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Finite Element Analysis for Free Vibration of Laminated Plates Containing Multi-Delamination

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ABSTRACT

In this proposed work, computational, finite element model for multi-delaminated plates will be developed. In the current analysis procedures of multi-delaminated plates, different elements are used at delaminated and undelaminated region separately. In the undelaminated region, plate element based on Mindlin plate theory is used in order to obtain accurate results of out-of-plane displacement of thick plate. And for delaminated region, plate element based on Kirchhoff plate theory is considered. To satisfy the displacement continuity conditions, displacement vector based on Kirchhoff theory is transformed to displacement of transition element. Element mass and stiffness matrices of each region (delaminated, undelaminated and transition region) will be assembled for global matrix.

1. Introduction

Delamination may result in the degradation of strength overall stiffness of the laminate, especially in the structure of the laminated construction under compression (Pavier and Clarke, 1995). Tracy and Pardoen (1989) show the effect of delamination on the natural frequencies of composite laminates. In their works, the frequency of a particular mode of vibration degraded by a delamination. Degradation of the frequency depends on the size and location of the delamination in the laminates.

During last two decades, many researches on one-dimensional delamination problems have been made (Wang et al., 1982; Shen and Grady, 1992; Lee et al., 2002; Lee et al., 2003; Park et al., 2003). However, studies on vibration analysis of delaminated plates are very few compared with one-dimensional analysis. The first work for vibration of laminated plates with delamination was Tenek et al. (1993). In their work, a three-dimensional finite element method was used to analyze the natural frequencies of delaminated composite.

Such a three-dimensional analysis is accurate and instructive, but is very computationally intensive. Thus, Ju et al. (1995) presented a two-dimensional finite element approach for the analysis of free vibration of laminated composite plates with multiple delaminations.

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In their work, an eight-noded, forty-degree of freedom isoparametric plate element is formulated based on Mindlin plate theory. Presented formulation by Ju et al. (1995) includes the effect of transverse shear deformations as well as the bending-extension coupling effect caused by the presence of delaminations.

Gim (1994) developed a plate finite element based on a lamination theory, which includes the effect of transverse shear deformation. In modeling 2-D delaminations in laminated plates, the global region is modeled by a single layer of plate elements while the delaminated region is modeled by two layers of plate elements whose interface contains the delamination. To ensure the compatibility of deformation and equilibrium of resultant forces and moments at the delamination crack tip, a multipoint constraint algorithm has been developed and incorporated into the finite element code. Parhi et al. (2000) extend the simple model proposed by Gim (1994) to the general case of a laminated composite plate having arbitrarily located multiple delaminations for analyzing its dynamic behavior. The finite element dynamic equations are accordingly derived based on eight-noded isoparametric quadratic elements.

Many finite elements for modeling of laminated composite plates have been formulated for a long time. There are many plate theories existed to develop the finite plate element. Many plate elements are based on Kirchhoff and Mindlin plate theory. The main difference between Kirchhoff and Mindlin plate theory is account of transverse shear deformation (Whitney and Pagano, 1970). Mindlin plate theory costs much computational efforts than Kirchhoff theory but gets more exact results especially thick laminated plates (Reddy, 1997).

In this proposed work, computational, finite element model for multi-delaminated plates is developed. In the current analysis procedures of multi-delaminated plates, elements used in the delaminated and undelaminated regions are based on Mindlin and Kirchhoff plate theories, respectively. To satisfy the displacement continuity conditions, displacement vector based on Kirchhoff theory is transformed to displacement of transition element. Element mass and stiffness matrices of each region, that is delaminated, undelaminated and transition regions are assembled for global matrix.

2. Element for Delaminated Region

Figure 1 shows a delaminated composite plate where the delaminations are presumed to be parallel to the mid-plane of the plate. The global coordinate system is located at the mid-plane of the plate with the z-axis perpendicular to it. Figure 1 shows the four elements used to model a region with three delaminations.

