Mathematics Model of Space Backside Resection Based on Condition Adjustment

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Abstract: This paper focuses on the image correction under few GCPs, utilizes the collinearity equation, and builds up this mathematics model of space backside resection based on condition adjustment. Then calculates the adjusted elements of exterior orientation by iteration algorithm, and evaluates the precision. And demonstrates the high-precision, affection and wide-supplying-perspective of this model.

Key words: GCP, collinearity equation, space backside resection, the elements of exterior orientation, iteration algorithm

1. Introduction:

In photogrammetry and remote sensing, the elements of exterior orientation（X_s、Y_s、Z_s、ϕ、ω、κ）are the very important parameters to fix the space position and attitudes of image rays at the moment of imagination. As soon as we get the elements of exterior orientation of each image, we can resume the space geometric relationship at the imaging moment, and rebuild DTM, then realize the high precision orientation. It is always the problem that how to acquire the elements of exterior orientation of images for photogrammetry and RS workers. The main ways are: using radar, GPS, INS, or satellite-borne camera to get these elements, and also can using some GCPs to obtain the elements according to collinearity equation. This kind of mathematics model is called space backside resection.

The traditional space backside resection model is based on the central projection imaging equations. These formulas are:

$$x = -f\bullet\frac{a_{11}(Xp-Xs)+a_{21}(Yp-Ys)+a_{31}(Zp-Zs)}{a_{13}(Xp-Xs)+a_{23}(Yp-Ys)+a_{33}(Zp-Zs)}$$

$$y = -f\bullet\frac{a_{12}(Xp-Xs)+a_{22}(Yp-Ys)+a_{32}(Zp-Zs)}{a_{13}(Xp-Xs)+a_{23}(Yp-Ys)+a_{33}(Zp-Zs)} \quad \cdots\cdots\cdots \quad (1)$$

among: quotients a_{ij} are functions about attitude angles.

The traditional space backside resection is take these 6 elements of exterior orientation as unknown values, and every pair of grip altogether points can set up two equations. If we have 3 GCPs, we can calculate these 6 elements. But in order to improve the dependability of the precision and result of calculation, there must be enough redundant observation, and uses parameter adjustment principle of least squares method to carry on the solution. In the actual computation, usually regards the geographical coordinates of GCPs as the true values, regards the corresponding pixel coordinates as the observation values, adds corresponding correction values. Each control point may set up two error equations, finally forms the total error equation. Then composites the normal equation, solves the correction values of these 6 elements, and obtains the adjusted values of 6 elements of exterior orientation at last.

This method requires at least 3 control points and more than 6 generally. If the control point quantity less than 3, it is unable to get fixed solution, and must seek the new calculation model .

With the full speed development space information technology, the measuring and controlling precision on RS satellite orbit and attitudes raises constantly. Orbit, attitudes and all kinds of parameter information are offered to users in company with RS datum together. Take the SPOT-5 satellite example, which is put into operation newly, the initial data file (Metadata.dim) supplies abundant parameter information about satellite operation, among which includes the elements of exterior orientation data. Adopt fitting method in numerical analysis and statistical method on analyzing these data, set up each high-accuracy parametric equations of these elements which is changed with time, and then fit and calculate the elements values at any satellite imaging moment. These values can use for rough orientation of images directly , and can use as initial values of the elements of exterior orientation in accurate orientation.

In aerial photogrammetry, can acquire high precision 3-dimensional coordinates of camera station on imaging moment using GPS areotriangulation technology.

In a word, in modern aerial survey and RS technology, the elements of exterior orientation can be got through various kinds of methods, but the precision can't yet meet the need for accurate orientation. If take these early got elements as virtual observation values that bring certain weight, and combine few GCPs, then set up the condition equations according to the collinear equation, and adopt the condition adjustment principle under least square theory, calculate out the correction of the 6 elements, and then obtain accurate elements. We call this method as space backside resection based on condition adjustment.

Compared with traditional space resection law to calculate the elements of exterior orientation, adopting condition adjustment model can reduce the number of GCPs. Theoretically, so long as there is one condition, can solve the correction of a lot of observation values. One grip GCP altogether, can set up two condition equations because of the collinear equation. The number of GCPts is smaller than or equaling 3.

