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Accelerated Subspace Iteration Method for Computing Natural Frequencies and Mode Shapes of Structures

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ABSTRACT

This paper proposes modified subspace iteration method for efficient frequency analysis of structures. Proposed method uses accelerated Lanczos vectors as starting vectors in order to reduce the number of iterations in the subspace iteration method. Proposed method has better computing efficiency than the conventional method when the number of desired frequencies is relatively small. The efficiency of proposed method is verified through numerical examples.

1. Introduction

The determination of natural frequencies and mode shapes is an essential step in the dynamic analysis or buckling analysis of structures. Natural frequencies and mode shapes of structures are determined from eigenvalue analysis. The subspace iteration method[1] and the Lanczos method[2] are efficient eigensolvers for structures. The subspace iteration method has been widely used and various improved versions of the method has been employed. Among them, the subspace iteration method with Lanczos starting subspace[3] is also efficient. The method uses Lanczos vectors as a starting subspace. This paper focuses on the improvement of the method. Improved method is based on squaring the dynamic matrix and similar technique was already applied to quantum problems[4] and simultaneous inverse iteration process in the subspace iteration method[5],[6].

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This paper applied the technique to generation of Lanczos vectors that are used as starting vectors to improve convergence of the subspace iteration method with Lanczos starting subspace.

2. Subspace iteration method with accelerated Lanczos starting subspace

The subspace iteration method with Lanczos starting subspace uses Lanczos vectors as starting iteration vectors and the Lanczos vectors are generated from the following Lanczos algorithm

$$\tilde{\mathbf{x}}_i = \bar{\mathbf{x}}_i - \alpha_i \mathbf{x}_i - \beta_{i-1}\mathbf{x}_{i-1}, \quad \bar{\mathbf{x}}_i = \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i \tag{1}$$

where $\mathbf{K}$, $\mathbf{M}$ and $\mathbf{x}_i$ are stiffness matrix, mass matrix and ith Lanczos vector, respectively, and $\mathbf{K}^{-1}\mathbf{M}$ is called the dynamic matrix. α_i and β_i are scalar coefficients obtained by

$$\alpha_i = \bar{\mathbf{x}}_i^T \mathbf{M}\mathbf{x}_i, \quad \beta_i = (\tilde{\mathbf{x}}_i^T \mathbf{M}\tilde{\mathbf{x}}_i)^{1/2} \tag{2}$$

then the next Lanczos vector is

$$\mathbf{x}_{i+1} = \tilde{\mathbf{x}}_i / \beta_i \tag{3}$$

In this paper, modified Lanczos alogrithm is proposed to generate accelerated Lanczos vectors that are used as staring vectors to improve convergence of the subspace iteration method with Lanczos starting subspace. Modified Lanczos algorithm uses the following modifed Lanczos recursion with squared dynamic matrix

$$\tilde{\mathbf{y}}_i = \bar{\mathbf{y}}_i - \gamma_i \mathbf{y}_i - \delta_{i-1}\mathbf{y}_{i-1}, \quad \bar{\mathbf{y}}_i = (\mathbf{K}^{-1}\mathbf{M})^2 \mathbf{y}_i \tag{4}$$

where $\mathbf{y}_i$ is modifed Lanczos vector. γ_i and δ_i are calculated by

$$\gamma_i = \bar{\mathbf{y}}_i^T \mathbf{M}\mathbf{y}_i, \quad \delta_i = (\tilde{\mathbf{y}}_i^T \mathbf{M}\tilde{\mathbf{y}}_i)^{1/2} \tag{5}$$

and the next Lanczos vector is

$$\mathbf{y}_{i+1} = \tilde{\mathbf{y}}_i / \delta_i \tag{6}$$

Squared dynamic matrix in (4) can separate Riz values more rapidly than the nonsquared dynamic matrix in (1). Therefore, proposed Lanczos starting vectors are closer to exact eigenvector space, resulting in reduction of the number of iterations, than conventional Lanczos starting vectors. Of course, squared dynamic matrix requires additional cost for forward reduction and back-substitution. However, the degree of cost reduction due to less iteration overwhelms that of cost increase due to additional forward reduction and back-substitution. Simultaneous inverse iteration and eigensolution of reduced system of the two methods have the same procedures as the standard subspace iteration algorithm. Algorithm for each method is summarized in Table 1. In Table 1, q is the number of iteration vectors (the size of subspace). If the number of desired eigenpairs is p, q is generally $2p$[7]. In this paper, the following error norm[7] is used to check

convergence.

