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FINITE ELEMENT METHOD - AN EFFECTIVE TOOL FOR ANALYSIS OF SHELL

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Abstract

This paper deals with the problems and their possible solutions in the development of finite element for analysis of shell. Based on these solution schemes, a series of flat shell elements are established which show no signs of membrane locking and other defects even though the coarse meshes are used. In the element formulation, non-conforming displacement modes are extensively used for improvement of element behaviors. A number of numerical tests are performed to prove the validity of the solutions to the problems involved in establishing a series of high performance flat shell elements. The test results reveal among others that the high accuracy and fast convergence characteristics of the elements are obtainable by the use of various non-conforming modes and that the 'Direct Modification Method' is a very useful tool for non-conforming elements to pass the patch tests. Furthermore, hierarchical and higher order non-conforming modes are proved to be very efficient not only to make an element insensitive to the mesh distortion but also to remove the membrane locking. Some numerical examples are solved to demonstrate the validity and applicability of the presented elements to practical engineering shell problems.

Introduction

Shells are regarded as one type of the most complicated structures to analyze in general. Each of a number of shell theories, i.e. the deep, shallow or flat shell theories, is associated with particular assumptions. Theoretical solutions obtained with these different shell theories may not coincide and the assumptions introduced in shell theories may cause significant errors in occasional cases. The finite element methods have been successfully used and become indispensable tool in the analysis of shell structures. The accuracy of finite element analysis much depends on the shell elements used and modeling schemes.

Due to the simplicity in formulation, the effectiveness of computation, and the flexibility in applications to both shells and folded plate structures, flat shell elements are used extensively in many engineering practices. The flat shell elements can be effective and accurate only when both the membrane and plate bending components of the shell are equally accurate and robust.

A number of researchers worked on the development of the perfect shell element in the past. Providas and Kattis (2000) examined two types of flat triangular shell elements which have six-degrees-of-freedom per node. The study of Batoz *et al.* (2000) was restricted to thin plate/shell and did not include the warped geometry, which can be effectively taken into account by 'rigid link correction' (Taylor 1987). Groenwold and Stander (1995) developed the 4-node flat shell element which has 6 DOF per node and uses 5-point quadrature scheme to reduce the membrane locking phenomenon. Choi *et al.* (1999) also presented an efficient 4-node flat shell element (NMS-4F), in which the Allman-type shape function is used to approximate the membrane behavior. Their elements do not show any spurious zero energy mode, show a good convergence, pass the patch test, and do not show shear locking phenomenon. A minor shortcoming of aforementioned flat shell elements (Groenwold and Stander 1995; Choi *et al.* 1999) appears to have a tendency toward the membrane locking especially when coarse meshes are used. Recently, Choi and Lee (2003) presented a scheme to remove the membrane locking completely using hierarchical and higher order non-conforming(NC) modes which is highly efficient even though coarse meshes are used.

In this paper, problems in developing the high-performance quadrilateral flat shell elements are reviewed and their possible solutions are discussed. Two new solution schemes were specifically introduced in addition to the existing schemes; (1) introduction of hierarchical and higher order non-conforming displacement modes in addition to the basic NC modes, and (2) application of Direct Modification Method [9] to the flat shell formulation. The appropriate remedial schemes, among the existing and/or newly introduced schemes, are selectively merged into the formulation of a new series of defect-free flat shell elements in a complementary manner. The hierarchical NC modes used in this study for the membrane component of flat shell element which replaces the Allman-type formulation, improve the membrane behavior significantly and thus eliminate the membrane locking.

A number of practical shell/folded plate problems, i.e. a roof shell, a box with holes, and in-ground LNG storage tank, were analyzed to show the validity of the presented high performance flat shell elements.

Problems and solutions in development of defect-free flat shell

A large number of researchers attempted to develop a defect-free flat shell element in the past (Taylor 1987; Cook 1994; Groenwold and Stander 1995; Choi *et al.* 1999; Providas and Kattis 2000). From these works it appears that the perfect flat shell element should satisfy the following requirements: 1) To be easy to formulate and implement, 2) To be highly accurate and converge fast to the exact solutions, 3) To be insensitive to element distortions, 4) To be free from patch test failure, 5) To show no shear locking, 6) To show no membrane locking, 7) To possess six degrees of freedom per node, 8) To contain no spurious zero-energy modes, and 9) To be able to handle the warped geometry.

