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Nonlinear Analysis of Cable-Stayed Bridges Using Energy Method

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ABSTRACT

This paper presents an energy method for the analysis of the in-plane ultimate load capacity of cable-stayed bridges considering deck and pylon connection. The potential energy of the whole bridge, including bridge deck, stayed cables, and pylons, and the work done by external loads are considered in the development of the bridge energy equation. Both geometric and material nonlinearities are taken into account in the analysis. The method is simple to use and has a high convergence rate.

1. INTRODUCTION

Under the action of external loading, the stiffening girders and pylons of cable-stayed bridges are subjected simultaneously to axial compressive forces and bending moments. They work as beam-columns and should be analyzed by the theory of beam-columns (Chajes 1974). As cable-stayed bridges possess large spans and are very slender, the analysis of such bridges using the theory of beam-columns is very complicated. Due to the complexity of the theory of beam-columns, the stability of stiffening girders, pylons, and the whole cable-stayed bridges has usually been checked by the bifurcation stability theory.

Tang (1976) first derived and calculated the buckling load of cable-stayed bridges using an energy method. Seif and Dilger (1990) conducted in-plane nonlinear analysis and collapse load calculation of prestressed concrete cable-stayed bridges by finite element method. Ermopoulos et al. (1992) performed an elastic stability analysis of a cable-stayed bridge with two pylons by the finite element method. Yan (1994) carried out the analysis of the in-plane ultimate load capacity of long span steel cable-stayed bridges by the finite element method. However, most of the researchers employed the finite element method to carry out the investigation of in-plane stability of cable-stayed bridges. All of the reported research by the energy method is limited to analyze buckling load of cable-stayed bridges by the bifurcation stability theory.

Because the general finite element packages are usually not convenient or cannot conveniently handle the

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special features of cable-stayed bridges, it is desirable to develop a special procedure for cable-stayed bridge analysis. The purpose of this paper is to present an energy method for analysis of the in-plane stability and determination of the overall stability limit load of cable-stayed bridges.

2. METHOD OF ANALYSIS

2.1 Analysis

Consider the typical cable-stayed bridge, as shown in Fig 1, consisting of the decks, two pylons, and cables, which is subjected to concentrated loads p_k and a distributed load q_d. For convenience, three coordinate systems, $X-Y$, Z_1-X_1, and Z_2-X_2 are used for the deck and two pylons in the analysis.

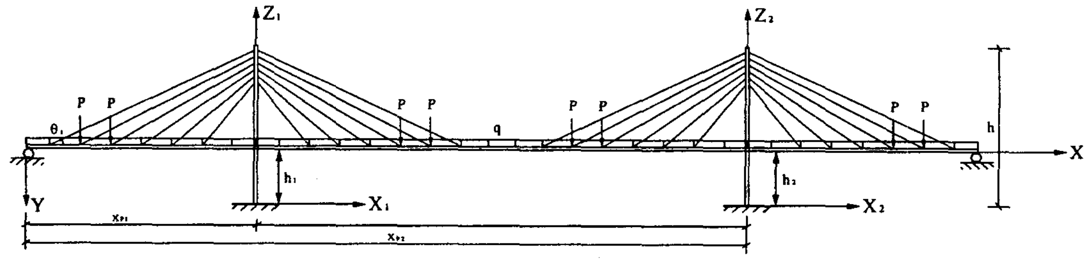

Fig. 1. A Cable-Stayed Bridge

The potential energy of the whole bridge considering the beam-column effect of the deck and pylon can be written as

$$U=\int_0^l \frac{1}{2} E_d I_d \left[y''(x)\right]^2 dx-\int_0^l \frac{1}{2} N_d(x) \left[y'(x)\right]^2 dx$$
$$+\int_0^l \frac{1}{2} E_p I_p(z_1)\left[x_1''(z_1)\right]^2 dz_1-\int_0^h \frac{1}{2} N_{p1}(z_1)\left[x_1'(z_1)\right]^2 dz_1+\int_0^l \frac{1}{2} E_p I_p(z_2)\left[x_2''\right]^2 dz_2-\int_0^h \frac{1}{2} N_{p2}(z_2)\left[x_1'(z_2)\right]^2 dz_2 \qquad (1)$$
$$+\frac{1}{2}\sum_{i=1}^{m_1} \frac{E_{ci}A_{ci}}{l_{ci}}(y_i \sin\theta_i \pm x_{1i}\cos\theta_i)^2+\frac{1}{2}\sum_{i=1}^{m_2} \frac{E_{ci}A_{ci}}{l_{ci}}(y_i \sin\theta_i \pm x_{2i}\cos\theta_i)^2-\sum_{k=1}^{num} P_k y_k-\int_0^l q_d\, y(x)dx$$