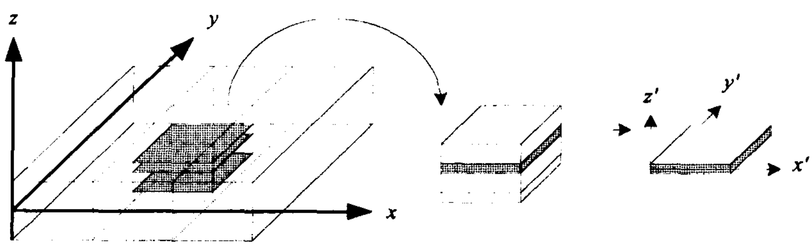


Figure 1. Laminated plate with rectangular delaminations, global (*xyz*) and local (*x'y'z'*) coordinates.

The displacement field of the element is assumed to be of the following form relative to its own local coordinate system.

$$u'(x', y', z') = z'\varphi_x$$
$$v'(x', y', z') = z'\varphi_y \quad (1)$$
$$w'(x', y', z') = w_0'(x', y')$$

where $\varphi_x = -\partial w_0'/\partial x'$ and $\varphi_y = -\partial w_0'/\partial y'$. $w_0'(x', y')$ is the displacement of the middle surface in the z'-direction.

Using the four nodes, twelve degree of freedoms plate element, shown in figure2, the displacement field can be interpolated as

$$w'(x', y', z') = \sum_{i=1}^{4} N_i(\xi, \eta) w_{0i}' \quad (2)$$

where φ_{xi} and φ_{yi} are the nodal rotations and w_{0i}' is the nodal translations. $N_i(\xi, \eta)$ is the shape functions.

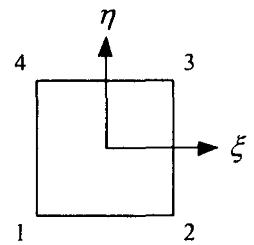


Figure 2. Four nodes plate element.

The strain can be expressed as

$$\boldsymbol{\varepsilon}' = \mathbf{B}'\mathbf{d}' \quad (3)$$

where $\boldsymbol{\varepsilon}'$, $\mathbf{d}'$ and $\mathbf{B}'$ are strain, displacement vectors and strain-displacement matrix, respectively, which are defined by

$$\boldsymbol{\varepsilon}' = \begin{bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} \end{bmatrix}^{\mathrm{T}} \quad (4)$$

$$\mathbf{d}' = \begin{bmatrix} \mathbf{d}_1' & \mathbf{d}_2' & \mathbf{d}_3' & \mathbf{d}_4' \end{bmatrix}^{\mathrm{T}} \quad (5)$$
$$\mathbf{B}' = \begin{bmatrix} \mathbf{B}_1' & \mathbf{B}_2' & \mathbf{B}_3' & \mathbf{B}_4' \end{bmatrix} \quad (6)$$

where, superscript T stands for transpose matrix. The stress-strain relation for the k-th lamina is

$$\boldsymbol{\sigma}' = \mathbf{Q}^{k}{}'\boldsymbol{\varepsilon}' \quad (7)$$

where $\boldsymbol{\sigma}'$ and $\mathbf{Q}^{k}{}'$ are stress vector and transformed reduced stiffness matrix of the k-th lamina, respectively.

The element strain energy $U^{e}{}'$ and element kinetic energy $T^{e}{}'$ are given by

$$U^{e}{}' = \mathbf{d}'^{\mathrm{T}}\mathbf{k}'\mathbf{d}' \quad (8)$$
$$T^{e}{}' = \dot{\mathbf{d}}'^{\mathrm{T}}\mathbf{m}'\dot{\mathbf{d}}' \quad (9)$$

where $\dot{\mathbf{d}}'$ is the element nodal velocity vector and dot stands for differentiation with respect to time of vectors. $\mathbf{k}'$ and $\mathbf{m}'$ are the element stiffness and mass matrixes, respectively, which are defined by

$$\mathbf{m}' = \int_{-1}^{1}\int_{-1}^{1}\left(\sum_{k=1}^{n}\int_{z_{k-1}'}^{z_k'} \mathbf{N}'^{\mathrm{T}} \rho^{k} \mathbf{N}' \mathrm{d}z'\right) \det|J| \,\mathrm{d}\xi\,\mathrm{d}\eta \quad (10)$$