At this time, the perfect shell element that satisfies all the above requirements is yet to be developed. Even the successful element may show one or more defects in the aforementioned requirements. Each of the above requirements is discussed briefly in the following sub-sections.

Easy formulation –Linear combination of membrane and plate element

The major advantage of flat shell is simplicity in its formulation and computer implementation. In a flat shell approach, the element formulation can be separated into two independent formulations, i.e. the plate bending element formulation and the membrane element formulation, and then the two elements are combined together to form a flat shell element.

One of the effective ways to achieve an improvement of the isoparametric based element is the addition of various NC modes to the basic isoparametric displacement mode of both the plate bending and the membrane elements. The displacement field in an element can be expressed by

$$\boldsymbol{u} = \sum N_i \boldsymbol{u}_i + \sum \bar{N}_j \bar{\boldsymbol{u}}_j \tag{1}$$

where N_i are the shape functions, $\boldsymbol{u}_i$ are the conforming displacements, $\bar{N}_j$ are the additional NC modes, and $\bar{\boldsymbol{u}}_j$ are the additional unknowns corresponding to the additional displacement modes.

Those NC modes considered are as shown in Eq.(2).

$$\bar{N}_1 = 1-\xi^2, \qquad \bar{N}_2 = 1-\eta^2, \qquad \bar{N}_3 = (1-\xi^2)(1-\eta^2) \tag{2a}$$

$$\bar{N}_4 = (1-\xi^2)\eta, \qquad \bar{N}_5 = (1-\eta^2)\xi \tag{2b}$$

$$\bar{N}_6 = (1-\xi^2)\xi\eta, \qquad \bar{N}_7 = (1-\eta^2)\xi\eta \tag{2c}$$

Modes in Eq.(2a) are defined as 'basic NC modes' and are used in the formulation to achieve the basic improvement of various NC elements. They are particularly effective in improving the behavior of regular mesh. Eq.(2b) and Eq.(2c) are respectively designated as 'hierarchical NC modes' and 'higher order NC modes' (Choi and Lee 2002b). Combined with the modes in Eq.(2a), the hierarchical NC modes (Eq.(2b)) improves the element behavior further to the extent of improvement achieved by the Allman-type formulation (Allman 1988).

As there are no loads corresponding to the internal degrees of freedom $\bar{\boldsymbol{u}}$, the load–deflection equations may be partitioned as (Choi and Schnobrich 1975)

$$\begin{bmatrix} \boldsymbol{K}_{cc} & \boldsymbol{K}_{cn} \\ \boldsymbol{K}_{cn}^T & \boldsymbol{K}_{nn} \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ \bar{\boldsymbol{u}} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{f} \\ \boldsymbol{0} \end{Bmatrix} \tag{3}$$

The enlarged element stiffness matrix in Eq.(3) can be condensed back to the same size as the stiffness matrix of original element before the addition of NC modes by static condensation.

$$K = K_{cc} - K_{cn} K_{nn}^{-1} K_{cn}^{T} \tag{4}$$

A series of plate-bending elements (Choi and Lee 2000a) can be established according to the integration schemes used and the NC modes added in the formulation of submatrices in Eq.(4). The higher order NC modes $\bar{N}_6$ and $\bar{N}_7$ in the elements NPB4-II (4-node Non-conforming Plate-Bending element) and NPB4-III are first introduced by Choi and Lee (2002a). The element NPB4-I is formulated by the same integration scheme for both the bending and shear parts, whereas NPB4-II and NPB4-III which have the higher order NC modes ($\bar{N}_6$ and $\bar{N}_7$) are formulated by selective integration schemes for K_{cn} and K_{nn}. The '4-node Conforming Plate-Bending element' (CPB4), which is identical to MITC4 element (Bathe and Dvorkin (1985), is also included for comparison.

