confidence interval on σ_α^2, that is, when $n_i = n$ for all i. A full description of this technique is given in Burdick and Graybill (1992, page 70). The corresponding approximate $(1-\alpha)100\%$ confidence interval on σ_α^2 is given by

$$\left[\frac{1}{n_0}(MS_\alpha - MS_E - \sqrt{\tau_1}), \quad \frac{1}{n_0}(MS_\alpha - MS_E + \sqrt{\tau_2})\right], \tag{2}$$

where MS_α and MS_E are the between-group and among-group mean squares, with $\nu_1 = k-1$, and $\nu_2 = n_. - k$ degrees of freedom, respectively. In equation (2), $n_0 = \frac{1}{k-1}(n_. - \frac{1}{n_.}\sum_{i=1}^k n_i^2)$, $n_. = \sum_{i=1}^k n_i$, $\tau_1 = g_1^2 MS_\alpha^2 + h_2^2 MS_E^2 + g_{12} MS_\alpha MS_E$, $\tau_2 = h_1^2 MS_\alpha^2 + g_2^2 MS_E^2 + h_{12} MS_\alpha MS_E$, $g_i = 1 - \frac{1}{F_{\frac{\alpha}{2},\nu_i,\infty}}$, $h_i = \frac{1}{F_{1-\frac{\alpha}{2},\nu_i,\infty}} - 1, i = 1, 2$, $g_{12} = \frac{(F_{\frac{\alpha}{2},\nu_1,\nu_2}-1)^2 - g_1^2 F_{\frac{\alpha}{2},\nu_1,\nu_2}^2 - h_2^2}{F_{\frac{\alpha}{2},\nu_1,\nu_2}}$, and $h_{12} = \frac{(1-F_{1-\frac{\alpha}{2},\nu_1,\nu_2})^2 - h_1^2 F_{1-\frac{\alpha}{2},\nu_1,\nu_2}^2 - g_2^2}{F_{1-\frac{\alpha}{2},\nu_1,\nu_2}}$.

2.2 The Thomas-Hultquist procedure

Thomas and Hultquist (1978) used the unweighted sum of squares for α_i, namely, $SS_\alpha^* = n_h \sum_{i=1}^k (\bar{y}_{i.} - \bar{y}_{..}^*)^2$, to obtain an approximate confidence interval on σ_α^2. Here, n_h denotes the harmonic mean of the n_i's, that is, $n_h = k\left[\sum_{i=1}^k \frac{1}{n_i}\right]^{-1}$, $\bar{y}_{i.} = \frac{1}{n_i}\sum_{j=1}^{n_i} y_{ij}$, and $\bar{y}_{..}^* = \frac{1}{k}\sum_{i=1}^k \bar{y}_{i.}$. This interval is given by

$$\left[\frac{SS_\alpha^* - (k-1)MS_E F_{\frac{\alpha}{2},k-1,n_.-k}}{n_h \chi^2_{\frac{\alpha}{2},k-1}}, \quad \frac{SS_\alpha^* - (k-1)MS_E F_{1-\frac{\alpha}{2},k-1,n_.-k}}{n_h \chi^2_{1-\frac{\alpha}{2},k-1}}\right]. \tag{3}$$

2.3 The modified harmonic mean procedure

Khuri (1999) used an alternative value, denoted by n^*, to n_h in the Thomas-Hultquist procedure. The new value is given by $n^* = \frac{2}{\lambda_{(1)} + \lambda_{(k-1)}}$, where $\lambda_{(1)} \geq \lambda_{(2)} \geq ... \geq \lambda_{(k-1)}$ are the ordered eigenvalues of the matrix $(\mathbf{I}_k - \frac{1}{k}\mathbf{J}_k)\mathbf{K}(\mathbf{I}_k - \frac{1}{k}\mathbf{J}_k)$, $\mathbf{K} = \text{diag}(\frac{1}{n_1}, \frac{1}{n_2}, \ldots, \frac{1}{n_k})$, and $\mathbf{I}_k$ and $\mathbf{J}_k$ are the identity matrix and matrix of ones, of orders $k \times k$, respectively.