$$\mathbf{k}' = \int_{-1}^{1}\int_{-1}^{1}\left(\sum_{k=1}^{n}\int_{z_{k-1}'}^{z_k'} \mathbf{B}'^{\mathrm{T}} \mathbf{Q}^{k}{}' \mathbf{B}' \mathrm{d}z'\right) \det|J| \,\mathrm{d}\xi\,\mathrm{d}\eta \quad (11)$$

where

$$\mathbf{N}' = \begin{bmatrix} \mathbf{N}_1' & \mathbf{N}_2' & \mathbf{N}_3' & \mathbf{N}_4' \end{bmatrix} \quad (12)$$

$$\boldsymbol{\rho}^k = \begin{bmatrix} \rho^k & 0 & 0 \\ 0 & \rho^k & 0 \\ 0 & 0 & \rho^k \end{bmatrix} \tag{13}$$

$$\mathbf{N}_i' = \begin{bmatrix} z'N_i & 0 & 0 \\ 0 & z'N_i & 0 \\ 0 & 0 & N_i \end{bmatrix}, \quad (i = 1 \sim 4) \tag{14}$$

and n, ρ^k and $\det|J|$ are the number of layers in the delaminated element, the average density of the k-th lamina and the determinant of the Jacobian transformation matrix.

3. Element for Undelaminated Region

When shear deformation is important, it cannot be assumed that normals to the middle surface remain normal to it. Based on the Mindlin plate theory, the displacement field is given by

$$\begin{aligned} u(x, y, z) &= z\theta_x \\ v(x, y, z) &= z\theta_y \\ w(x, y, z) &= w_0(x, y) \end{aligned} \tag{15}$$

where θ_x , θ_y and $w_0(x, y)$ are the rotations of a transverse normal about the y- and x-axes, and displacement of the middle surface in the z-direction, respectively.

The strain-displacement and stress-strain relation for the k-th lamina are expressed as

$$\boldsymbol{\varepsilon} = \mathbf{Bd} \tag{16}$$

$$\boldsymbol{\sigma} = \mathbf{Q}^k \boldsymbol{\varepsilon} \tag{17}$$

where $\boldsymbol{\varepsilon}$, $\mathbf{d}$, $\boldsymbol{\sigma}$, $\mathbf{B}$ and $\mathbf{Q}^k$ are strain, displacement, stress vectors, strain-displacement matrix and transformed reduced stiffness matrix of the k-th lamina based on Mindlin plate theory, respectively. Strain and stress vectors of the undelaminated region have transverse shear strain and stress terms, respectively, which are neglected in the delaminated region. Thus, reduced stiffness matrix of the undelaminated region has 5×5 size.

The element stiffness and mass matrixes based on the Mindlin plate theory can be obtained as

$$\mathbf{m} = \int_{-1}^{1} \int_{-1}^{1} \left(\sum_{k=1}^{m} \int_{z_{k-1}}^{z_k} \mathbf{N}^{\mathrm{T}} \boldsymbol{\rho}^k \mathbf{N} \, \mathrm{d}z \right) \det|J| \, \mathrm{d}\xi \, \mathrm{d}\eta \tag{18}$$

$$\mathbf{k} = \int_{-1}^{1} \int_{-1}^{1} \left(\sum_{k=1}^{m} \int_{z_{k-1}}^{z_k} \mathbf{B}^{\mathrm{T}} \mathbf{Q}^k \mathbf{B} \, \mathrm{d}z \right) \det|J| \, \mathrm{d}\xi \, \mathrm{d}\eta \tag{19}$$

where m is the number of layers in the undelaminated element.

4. Element for Transition Region

In the previous sections, the elements in the delaminated region and undelaminated region are derived separately. However, at the boundary connecting two regions, the continuity conditions for the displacement field must be satisfied.

Displacements of the middle surface in the z-direction of the two regions are same. Let the common nodes at the connection boundary are denoted i. For common nodes i, generalized displacement of the delaminated region is transformed as follows.

$$\mathbf{d}_{Ti} = \mathbf{T}_i \mathbf{d}_{Mi} \tag{20}$$

where subscript T denotes the transition region. $\mathbf{T}_i$ and $\mathbf{d}_{Ti}$ are the nodal transformation matrix and nodal displacement vector of the transition element, respectively. Subscript M stands for the Kirchhoff elements.