For the membrane component of the flat shell elements, NMD4-series elements (4-node Non-conforming Membrane element with Drilling degrees of freedom) are developed by Choi *et al.* (2002). The modes $\bar{N}_4$ and $\bar{N}_5$ in the element NMD4-III are also first used by Choi *et al.* (2002) and found to be highly effective in the elimination of locking for distorted elements.

Table 1 shows a series of flat shell elements (NFS4-I~VII) established by the linear combination of plate bending elements (NPB4-series) and membrane elements (NMD4-series) where 'NFS4' stands for '4-node Non-conforming Flat Shell element'. The NPB4-III element and the NMD4-III element show the best performance in plate bending and in-plane problems, respectively (Choi and Lee 2002b, Choi *et al.* 2002). Thus, the NFS4-IV element is judged to be the best flat shell element (Table 1). Hereafter the NFS4-IV element is designated in a simple manner as NFS4 in this study.

Table 1. Types of presented flat shell elements

Elements	Plate			Membrane		
	Elements	NC modes		Elements	NC modes	
		w	θ_x, θ_y		u, v	θ_z
NFS4-I	CPB4	-	-	NMD4-III	$\bar{N}_1, \bar{N}_2, \bar{N}_4, \bar{N}_5$	-
NFS4-II	NPB4-I	-	$\bar{N}_1, \bar{N}_2$			
NFS4-III	NPB4-II	-	$\bar{N}_1, \bar{N}_2, \bar{N}_6, \bar{N}_7$			
NFS4-IV	NPB4-III	-	$\bar{N}_1 \sim \bar{N}_3, \bar{N}_6, \bar{N}_7$			
NFS4-V	NPB4-I	-	$\bar{N}_1, \bar{N}_2$	NMD4-I	$\bar{N}_1, \bar{N}_2$	-
NFS4-VI	NPB4-II	-	$\bar{N}_1, \bar{N}_2, \bar{N}_6, \bar{N}_7$			
NFS4-VII	NPB4-III	-	$\bar{N}_1 \sim \bar{N}_3, \bar{N}_6, \bar{N}_7$			

High accuracy and fast convergence – Use of non-conforming modes

The performance of the conventional lower-order isoparametric elements is not so good since they are too stiff in bending due to the parasitic shear deformation and therefore the slow convergence is resulted. Among the techniques proposed for improvement of the basic behavior of this type of elements, the use of NC modes may be one of the most successful approaches that improve the element behavior and thus ensure the fast convergence of the analysis. The basic concepts and some details of this approach can be found in the references by Choi and his coworkers (Choi and Schnobrich 1975; Choi and Lee 2002a).

Insensitivity for mesh distortion – Combined use of hierarchical/higher order and basic NC modes

The element performance is generally at its best if its shape is compact and regular. However, it is also desirable to have an element that is insensitive to mesh distortions for the practical use. Thus the element can produce solutions without significant losses in accuracy even though the distorted mesh is used.

Choi and Lee (2002b) suggested that a more general configuration of deformation can be described for NC elements by the addition of hierarchical NC modes ($\bar{N}_4$ and $\bar{N}_5$) to the basic NC modes ($\bar{N}_1 \sim \bar{N}_3$). Choi *et al.* (2002) showed that the addition of hierarchical NC modes are very effective to improve further the behavior of distorted elements of the membrane and the hexahedral elements, especially when the selective integration techniques are used simultaneously. The similar combined effects for plate elements, in which the basic and higher order NC modes are used for rotational fields, can be expected.