where $E_d I_d$ = bending stiffness of the bridge deck in elastic zone; $E_p I_p(z)$ = bending stiffness of the pylon; $N_d(x)$ = axial force in the deck; $N_{p1}(z)$ = axial force in the first pylon; $N_{p2}(z)$ = axial force in the second pylon; $y(x)$ =vertical deflection of the deck at a point distance x from the left end abutment; $x_1(z_1)$ =horizontal deflection of the first pylon at a point distance z_1 from the fixed end of the pylon; $x_2(z_2)$ =horizontal deflection of the second pylon at a point distance z_2 from the fixed end of the pylon; θ_i =slope of cable i to horizontal; m_1 =total number of cables of the first pylon; m_2 =total number of cables of the second pylon; y_i =deck deflection at a point where cable i; y_k =deck deflection at a point where the concentrated load; P_k =concentrated load; q_d =distributed load; *num* =number of concentrated loads; l =deck span between two end abutments; and h =height of the first pylon and the second pylon; m_1 = cable number of the first pylon; m_2 = cable number of the second pylon. The plus sign of the seventh term and the eighth term in (1) is used when the pylon moves horizontally away from cable i, while the minus sign is used when the pylon moves toward cable i (Xi and Kuang, 1999).

For considering the geometric nonlinearity of the cable sag, the equivalent modulus of elasticity of the cable suggested by Ernst(1965) is used

$$E_{ci} = \frac{E}{1 + \frac{E\gamma^2 (l_{ci} \cos\theta_i)^2}{12\sigma^3}} \tag{2}$$

where E is the modulus of elasticity of the cable, γ is the density of the cable, and σ is the cable stress.

The deck deflection y at any point distance x from the left end abutment may be expressed by a trigonometric series

$$y = \sum_{i=1}^{n_1} a_i \sin\frac{i\pi x}{l} \tag{3}$$

and the deflection of the first pylon and second pylon may be expressed by the rigonometric function

$$x_1 = \sum_{i=1}^{n_2} b_i\left(1 - \cos\frac{i\pi z_1}{2h}\right), \qquad x_2 = \sum_{i=1}^{n_3} c_i\left(1 - \cos\frac{i\pi z_2}{2h}\right) \tag{4}$$

where a_i, b_i, and c_i are coefficients. Equation (5) is cable force of in the first pylon and cable force of in the second pylon;

$$c_{f1} = \sum_{i=1}^{n_1} a_i \sin\frac{i\pi x_i}{l}\sin\theta_i \pm \sum_{i=1}^{n_2} b_i\left(1 - \cos\frac{i\pi z_i}{2h}\right)\cos\theta_i \tag{5}$$

$$c_{f2} = \sum_{i=1}^{n_1} a_i \sin\frac{i\pi x_i}{l}\sin\theta_i \pm \sum_{i=1}^{n_3} c_i\left(1 - \cos\frac{i\pi z_i}{2h}\right)\cos\theta_i$$

Equation (6) is axial force of deck in the first pylon and axial force of deck in the second pylon;

$$N_{d1}(x) = \sum_{i=1}^{k} \frac{E_{ci}A_c}{l_{ci}}[c_{f1}]\cos\theta_i \qquad N_{d2}(x) = \sum_{i=1}^{k} \frac{E_{ci}A_c}{l_{ci}}[c_{f2}]\cos\theta_i \tag{6}$$

Equation (7) is axial force of pylon in the first pylon and axial force of pylon in the second pylon;

$$N_{p1}(z_1)=\sum_{i=1}^{k}\frac{E_{ci}A_c}{l_{ci}}\left[c_{f1}\right]\sin\theta_i \qquad N_{p2}(z_2)=\sum_{i=1}^{k}\frac{E_{ci}A_c}{l_{ci}}\left[c_{f2}\right]\sin\theta_i \tag{7}$$

When the bridge deck is simply supported by the pylon, the following constraint conditions at the deck and pylon connection should the satisfied.