$$\varepsilon_i = \frac{\| \mathbf{K}\phi_i - \lambda_i \mathbf{M}\phi_i \|_2}{\| \mathbf{K}\phi_i \|_2} \tag{7}$$

Table 1. Summary of algorithm for conventional and proposed subspace iteration methods

Operation	Conventional	Proposed
Factorization	$\mathbf{K} = \mathbf{LDL}^T$	
Generation of starting vectors	Eqs. (1) ~ (3)	Eqs. (4) ~ (6)
Starting subspace	$\mathbf{\Phi}_1 = [\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_q]$	$\mathbf{\Phi}_1 = [\mathbf{y}_1\ \mathbf{y}_2 \cdots \mathbf{y}_q]$
Iteration	$k = 1, 2, \cdots$	
Simultaneous inverse iteration	$\overline{\mathbf{\Phi}}_{k+1} = \mathbf{K}^{-1}\mathbf{M}\mathbf{\Phi}_k$	
System reduction	$\mathbf{K}_{k+1} = \overline{\mathbf{\Phi}}_{k+1}^T \mathbf{K} \overline{\mathbf{\Phi}}_{k+1}$, $\mathbf{M}_{k+1} = \overline{\mathbf{\Phi}}_{k+1}^T \mathbf{M} \overline{\mathbf{\Phi}}_{k+1}$	
Eigensolution for reduced system	$\mathbf{K}_{k+1}\mathbf{Q}_{k+1} = \mathbf{M}_{k+1}\mathbf{Q}_{k+1}\mathbf{\Lambda}_{k+1}$	
Approximate eigenvector space	$\mathbf{\Phi}_{k+1} = \overline{\mathbf{\Phi}}_{k+1}\mathbf{Q}_{k+1}$	
Check convergence	ε_i vs tolerance(usually 10^{-6})	

3. Numerical examples

Two large building structures shown in Fig. 1 are analyzed to verify the effectiveness of the proposed subspace iteration method. The results are compared with those of the subspace iteration method with conventional Lanczos starting subspace. The number of iterations and computing time are examined to compare each method.

Some results are shown in Table 2 and Fig. 2. Table 2 shows iteration counts of the two methods and Fig. 2 compares computing time graphically. As shown in Table 2, subspace iteration method with proposed Lanczos starting subspace has smaller number of iterations than the subspace iteration method with conventional Lanczos starting subspace. Fig. 2 shows that proposed method generally has less computing time than the conventional method.

Proposed method is mucn more efficient when the number of desired eigenpairs is small. However, the proposed method is not better than the conventional method when iteration counts of the two methods are identical. This phenomenon occurs when the number of desired eigenpairs are large. When the number of desired eigenpairs is large, conventional Lanczos starting subspace has already good approximations of exact eigenvector space because the size of starting subspace

is large. In that case, accelerated Lanczos starting subspace has no gains. However, that case occurs only when the number of desired eignepairs is somewhat large. In practical dynamic analysis, the number of required eigenpairs is generally small because a few lower mode shapes are dominant. Therefore, proposed method is practically useful since it has better efficiency when the number of desired eigenpairs is small.