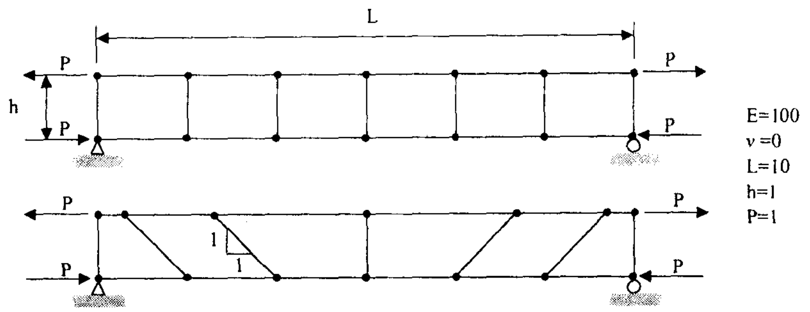


Figure 1. Higher order patch test

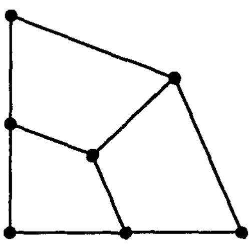
To show the behavior of highly distorted membrane elements with drilling DOF, a simple beam with a length to depth ratio of 10, subjected to a pure bending state, is modeled by one row of six elements for both the regular and distorted mesh as shown in Figure 1. The exact solutions are 1.5 and 0.6 for the displacement at center and the tip rotation at both ends, respectively. The numerical results are tabulated in Table 2, which reveals that the end rotations are sensitive to element distortion. When the basic NC modes ($\bar{N}_1$ and $\bar{N}_2$) are added, the behavior of elements ('NMD4-I' and 'NMD4-III') is significantly improved. Furthermore, for the case of severe distortion, the NMD4-III element produces highly accurate results by virtue of combined effect of basic and hierarchical NC modes and selective integration scheme.

Table 2. Results of higher order patch test

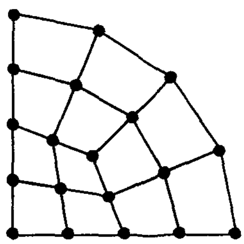
Mesh / Elements	Vertical displacement		End rotation		Normalized Values				Remark
					Vertical displacement		End rotation		
	R	D	R	D	R	D	R	D	
CMD4	0.547	0.222	0.210	0.095	0.365	0.148	0.350	0.158	Choi *et al.* (2002)
NMD4-I *	1.500	1.166	0.600	0.469	1.000	0.777	1.000	0.782	
NMD4-III **	1.500	1.382	0.600	0.590	1.000	0.921	1.000	0.983	
Groenwold and Stander (1995)	1.500	1.436	0.600	0.565	1.000	0.957	1.000	0.942	
Iura and Atluri (1992)	1.500	1.091	0.600	0.498	1.000	0.727	1.000	0.830	
Theory	1.5		0.6		1.000		1.000		

* Basic NC modes ** Basic and Hierarchical NC modes
R Regular mesh D Distorted mesh

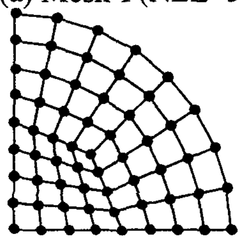
Figure 2 shows the typical example for the distorted mesh in which the symmetric quadrant of a clamped circular plate under uniform loading ($q = 100$) is idealized by meshes of different number of elements. The radius of the plate is 5 and thickness is 0.1, and the material properties are $E = 10.92$ and $\nu = 0.3$. The central displacements obtained by using various meshes are compared with the exact solution and results by other authors (Hinton and Huang 1986) in Table 3. The elements 'NPB4-II' and 'NPB4-III', which have higher order NC modes ($\bar{N}_6$ and $\bar{N}_7$) in addition to the basic NC modes, produced better results than those of other elements.



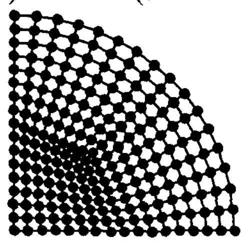
(a) Mesh-I (NEL=3)



(b) Mesh-II (NEL =12)



(c) Mesh-III (NEL =48)



(d) Mesh-IV (NEL =192)

Figure 2. Clamped circular plate

Table 3. Results of circular plate

Elements \ Mesh	Central deflections				Normalized Values				Remark
	Mesh -I	Mesh -II	Mesh -III	Mesh -IV	Mesh -I	Mesh -II	Mesh -III	Mesh -IV	
CPB4	9068	9693	9765	9779	0.9165	0.9797	0.9870	0.9884	
NPB4-I *	9280	9742	9777	9782	0.9380	0.9847	0.9882	0.9887	Choi and Lee (2002a)
NPB4-II **	9316	9752	9780	9783	0.9416	0.9857	0.9885	0.9888	
NPB4-III **	9316	9752	9780	9783	0.9416	0.9857	0.9885	0.9888	
Hinton and Huang (1986)	8894	9581	9738	N/A	0.8990	0.9684	0.9843	N/A	
Theory	9893.75				1.000				

* Basic NC modes ** Basic and Higher order NC modes

When the shape of element is distorted, it is shown from the numerical tests (Table 2, Table 3) that the addition of hierarchical and higher order NC modes in addition to basic NC modes is highly effective to improve the behaviors of elements.

