A Simulation Study on the Use of GPS Signals to Infer 3-D Atmospheric Wet Refractivity Structure

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Abstract: Atmospheric water vapor is a key variable in numerical weather prediction (NWP) models, but it is a crucial factor to limit the accuracy of high-precision GPS positioning technique. For both issues, knowledge about the amount of water vapor is extremely important. In this study, we perform a simulation study to utilize GPS signals through a developed tomographic scheme to retrieve 3D structure of atmospheric wet refractivity, which may be assimilated into NWP models for advancing forecasting or position calculation for improving GPS positioning accuracy. For the purpose of knowing the absolute accuracy of the developed tomographic method, a well-defined temporal and spatial varying state of atmospheric profile is utilized. Under such circumstance, several factors that may influence the retrievals can be easily examined and their impacts may be clearly quantified. They include the values of the positional dilution of precision (PDOP) factors of the GPS signals, … etc. Based upon the use of a variety spectrum of adjustable factors, many interesting findings are obtained. For example, the more is the number of the observed GPS signals the better becomes the retrievals as expected. Also, the smaller is the PDOP value the better becomes the retrievals..
Keywords: Wet refractivity, Tomography, GPS

1. Introduction

There have been a lot of efforts to develop tomographic methods to retrieve 3D structure of water vapor in the troposphere for the past few years[2][3]. In this study, we present our tomographic model and validate its performance in retrieving 3D wet refractivity of the troposphere by using a well-defined temporally and spatially varying state of atmospheric profile. Several factors that may influence the retrievals are examined and their impacts are quantified.

2. Tropospheric Tomography

Tomographic methods have been developed to reconstruct the distribution of the electron density in the ionosphere[1] for years. Recently, the same concept was used to retrieve the 3D structure of atmospheric wet delay and water vapor [5][6], while due to some intrinsic characteristics of the GPS sensing scheme, several simplifications and assumptions are generally imposed upon the tomographic methods without exceptions. Similarly, we inevitably use some simplifications in our tomographic model. First, in order to prevent errors from the wet slant delay of the atmosphere, a well-defined temporally and spatially varying state of the atmospheric profile is utilized. The wet refractivity can be described as:

$$N_w = \frac{N_{ws}}{Z_w^4}(Z_w - Z)^4 + a(X - X_0) + b(Y - Y_0) \quad (1)$$

where (X,Y,Z) is the coordinate of any point in the study area that we called “voxel”; (X_0,Y_0) represents the coordinate of the center of the simulation area, i.e., National Central University; N_w means the wet refractivity at the coordinate of any point in the voxel; N_{ws} is the wet refractivity on the ground; and Z_w represents the altitude when the wet refractivity is zero. The first term on the right side of the equation (1) is the atmospheric wet refractivity model proposed by Hopfield et al.[4]. The second and third terms on the right side of the equation (1) represent the horizontal gradient for the wet refractivity, where *a* and *b* are arbitrary numbers. Second, the wet refractivity is assumed to be invariant within a certain simulation time interval in order to collect enough observations for performing tomography. In this study, 1 hour is chosen as the time interval.

To reconstruct the 3-D distribution of the atmospheric wet delay, a least square method is utilized to solve the system equations for the unknown vector (matrix) of refractivity within a uniformly-distributed network with 5 by 5 GPS receivers of concern. The reconstruction is possible because a ground network of many GPS receivers that record integrated slant water along ray paths of the electromagnetic waves provide the required information. Distance between two consecutive GPS receivers is 4 km as shown in Figure 1 for a view of 5 consecutive GPS receivers along a straight line. The atmosphere is divided into 8 layers whose thickness is 1 km under 4 km height; and 2 km between 4 km and 10 km to allow more signals passing through the voxels at higher layers. Hence, it is divided into 112 voxels and there are 200 (=5 by 5 by 8) unknowns. The GPS signals that pass through the 112 voxels intersect all over during a certain time interval of interest, and permit the reconstruction of the wet delay. Their paths may be determined given the IGS precise ephemeris data. Then the profile of wet delay along the path can be computed. For easy understanding, a 2-D plot of the GPS signals transecting the voxel is shown in Fig. 2. This straight line can be described as:

$$\frac{x-x_0}{a}=\frac{y-y_0}{b}=\frac{z-z_0}{c}=k \qquad (2)$$

where (x_0,y_0,z_0) represents a reference coordinate; (x, y, z) is the coordinate of any point on the line; (a, b, c) is the unit vector of the line; and k represents an arbitrary point on the straight line. Then, any point on the straight line in the 3-D cube can be determined by three parameters p, q, and r, where:

x component: $$p=\frac{x-x_1}{x_2-x_1}=\frac{x_0+ak-x_1}{x_2-x_1}$$

y component: $$q=\frac{y-y_1}{y_2-y_1}=\frac{y_0+bk-y_1}{y_2-y_1} \qquad (3)$$

z component: $$r=\frac{z-z_1}{z_2-z_1}=\frac{z_0+ck-z_1}{z_2-z_1}$$

Therefore, the atmospheric wet refractivity of any point along the ray path in the voxel of interest can be described as a function of the to-be-solved wet refractivities at the eight corners of the voxel as shown in Fig. 3. That is,

$$N(p,q,r)=\{[(1-p)\times N_{111}+p\times N_{211}](1-q)+ [(1-p)\times N_{121}+p\times N_{221}]\times q\}\times(1-r)+\{[(1-p)\times N_{112}+p\times N_{212}](1-q)+[(1-p)\times N_{122}+p\times N_{222}]\times q\}\times r \qquad (4)$$

where N_{ijk} is the wet refractivity at the position of spatial index (i, j, k); and i, j, k=1, or 2.

3. Initial Set-up and Simulation Results

The location of National Central University is the center of the tomographic GPS network. There are 25 GPS receivers in the simulation area. The size of the study cube is 16 by 16 by 10 km^3. It contains 112 (4 by 4 by 7) voxels. In order to collect enough slant observations for performing tomography, the interval of sampling is chosen to be 1 hour. Given the precise GPS orbital information for the date of investigation June 19, 2001, the paths of slant GPS signals may be determined.

Fig. 4 shows the difference between retrieval and ground truth at every layer for the first 1-hour interval. It shows that the accuracy of the retrievals for the lower layers is better than that for the higher layers. Our explanation is that there are fewer intersections of the GPS ray signals within the higher voxels. These retrievals appear better than those from a case study when the thickness of all layers is assumed to be 1 km, as shown in Fig. 5. Since the water vapor essentially becomes negligible at a height of 10 km, and since the surface meteorological conditions are typically known, two constraints are employed, namely known surface meteorological conditions and null atmospheric wet refractivity conditions at a 10 km height. Fig. 6 shows the results of the simulation with constraints. The accuracy for the top layer is increased obviously. Fig. 7 gives some information about the configuration of the tomography set-up during 24 hours of the simulation. The horizontal axis has two scales -- the bottom is the number of grid points; and the top is PDOP value. The vertical axis represents time. The red straight line indicates that the GPS signals pass through the cube in a simulation interval. The green line is the difference between retrieval and ground truth. The blue line is the PDOP value. The pink dotted line is the number of the observable GPS satellites in the sky.

We found that the more is the observable GPS satellites, the smaller becomes the PDOP value during the same time, the larger is the number of slant GPS observation, and then the better is the accuracy in that interval.

4. Conclusions

1. A better accuracy may be achieved when the size of the voxel at higher altitude becomes larger to allow more GPS signals passing through.
2. Retrievals near surface layers may be improved with proper constraints using known surface meteorological conditions.
3. The more is the observable GPS satellites in the sky, the larger becomes the number of the slant GPS observations, the smaller is the PDOP value at that time, and then the better is the retrieval.

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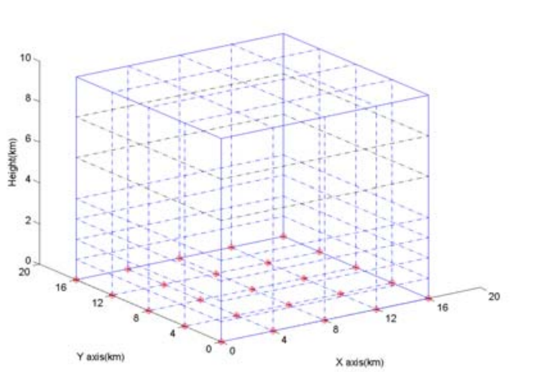

Figure 1. A schematic diagram of the simulation network. Red points represent locations of the GPS receivers.