$$\psi_1=\frac{1}{2}\int_0^{x_{p1}}\left(y'\right)^2dx+x\Big|_{z=h_1}=0 \qquad \psi_2=\frac{1}{2}\int_0^{h_1}\left(x_1'\right)^2dz_1-y\Big|_{x=x_{p1}}=0 \tag{8a}$$

$$\psi_3=\frac{1}{2}\int_0^{x_{p2}}\left(y'\right)^2dx+x\Big|_{z=h_2}=0 \qquad \psi_4=\frac{1}{2}\int_0^{h_2}\left(x_2'\right)^2dz_2-y\Big|_{x=x_{p2}}=0 \tag{8b}$$

Substituting Eqs (3) and (4) into Eq. (8), the constraint conditions are not satisfied. Therefore, a solution may be the obtained by making a new function as follows

$$V=U-\lambda_1\psi_1-\lambda_2\psi_2-\lambda_3\psi_3-\lambda_4\psi_4 \tag{9}$$

which is required to be the stationary with respect to the constraints of coefficients of a_i, b_i, and c_i, and Lagrangian multiplies of λ_1, λ_2, λ_3, and λ_4.

Equation (9) results in

$$V=U-\lambda_1\left[\frac{1}{2}\int_0^{x_{p1}}\left(\frac{\pi}{l}\sum_{i=1}^{n_1}a_i i\cos\frac{i\pi x}{l}\right)^2dx+\sum_{i=1}^{n_2}b_i\left(1-\cos\frac{i\pi h_1}{2h}\right)\right]-\lambda_2\left[\frac{1}{2}\int_0^{h_1}\left(\frac{\pi}{2h}\sum_{i=1}^{n_2}b_i\sin\frac{i\pi z_1}{2h}\right)^2dz_1-\sum_{i=1}^{n_1}a_i\sin\frac{i\pi x_{p1}}{l}\right]$$

$$-\lambda_3\left[\frac{1}{2}\int_0^{x_{p2}}\left(\frac{\pi}{l}\sum_{i=1}^{n_1}a_i i\cos\frac{i\pi x}{l}\right)^2dx+\sum_{i=1}^{n_3}c_i\left(1-\cos\frac{i\pi h_2}{2h}\right)\right]-\lambda_4\left[\frac{1}{2}\int_0^{h_2}\left(\frac{\pi}{2h}\sum_{i=1}^{n_3}c_i i\sin\frac{i\pi z_2}{2h}\right)^2dz_2-\sum_{i=1}^{n_1}a_i\sin\frac{i\pi x_{p2}}{l}\right] \tag{10}$$

The stationary conditions require that

$$\frac{\partial V}{\partial a_j}=0 \qquad \frac{\partial V}{\partial b_j}=0 \qquad \frac{\partial V}{\partial c_j}=0 \qquad \frac{\partial V}{\partial \lambda_j}=0 \qquad (j=1,2,3,4) \tag{11}$$

Equation (11) represent a set of $(n_1+n_2+n_3+4)$ equations and can be expressed as matrix form.

2.2 Material Nonlinearity

Deck and Pylon

The cross section of the bridge deck can be divided into many elements, as shown in Fig. 2. The strain of and point in the cross section is expressed as

$$\varepsilon_i = \varepsilon_0 + \phi y_i \tag{12}$$

where ε_i is the strain of any point in the cross section; ε_0 is the axial strain of the deck; ϕ is the curvature of the cross section. The stress-strain relationships are given by

$$\sigma_i = \varepsilon_i E \ \text{ for} \left|\varepsilon_i\right| \le \varepsilon_y \qquad \sigma_i = \sigma_y \ \text{ for } \varepsilon_i > \varepsilon_y \qquad \sigma_i = -\sigma_y \ \text{ for } \varepsilon_i < -\varepsilon_y \tag{13}$$

where σ_y is the yield stress; ε_y is the yield strain.

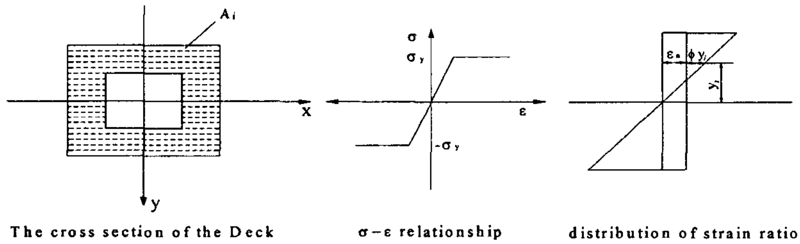

Fig. 2. $\varepsilon_i - \sigma_i$ Relationship

The stresses and strains on any element A_i are approximated by the stress and strain of the center of the element. The equilibrium conditions

$$N = \sum_A \sigma_i A_i \qquad\qquad M = \sum_A \sigma_i A_i y_i \tag{14}$$

should be satisfied. The number of elements will affect the precision of calculation results and work load.