2.4 The Burdick and Eickman (1986) procedure

The interval based on this procedure is given by

$$\left[\frac{SS_\alpha^* L}{\chi^2_{\frac{\alpha}{2},k-1}(1+n_h L)}, \ \frac{SS_\alpha^* U}{\chi^2_{1-\frac{\alpha}{2},k-1}(1+n_h U)}\right], \tag{4}$$

where SS_α^* and n_h are the same as in Section 2.2, $L = \frac{SS_\alpha^*}{n_h(k-1)MS_E F_{\frac{\alpha}{2},k-1,n.-k}} - \frac{1}{n_{(1)}}$, and $U = \frac{SS_\alpha^*}{n_h(k-1)MS_E F_{1-\frac{\alpha}{2},k-1,n.-k}} - \frac{1}{n_{(k)}}$. Here, $n_{(1)}$ and $n_{(k)}$ are the smallest and largest of the n_i's, respectively.

We shall refer to the confidence intervals in Sections 2.1 – 2.4 as the MLS, TH, MHM, and BE intervals, respectively.

3 Modeling the Coverage Probability

The coverage probability of any of the confidence intervals in Section 2 depends on the design, $D = \{n_1, n_2, \cdots, n_k\}$, and on the true values of σ_α^2 and σ_ϵ^2. The degree of imbalance of D is determined by a measure of imbalance given by $\phi = \frac{n.^2}{k\sum_{i=1}^k n_i^2}$, where $\frac{1}{k} < \phi \le 1$ (see Ahrens and Pincus, 1981). A small value of ϕ indicates a high degree of imbalance. The value $\phi = 1$ is attained when the data set is balanced. For a given value of k, there are many designs that can be generated with specified values of $n.$ and ϕ. A method for generating such designs is given in Khuri (1996). We refer to k, $n.$ and ϕ as design parameters.

Let S be a specified region of interest for σ_α^2 and σ_ϵ^2. For each generated design and selected values of σ_α^2 and σ_ϵ^2 from S, four confidence intervals can be obtained as was described in Section 2. Let $\hat{\pi}$ denote an estimated value of the true coverage probability for a given confidence interval. Such a value can be obtained by computer simulation. Our objective here is to develop an empirical relationship between $\hat{\pi}$, on one hand, and k, $n.$, ϕ, and ρ on the other hand, where $\rho = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\epsilon^2}$. In such a relationship, $\hat{\pi}$ is treated as a response variable, and k, $n.$, ϕ, and ρ are considered as control variables. Note that the specification of k, $n.$ and ϕ does not uniquely determine the design D. Several replications on $\hat{\pi}$ can therefore be generated for each assignment of the quadruple (k, $n.$, ϕ, ρ). These replications will be useful in the construction of the aforementioned relationship.

Given the nature of the response $\hat{\pi}$, it would be appropriate to model $\hat{\pi}$ against k, $n.$, ϕ, and ρ using generalized linear models techniques. Instead of dealing with $\hat{\pi}$ directly, let us consider the quantity ω_λ, where

$$\omega_\lambda = \left(\frac{\hat{\pi}}{1-\hat{\pi}}\right)^\lambda, \lambda \geq 1. \tag{5}$$

This transformation maps the interval $[0, 1)$ onto $[0, \infty)$. The value of λ is chosen in a manner that facilitates the approximate identification of the distribution of ω_λ using the $\hat{\pi}$-replications at the point $(k,\ n.,\ \phi,\ \rho)$. Let $\mu_\omega(\mathbf{x})$ denote the mean of the distribution of ω_λ at the point $\mathbf{x} = (x_1, x_2, x_3, x_4)'$, where $x_1 = k$, $x_2 = n.$, $x_3 = \phi$, and $x_4 = \rho$. Let $g(\cdot)$ be a chosen link function such that $\eta(\mathbf{x}) = g[\mu_\omega(\mathbf{x})]$, where $\eta(\mathbf{x})$ is an appropriately chosen linear predictor of the form

$$\eta(\mathbf{x}) = \mathbf{f}'(\mathbf{x})\boldsymbol{\beta}. \tag{6}$$

The right-hand side of (6) is a polynomial of a certain degree in the elements of $\mathbf{x}$ and $\boldsymbol{\beta}$ is a vector of unknown parameters. An estimate of $\mu_\omega(\mathbf{x})$ is given by

$$\hat{\mu}_\omega(\mathbf{x}) = g^{-1}[\mathbf{f}'(\mathbf{x})\hat{\boldsymbol{\beta}}], \tag{7}$$

where $\hat{\boldsymbol{\beta}}$ is the maximum likelihood estimate of $\boldsymbol{\beta}$, and g^{-1} is the inverse function of g, which is assumed to be a strictly monotone function. The estimating equation (7) can then be used to obtain an empirical relationship between $\hat{\pi}$ and the elements of $\mathbf{x}$.