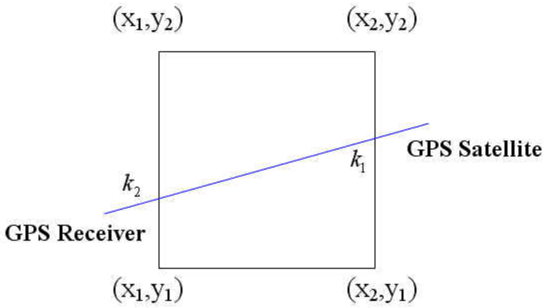

Figure 2. A 2D view of a GPS signal transecting a voxel.

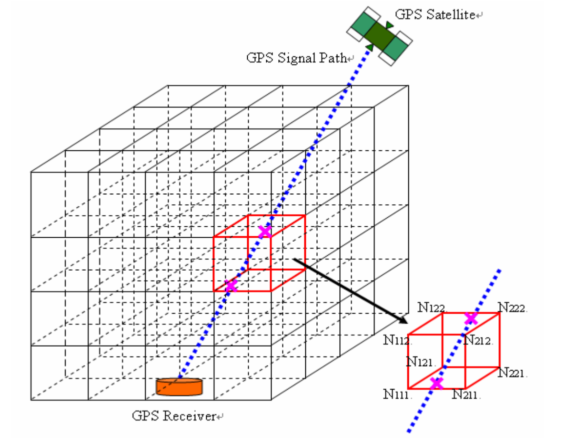

Figure 3. A 3D view of a GPS signal transecting a voxel. N_{ijk} is the wet refractivity at any point of spatial index (i, j, k).

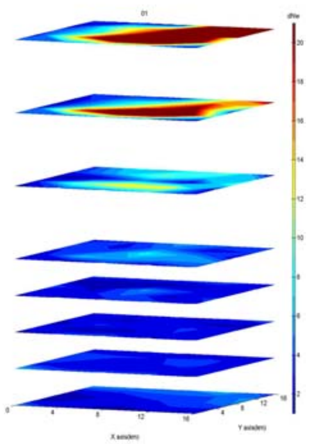

Figure 4. The difference between retrieval and ground truth at 8 layers for the first hour simulation.

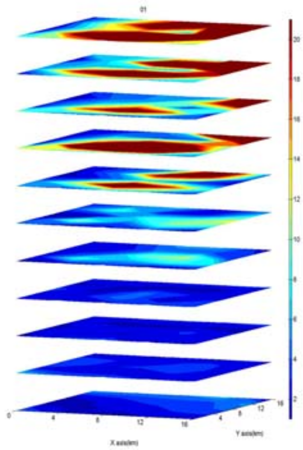

Figure 5. The difference between retrieval and ground truth at 11 layers for the first hour simulation.

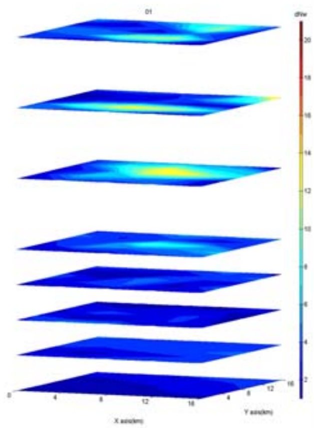

Figure 6. The difference between retrieval and ground truth at 8 layers with constraints for the first hour simulation interval.

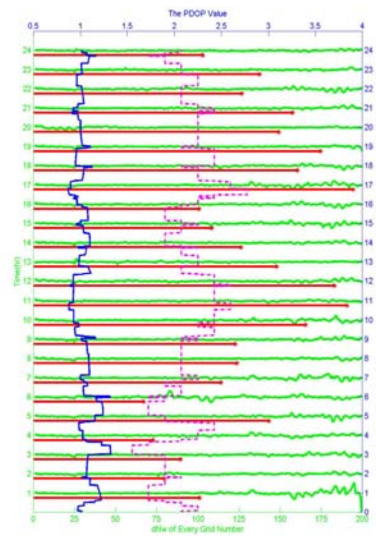

Figure 7. Configuration of the tomography set-up during the 24-hour simulation period:the difference between retrieval and ground truth (green line); PDOP value(blue line); observable GPS satellite number (pink dotted line); GPS bservation number(red straight line)