Using the displacement field of the element, transformation of the displacement vector can be written as

$$\mathbf{d}_T = \mathbf{T}\mathbf{d}_M \tag{21}$$

where, if the second and third nodes of the element are common nodes, transformation matrix $\mathbf{T}$ is expressed as follows.

$$\mathbf{T} = Diag[\mathbf{I} \quad \mathbf{T}_2 \quad \mathbf{T}_3 \quad \mathbf{I}] \tag{22}$$

where $\mathbf{I}$ and $Diag$ stand for the identity matrix and diagonal matrix, respectively.

Stiffness and mass matrices are expressed as follows for each region.

$$\mathbf{K} = \sum_{k=1}^{l} \left\{ \begin{array}{ll} \mathbf{T}^T \mathbf{k}_K \mathbf{T} & \text{(for trasition region)} \\ \mathbf{k}_M & \text{(for undelaminated region)} \\ \mathbf{k}_K & \text{(for delaminated region)} \end{array} \right\} \tag{23}$$

$$\mathbf{M} = \sum_{k=1}^{l} \left\{ \begin{array}{ll} \mathbf{T}^T \mathbf{m}_K \mathbf{T} & \text{(for trasition region)} \\ \mathbf{m}_M & \text{(for undelaminated region)} \\ \mathbf{m}_K & \text{(for delaminated region)} \end{array} \right\} \tag{24}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the global stiffness matrix and mass matrix, respectively. Subscript K and l stand for the Kirchhoff elements and the total number of element in the global model.

5. Eigenvalue Equation

The application of Hamilton's principle to the Lagrange's equations gives

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{K}\mathbf{d} = 0 \tag{25}$$

From above equations, the following eigenvalue equation is obtained.

$$(\mathbf{K} - \omega^2 \mathbf{M})\Delta = 0 \tag{26}$$

where ω and Δ are the natural frequency and corresponding mode shape, respectively.

6. Numerical Analysis

Using the commercial finite element program ABAQUS (Hibbitt, Karlsson and Sorensen, 2003), eigenvalue analysis of the laminated plate is performed. Length of square plate is 1,800 mm. The plate is made of 20 plies of graphite-epoxy in a $[(+45/-45)_5]_s$ lay-up. Each ply has a thickness of 18 mm; thus, the total plate thickness is 360 mm. The initial elastic ply properties are longitudinal modulus 135,000 MPa, transverse modulus 13,000 MPa, shear modulus 6,400 MPa, and Poisson's ratio 0.38. Four edges of the plate are all clamped. Delamination is located at the center of plate, and delamination size is 600×600 mm. Figure 3 shows the effect of delamination to eigenvalue of plates. The eigenvalue of delaminated plate are lower than sound model. In the figure 4, only one delamination is located at the middle height of the plate. For the application of the presented theory to numerical model, S4R and S4R5 element of the ABAQUS are used. S4R element is the general-purpose elements that are valid for thick problems. However, S4R5 elements formulation is based on Kirchhoff plate theory and neglects the transverse shear flexibility. Thus, S4R and S4R5 elements are used for undelaminated and delaminated regions, respectively. To employ the present theory to numerical model, keyword *MPC (Hibbitt , Karlsson and Sorensen, 2003) in ABAQUS is used. The degrees of freedoms of the Mindlin based plate element at the connecting boundary are constrained to degree of freedoms of Kirchhoff based plate element. Figure 4 shows the eigenvalue of the plate analyzed by the Kirchhoff plate theory, Mindlin plate theory and presented hybrid theory.

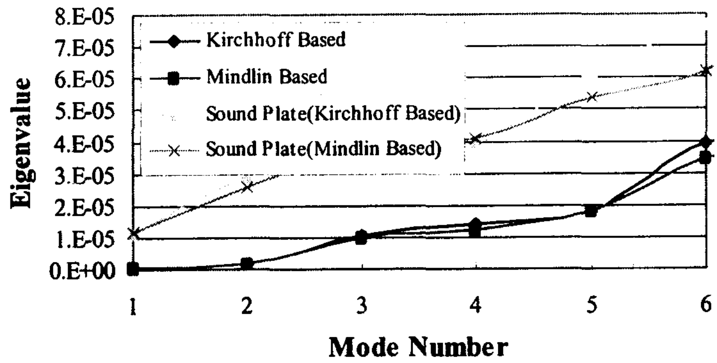


Figure 3. The effect of delamination on the eigenvalue of the plates.