For model (1) with a design $D = \{n_1, n_2, \cdots, n_k\}$, several combinations of levels of k, $n.$, ϕ, and ρ are chosen according to a 3^4 factorial design. The chosen levels are $k = 4, 7, 10$; $n. = 50, 100, 500$; $\phi = 0.30, 0.65, 0.95$; $\rho = 0.10, 0.60, 0.90$. For each combination, several designs D are generated using Khuri's (1996) method, such that a total of 900 designs are used. Note that since the values of both ϕ and ρ fall inside the unit interval $(0, 1]$, k is replaced by a scaled value, namely, $k_s = \frac{k-4}{6}$, and $n.$ is replaced by a scaled value, namely, $n_s = \frac{n.-50}{450}$. This way, the ranges of k_s and n_s for the selected values of k and $n.$, respectively, are equal to one, which matches the ranges of ϕ and ρ.

For the chosen levels of k_s, n_s, ϕ, and ρ, the region of interest is therefore of the form

$$S = \left\{(k_s, n_s, \phi, \rho) \middle| 0 \leq k_s \leq 1, 0 \leq n_s \leq 1,\ .3 \leq \phi \leq .95,\ .1 \leq \rho \leq .9\right\}. \tag{8}$$

Some of the generated designs are listed in Table 1 along with the actual value of ϕ, ϕ_a, for an $(k, n., \phi)$-generated design. For each design and a chosen value of ρ, the coverage probability of each of the four confidence intervals in Section 2 is estimated by Monte-Carlo simulation. To estimate the coverage probability, $10,000$ $\mathbf{y}$ vectors are generated for each specification of D and ρ. The estimated coverage probabilities, that is, values of $\hat{\pi}$, corresponding to the MLS, TH, MHM, and BE intervals are also given in Table 1. We denote such values by $\hat{\pi}_m$ for $m = MLS, TH, MHM, BE$.

The estimated coverage probability at the point $\mathbf{x} = (k_s, n_s, \phi, \rho)'$ is denoted by $\hat{\pi}_m(\mathbf{x})$. The values of $\hat{\pi}_m$ in Table 1 are used to obtain the corresponding values of ω_λ in (5). To determine an appropriate value for λ, we do the following: using the replications on $\hat{\pi}_m$ (and hence on ω_λ) at each combination of k_s, n_s, ϕ and ρ from the 3^4 factorial design, the sample mean, $\bar{\omega}_\lambda$, and sample standard deviation, s_{ω_λ}, are obtained. Let us now determine if there is an approximate linear relationship between $\bar{\omega}_\lambda$ and s_{ω_λ} by fitting a simple linear regression model with no intercept between $\bar{\omega}_\lambda$ and s_{ω_λ}, using several values of λ for each method. A satisfactory linear relationship was observed with $\lambda = 4$ for all four methods (the R^2 values are 0.83 for MLS, 0.83 for TH, 0.82 for MHM, and 0.97 for BE). This suggests assuming a gamma distribution for the ω_4 random variable (see McCullagh and Nelder, 1989, page 30; see also pp. 285–286). Let $\hat{p}_m(\mathbf{x})$ denote the predicted value of $\hat{\pi}_m(\mathbf{x})$ inside the region S in (8). Using the chosen gamma distribution for ω_4 along with a logarithmic link function, $\hat{p}_m(\mathbf{x})$ is given by

$$\hat{p}_m(\mathbf{x}) = \frac{1}{1 + exp\big[-\frac{1}{4}\mathbf{f}'(\mathbf{x})\hat{\boldsymbol{\beta}}\big]}, \quad m = MLS, TH, MHM, BE, \tag{9}$$

where $\hat{\boldsymbol{\beta}}$ is the maximum likelihood estimate of $\boldsymbol{\beta}$ in (6). It should be noted that the logarithmic link function was used here instead of the canonical link function for the gamma distribution, namely, the reciprocal link. The latter produced infeasible results since some of the values of $\hat{p}_m(\mathbf{x})$ at some points of the 3^4 factorial design did not fall inside the interval $[0, 1]$. After examining the scaled deviance values (and also scaled chi-squared values) for all possible nested models,a model with three-factor interaction terms of $k_s\phi\rho$ and $n_s\phi\rho$, together with second order terms of ϕ^2 and ρ^2, was selected for all four methods.