2. Mathematics model of space backside resection based on condition adjustment:

set:

$L=(l_1 \quad l_2 \quad l_3 \quad l_4 \quad l_5 \quad l_6)^T$ as corresponding elements of exterior orientation vector;

（X_s、Y_s、Z_s、ϕ、ω、κ）as observation value vector;

$V=(v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6)^T$ as corresponding correction vector;

$$P=\begin{pmatrix} p_{11} & p_{12} & \cdots & p_{16} \\ p_{21} & p_{22} & \cdots & p_{26} \\ \cdots & \cdots & \cdots & \cdots \\ p_{61} & p_{62} & \cdots & p_{66} \end{pmatrix}$$ □ as observation value weight matrix;

linear the (1) formula:

$$\frac{\partial x}{\partial l_1}v_1+\frac{\partial x}{\partial l_2}v_2+\frac{\partial x}{\partial l_3}v_3+\frac{\partial x}{\partial l_4}v_4+\frac{\partial x}{\partial l_5}v_5+\frac{\partial x}{\partial l_6}v_6+w_x=0$$

$$\frac{\partial y}{\partial l_1}v_1+\frac{\partial y}{\partial l_2}v_2+\frac{\partial y}{\partial l_3}v_3+\frac{\partial y}{\partial l_4}v_4+\frac{\partial y}{\partial l_5}v_5+\frac{\partial y}{\partial l_6}v_6+w_y=0$$

To every GCP, can set up two condition equations. The form of matrix as follow:

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}+\begin{bmatrix} w_x \\ w_y \end{bmatrix}=0$$

b_{ij} are the functions of three attitude angles（omited）

set: $\overline{X}=a_1(Xp-Xs)+b_1(Yp-Ys)+c_1(Zp-Zs)$

$\overline{Y}=a_2(Xp-Xs)+b_2(Yp-Ys)+c_2(Zp-Zs)$

$\overline{Z}=a_3(Xp-Xs)+b_3(Yp-Ys)+c_3(Zp-Zs)$

C1= $\overline{X}$ •f+ $\overline{Z}$ •x

C2= $\overline{Y}$ •f+ $\overline{Z}$ •y

Simplify and arrange the formulas then can get simple form condition equations (omited)
According to condition adjustment theory, can get normal equation Naa•K+W=0

among, K is correlate vector, Naa is quotients of normal equations Naa= $BP^{-1}B^T$,W is $\begin{bmatrix} C1 \\ C2 \end{bmatrix}$

then correlate vector K=-Naa $^{-1}$ •W, solve correction vector V=P $^{-1}$ A ^{T} K, and obtain maximum likelihood value of each observation value using $\hat{L} = L + V$

suppose there are **m** GCPs, then can set up **n** condition equations, and **n** = 2×**m**; finally compose total condition equation:

$$\underset{N\times 6}{B}\ \underset{6\times 1}{V} + \underset{N\times 1}{W} = 0$$

according to least square condition adjustment theory, carry out normal equation:

$$N_{aa} \cdot K + W = 0$$

K is correlate vector, Naa is quotients of normal equations Naa= $BP^{-1}B^T$,then correlate vector K=-Naa $^{-1}$ •W
then solve the correction vector V=P $^{-1}$ A ^{T} K, and obtain maximum likelihood value of each observation value using $\hat{L} = L + V$. In order to improve procession, adopt successive adjustment, it means that take the last adjustment values as new observation values and re-adjust, through the iteration algorithm to obtain accurate elements of exterior orientation.

3. Experiment analysis:

1. data resource:

Experiments adopt the certain aero photographs imaged in 2001, the scale of photographs 1/3300. The initial elements of exterior orientation are monocompared by Vizoo 3.2 software of Supersoft Company.

2. results of calculation:

Utilize above-mentioned condition adjustment model to do iterate calculation. Convergence speed of this algorithm is very fast, and the results of the second and third iteration have been 6 steps unanimous behind decimal point.

3. precision of check points:

Select some pass points to evaluate the points precision, and the absolute errors of points are less than 0.5meter.

4. evaluation of object precision:

Utilize the adjusted elements, realize digital difference rectification by program, obtain orthophotos by resampling, and superimpose these orthophotos on corresponding DLG got from digital photogrammetry, select certain number of objects to compare the precision, and then can conclude that the absolute errors of points are less than 0.5meter after image correction.

4. Conclusion:

1.Utilizing this condition adjustment model, can solve the elements of exterior orientation under less than 3 GCPs.
2.Experiments prove the high solution precision of this method, fast convergence speed. And the photos can get better correction using adjusted elements to do digital difference rectification.
3.This method can be extended to the RS image correction under few GCPs, and have important applying means.