When pylons work from the elastic stage to elasto-plastic stage during the procedure of loading, the problem can be solved by dividing the pylons into a number of elements along the z direction, with similar treatment for the deck.

Cable

When the first cables work at the elasto-plastic stage, the cable energy caused by the cables will be

$$\alpha\frac{1}{2}\sum_{i=1}^{n}\frac{E_{ci}A_c}{l_{ci}}(y_i\sin\theta_i \pm x_{li}\cos\theta_i)^2+\beta\sum_{i=1}^{n}\left[(y_i\sin\theta_i \pm x_{1i}\cos\theta_i)\sigma_{cy}-\frac{1}{2}\sigma_{cy}\varepsilon_{cy}l_{ci}\right]A_c \qquad (15)$$

at the elastic stage: $\alpha=1$, $\beta=0$

at the elasto-plastic stage: $\alpha=0$, $\beta=1$

where σ_{cy} is the yield stress of cables; ε_{cy} is the yield strain of cables.

3. A Cable-Stayed Bridge with Two Pylon of Considering Connection

This example haves eight concentrated loads of $p=5kN$ at a distance of 0.8m, 1.2m, 4.8m, 5.2m, 7.2m, 7.6m, 11.2m and 11.6m from the left abutment, respectively, and a uniformly distributed load of $q=3kN/m$ over the full span are applied to the bridge deck, as shown in Fig. 1. Note $X_{P1}=3.0$m, $X_{P2}=$ 9.4m and the total length is 12.4m. The stiffness of structural components and geometric data of the bridge are listed in Table 1. In these comparisons, Program considers nonlinearities of beam-column effect and cable sag, while MIDAS program uses linear analysis method.

Table 1. Material and Sectional Properties of Bridge (2-Pylon System)

		Elastic modulus $E\ (MPa)$	Area $A\ (m^2)$	Moment of inertia $I\ (m^4)$
cable		1.967×10^5	3.926×10^{-5}	-
deck		2.074×10^5	1.02×10^{-3}	1.66×10^{-7}
pylon	upper	2.074×10^5	2.04×10^{-3}	2.012×10^{-6}
	lower		3.7×10^{-3}	7.253×10^{-6}

The maximum deflection occurs in the mid-span of the bridge as shown in Fig. 3. However, the bridge deck deflects upwards in the side parts. This is caused by the effect of cable tension force. Figure 4 shows the bending moment of bridge deck. Due to the symmetry of the loading, The both side values of bending moment are almost the same. Figure 5 and 6 show the axial force of bridge deck and tension force of cables. Areas of pylon have maximum values due to the action of horizontal components of the cable forces. Figure 7 and 8 show the deflection and axial force of pylon. The deflection at the connection between deck and pylon has values of zero.

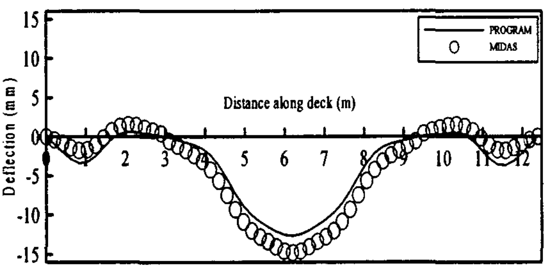

Fig. 3. Deflection of Bridge Deck Fig.