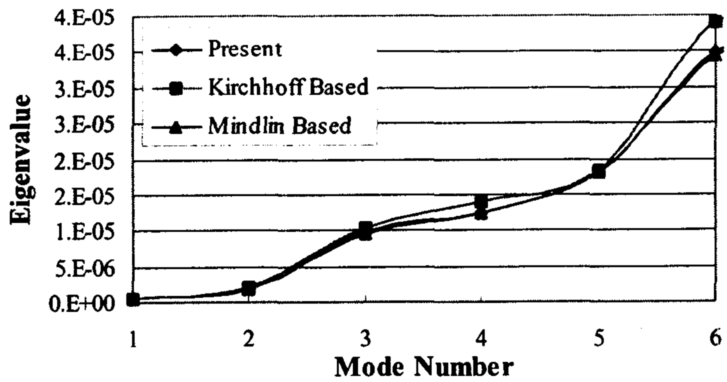


Figure 4. Eigenvalue analysis.

From the figure 4, Kirchhoff based model have higher eigenvalue than Mindlin based model. Eigenvalue of the presented model is closed to Mindlin based model. Total computation time of present model is more than 14% lower than the Mindlin based model. Present model is more accurate than Kirchhoff based model and more efficient than Mindlin based model.

Figure 5 shows the first four mode shapes of a plate.

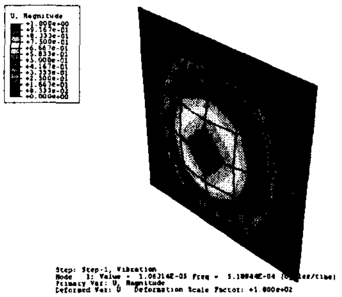
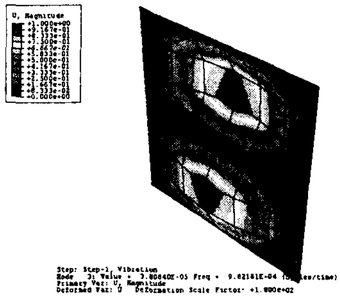
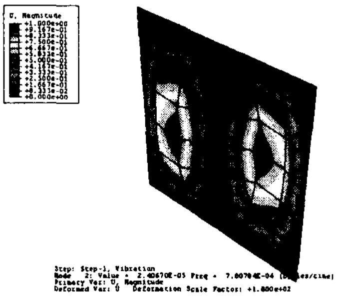
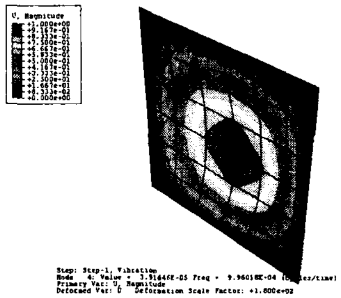
Mode Number	Mode Shape	Mode Number	Mode Shape
1		3	
2		4	

Figure 5. Mode shapes of plate with delamination.

7. Conclusions

In the current analysis procedures of multi-delaminated plates, different elements are used at delaminated and undelaminated region separately. In the undelaminated region, finite element based on Mindlin plate theory is used in order to obtain accurate results of out-of-plane displacement of thick plate. And for delaminated region, plate element based on Kirchhoff plate theory is considered. To satisfy the displacement continuity conditions, displacement vector based on Kirchhoff theory is transformed to displacement of transition element. Element mass and stiffness matrices of each region, that is delaminated, undelaminated and transition regions are assembled for global matrix. From the numerical analysis, Kirchhoff based model have higher eigenvalue than Mindlin based model and eigenvalue of the presented model is closed to Mindlin based model. Present model is more accurate than Kirchhoff based model and more efficient than Mindlin based model. Forthcoming papers, parametric studies for various models including boundary conditions, size and position of delaminations and lay-up effects will be presented.

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