Prediction of $\hat{\pi}_m$ using model (9) is restricted to the region S in (8). Since there are several replications on $\hat{\pi}_m$ at each quadruple (k_s, n_s, ϕ, ρ), the maximum difference between the replicated values of $\hat{\pi}_m$ and the corresponding predicted value $\hat{p}_m$ is used to check the adequacy of fit of the model. These values along with those of $\hat{p}_m$ for all four methods for designs with $k = 7$ are shown in Table 2. For other designs with $k = 4$ and $k = 10$ along with Table 2, the maximum difference values range from -0.031 to 0.067 for $m = MLS$; from -0.016 to 0.071 for $m = TH$; from -0.028 to 0.076 for $m = MHM$; and from -0.014 to 0.023 for $m = BE$. These values provides a good fit to the coverage probability data.

Contour plots of $\hat{p}_m(\mathbf{x})$ for the MLS, TH, BE and MHM methods for fixed values of k and $n.$ are made. For example, Figure ?? is those for the TH method. On the basis of these plots with others, the following conclusions can be made.

Although no single method is best in all situations, the TH and MHM intervals perform well for moderate to large ϕ and ρ values regardless of the sizes of k and $n.$. When both ϕ and ρ values are small, the TH and MHM intervals become liberal in the sense that they produce smaller coverage probabilities than the nominal value. In that case, the MLS interval is useful only if both k and $n.$ are small. The BE interval performs well, but only when ϕ and ρ values are large. Other than that, the BE interval is too conservative. Therefore, we recommend using the TH or

MHM intervals unless both ϕ and ρ values are small. When they are small and the design has small k and $n.$, the MLS interval is recommended.

4 Conclusion

The modeling of the coverage probability of a confidence interval on σ_α^2, and the subsequent plotting of its predicted values provide an effective procedure for comparing designs as well as different methods for constructing such an interval. The plots enable one to visualize the effects of design and values of the variance components on the coverage probability of a particular confidence interval without having to rely on cumbersome or lengthy tabulations of Monte Carlo simulations. The plots can also be helpful in identifying conditions for improving the coverage probability within a region of interest.

Although, in this article, emphasis has been placed on interval estimation of σ_α^2 for the one-way random model, the proposed methodology can be easily extended to higher-order models.

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Table 1: Generated designs and the estimated coverage probabilities for the four confidence intervals on σ_α^2 (nominal confidence coefficient is 0.95).

	Design*		$\hat{\pi}_m$			
(k, n, ϕ, ρ)	$\{n_1, n_2, \cdots, n_{10}\}$	$\phi_\alpha^\dagger$	MLS	TH	BE	MHM
(4, 50,.30,.1)	{ 1, 46, 1, 2 }	.295	.948	.940	.983	.936
(4, 50,.30,.6)	{ 1, 2, 1, 46 }	.295	.932	.952	.961	.951
(4, 50,.30,.9)	{ 1, 2, 46, 1 }	.295	.924	.953	.953	.952
...	...				...	
(4,100,.65,.1)	{ 24, 18, 54, 4 }	.652	.941	.946	.973	.950
(4,100,.65,.6)	{ 31, 12, 5, 52 }	.652	.928	.953	.953	.953
(4,100,.65,.9)	{ 56, 20, 11, 13 }	.653	.935	.949	.949	.949
...	...				...	
(4,500,.95,.1)	{ 81, 127, 161, 131 }	.950	.950	.953	.953	.953
(4,500,.95,.6)	{ 108, 173, 120, 99 }	.950	.948	.950	.950	.950
(4,500,.95,.9)	{ 172, 97, 108, 123 }	.950	.948	.950	.950	.950
...	...				...	
(7, 50,.30,.1)	{ 2, 1, 32, 12, 1, 1, 1}	.304	.938	.934	.990	.913
(7, 50,.30,.6)	{ 3, 6, 2, 34, 2, 2, 1}	.294	.899	.954	.967	.958
(7, 50,.30,.9)	{ 4, 33, 1, 1, 9, 1, 1}	.300	.848	.950	.950	.949
...	...				...	
(7,100,.65,.1)	{ 4, 28, 20, 14, 2, 4, 28}	.649	.933	.932	.981	.929
(7,100,.65,.6)	{ 18, 1, 14, 34, 4, 21, 8}	.650	.910	.949	.962	.953
(7,100,.65,.9)	{ 21, 8, 1, 31, 21, 17, 1}	.650	.891	.950	.950	.951
...	...				...	
(7,500,.95,.1)	{ 79, 64, 50, 81, 101, 53, 72}	.950	.944	.947	.948	.947
(7,500,.95,.6)	{ 40, 76, 67, 90, 90, 60, 77}	.950	.946	.951	.951	.951
(7,500,.95,.9)	{ 56, 100, 86, 64, 74, 72, 48}	.950	.947	.953	.953	.953
...	...				...	
(10, 50,.30,.1)	{ 1, 1, 27, 1, 2, 1, 4, 7, 1, 5}	.302	.936	.929	.994	.916
(10, 50,.30,.6)	{ 12, 1, 2, 1, 1, 2, 2, 26, 2, 1}	.298	.854	.952	.963	.950
(10, 50,.30,.9)	{ 4, 3, 2, 5, 1, 2, 2, 2, 28, 1}	.293	.878	.951	.952	.952
...	...				...	
(10,100,.65,.1)	{ 4, 3, 3, 15, 9, 12, 8, 29, 7, 10}	.650	.941	.944	.986	.944
(10,100,.65,.6)	{ 13, 17, 26, 3, 2, 10, 12, 1, 11, 5}	.650	.909	.948	.962	.951
(10,100,.65,.9)	{ 14, 1, 6, 2, 20, 18, 5, 1, 18, 15}	.651	.890	.946	.947	.947
...	...				...	
(10,500,.95,.1)	{ 55, 56, 33, 46, 41, 63, 39, 51, 73, 43}	.950	.948	.951	.953	.951
(10,500,.95,.6)	{ 30, 56, 66, 49, 46, 61, 32, 61, 54, 45}	.950	.946	.950	.950	.950
(10,500,.95,.9)	{ 35, 30, 62, 70, 42, 50, 57, 54, 47, 53}	.950	.945	.949	.949	.949