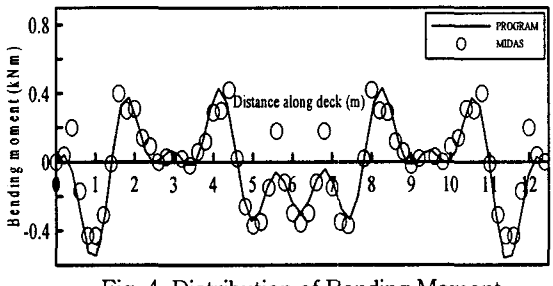

Fig. 4. Distribution of Bending Moment

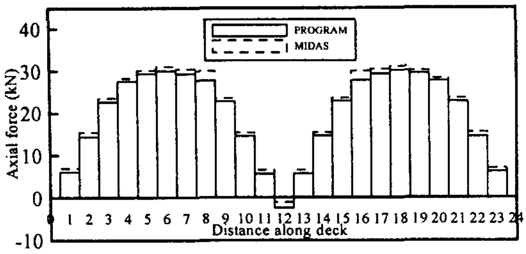

Fig. 5. Distribution of Axial Force

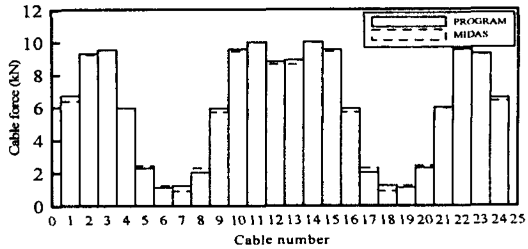

Fig. 6. Tension Force of Cable

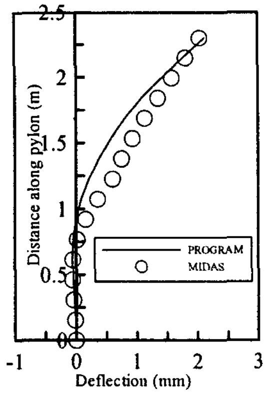

Fig. 7. Deflection of Bridge Pylon

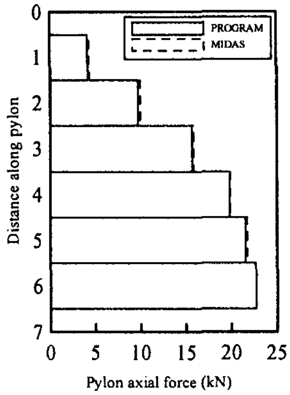

Fig. 8. Axial Force of Bridge Pylon

4. Load-Carrying Capacity of a Cable-Stayed Bridge

An example cable-stayed bridge was used to analyze its load-carrying capacity. This is a steel cable-stayed bridge model. Figure 9 shows the vertical deflection curves of the bridge deck under different load levels considering geometric and material nonlinearities. The displacement near the load positions has the same direction as that of the loads. The extreme fibber yields in $p = 16.45kN$. It can be seen that the loads could not be increased more than $p = 16.45kN$, but the deflections increase at this load level.

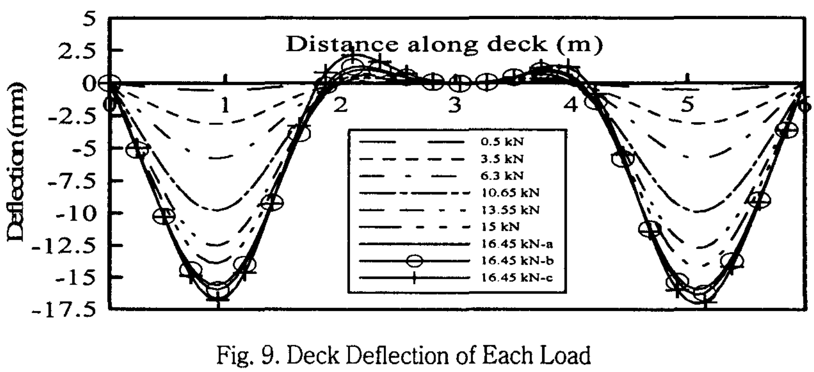

Fig. 9. Deck Deflection of Each Load

5. Conclusion

The proposed energy method may be considered as a simple and efficient, yet accurate, method for the in-plane nonlinear analysis of cable-stayed bridge. This approach can be employed to calculate effectively the in-plane load-carrying capacity of steel cable-stayed bridges. In this study, the potential energy of the whole bridge, including the bridge deck, stayed cables and pylons, and the work done by external loads are considered in the development of the bridge energy equation. A trigonometric series has been used to express the deformation of bridge deck. It satisfies all the boundary conditions of the bridge deck. The trigonometric function has been used to express the deflection of the bridge pylons. It satisfies the boundary condition of the bridge pylon.

Iteration procedures have been adopted to obtain the coefficients of trial functions for the deflection of the bridge deck and pylons. This paper describes problem solution for the connection of deck and pylon. Also, this paper presents method to solve nonlinear simultaneous equations that occur when considering the connection deck and pylon.

This paper compares results of energy method with common finite element method program. The results that compare linear analysis and nonlinear analysis considering beam-column effect and cable sag are evaluated. Analysis results of considering connection deck and pylon are presented. The method that analyze ultimate strength considering geometric and material nonlinearies is presented.

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