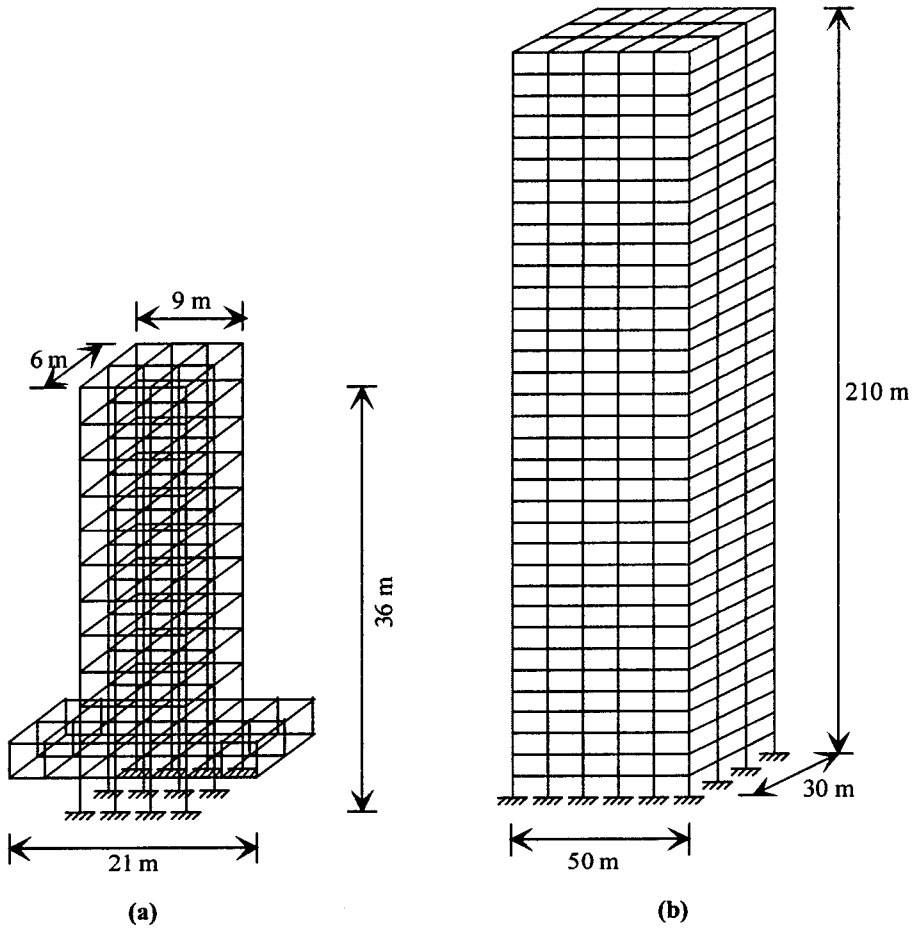

Fig. 1. Example building structures: (a) $E = 2.1\times10^{11}$ Pa, $I = 8.3\times10^{-6}$ m^4, $A = 0.01$ m^2, $\rho = 7850$ kg/m^3, DOF = 1008 (b) $E = 2.1\times10^{11}$ Pa, $I = 8.3\times10^{-6}$ m^4, $A = 0.01$ m^2, $\rho = 7850$ kg/m^3, DOF = 5040

Table 2. Number of iterations

Sturcture (a)			Structure (b)		
No. of desired eigenpairs	Conventional	Proposed	No. of desired eigenpairs	Conventional	Proposed
5	13	13	10	8	4
10	12	8	20	16	5
15	10	2	30	12	1
20	5	1	40	5	1
25	9	1	50	23	1
30	14	1	60	13	1
35	7	1	70	7	1
40	3	1	80	12	1
45	21	1	90	13	1
50	14	1	100	13	1
55	16	1	110	7	1
60	15	1	120	15	1
65	10	1	130	3	1
70	4	1	140	1	1
75	1	1	150	1	1
80	1	1	160	1	1
85	1	1	170	1	1
90	1	1	180	1	1
95	1	1	190	1	1
100	1	1	200	1	1

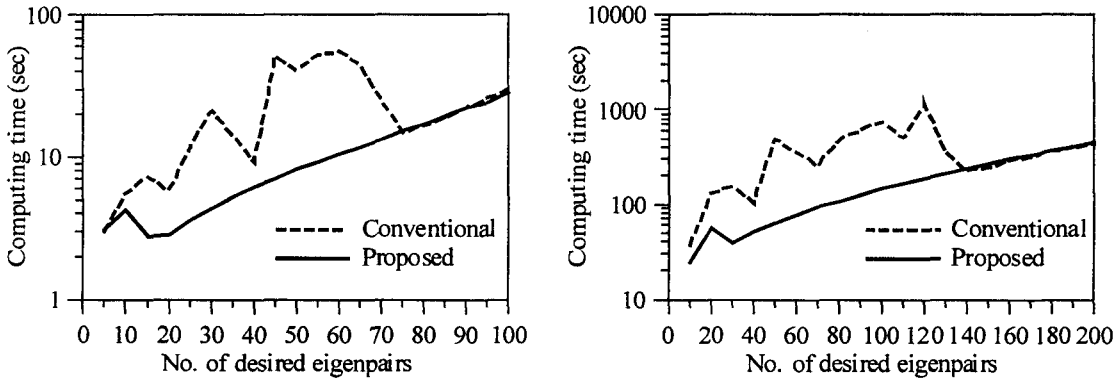

Fig. 2. Comparison of computing time

4. Conclusions

Subspace iteration method with accelerated starting Lanczos subspace is proposed for efficient eigenvalue analysis of structures. From numerical analysis, the characteristics of proposed method

can be summarized as follows:

(1) Subspace iteration method with proposed Lanczos starting subspace has smaller number of iterations than the subspace iteration method with conventional Lanczos starting subspace because squared dynamic matrix in proposed algorithm can accelerate convergence.

(2) Since proposed method has less computing time than the conventional method when the number of desired eigenpairs is small, proposed method is practically efficient.

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