* A total of 900 designs such that 210 designs for $k = 4$,

300 designs for $k = 7$ and 390 designs for $k = 10$ are generated.

† ϕ_a denotes the actual value of ϕ for a ϕ-generated design.

Table 2: The predicted coverage probability, $\hat{p}_m$, m =MLS, TH, BE and MHM, and its maximum difference(MD) from the estimated coverage probabilities, $\bar{\pi}_m$, obtained by simulation when $k = 7$.

	Method							
$k = 7$	MLS		TH		BE		MHM	
$n.$, ϕ, ρ	$\hat{p}_{MLS}$	MD	$\hat{p}_{TH}$	MD	$\hat{p}_{BE}$	MD	$\hat{p}_{MHM}$	MD
50 .30 .1	.941	.005	.929	.006	.991	.004	.924	.015
50 .30 .6	.903	.021	.948	-.006	.967	.007	.948	-.009
50 .30 .9	.880	.031	.953	.005	.946	-.008	.955	.007
50 .65 .1	.941	-.007	.940	.025	.985	-.004	.938	.035
50 .65 .6	.922	.014	.949	-.004	.962	.010	.950	-.006
50 .65 .9	.915	.024	.950	-.003	.952	-.002	.951	-.004
50 .95 .1	.950	.004	.950	-.006	.971	.005	.951	-.007
50 .95 .6	.945	.004	.953	.005	.951	-.004	.954	.004
50 .95 .9	.947	.003	.950	.003	.949	-.002	.950	.003
100 .30 .1	.937	.011	.930	.020	.990	.005	.925	.020
100 .30 .6	.900	.053	.948	.004	.966	.012	.949	-.006
100 .30 .9	.879	.050	.953	.006	.946	-.008	.954	.006
100 .65 .1	.939	.007	.940	.013	.984	.003	.939	.012
100 .65 .6	.921	.021	.949	-.004	.961	.013	.950	-.005
100 .65 .9	.915	.023	.950	.003	.952	.005	.951	-.004
100 .95 .1	.950	.006	.950	.005	.970	.012	.951	-.004
100 .95 .6	.945	.004	.953	.007	.950	.005	.954	.008
100 .95 .9	.947	.003	.950	-.003	.950	-.003	.950	-.003
500 .30 .1	.899	.026	.937	-.011	.978	.007	.938	.017
500 .30 .6	.877	.057	.949	.004	.955	.006	.951	-.003
500 .30 .9	.871	.062	.952	.003	.949	-.005	.952	.003
500 .65 .1	.922	.013	.943	.010	.969	.014	.943	.015
500 .65 .6	.912	.020	.949	-.004	.954	.008	.951	.004
500 .65 .9	.913	.020	.949	-.005	.956	.009	.949	-.005
500 .95 .1	.948	.006	.950	-.005	.953	.006	.950	-.005
500 .95 .6	.945	-.005	.952	.005	.945	-.009	.953	.006
500 .95 .9	.948	.005	.950	-.005	.954	.008	.949	-.005