Free from patch test failure – Direct Modification Method

As seen in the previous two sections, the use of NC modes improves the element behavior significantly. However, at the same time it may create another problem that the resulting elements do not always pass the patch test (Choi *et al.* 2001). Therefore, it is necessary to use some modification schemes for NC modes in order to obtain the elements that always pass the patch tests. The recent approach, 'Direct Modification Method (DMM)' set the NC modes entirely free from patch test failure with less computing time than 'B-bar method'. The fundamental concept of this method and the applications to 4-node membrane elements and 8-node hexahedral elements with drilling degrees of freedom can be found in the published literatures (Choi *et al.* 2001; Choi *et al.* 2002). This method is also adopted in this paper.

No shear locking –Substitute shear strain field

Most Reissner-Mindlin type elements become very stiff when used to model thin structures. A lot of effort has been devoted to identify and eliminate the source of this 'shear locking' problem. One of the successful techniques for solving the problems of shear locking is to use of the constrained substitute shear strain fields (Bathe and Dvorkin 1985; Hinton and Huang 1986; Oñate *et al.* 1992). In this method, the standard Mindlin plate theory is used to calculate the bending stiffness matrix except the way transverse shear strains are obtained. The plate-bending elements developed by Choi and Lee (2002a) are used as a plate bending component of flat shell element in this study. The shear strain matrix of these elements are substituted by the assumed shear strain matrix by mixed formulation (Oñate *et al.* 1992). Details can be found in the reference (Choi and

Lee 2002a).

No Membrane locking –Hierarchical NC modes

Problems related to membrane locking of flat shell elements were discussed by Taylor (1987), Cook (1994), Groenwold and Stander (1995), Sydenstricker and Landau (2000), and Providas and Kattis (2000). Recently, Choi and Lee (2003) suggested an efficient way to remove membrane locking using hierarchical and higher order NC modes. The latest study for removal of membrane locking is adopted in this paper. Detailed discussions and numerical validations can be found in the reference (Choi and Lee 2003).

Six DOF per node –Drilling degrees of freedom

A flat shell element is obtained with a relative ease by combining a membrane element for plane elasticity and a bending element for flat plates. Therefore, for a 6 DOF per node finite element formulation, it is required for membrane element to have drilling degrees of freedom. The presence of the sixth degree of freedom (or drilling degree of freedom) in shell analysis completes the shell theory and gives significant advantages such as an easy construction of structural models for the ridge-like connections of folded plates and beam-membrane connections.

Numerous efforts have been made to develop such membrane elements with drilling degrees of freedom. Allman (1988) derived a displacement function with a corner rotation taken as an independent degree of freedom. In this case the rotational degrees of freedom actually induce in-plane deformation. An approach to derive a functional in which the drilling rotations are introduced as variables independent of the in-plane deformation were used by Reissner (1965), Iura and Atluri (1992), and Choi *et al.* (2002). The membrane elements with drilling degrees of freedom developed by Choi *et al.* (2002) based on the aforementioned approach were used for the membrane part of quadrilateral flat shell elements in this study.

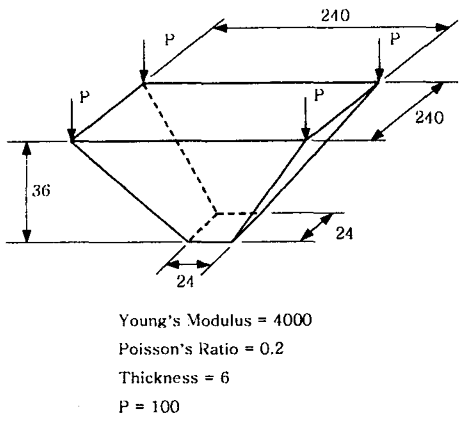


Figure 3. Faceted capital

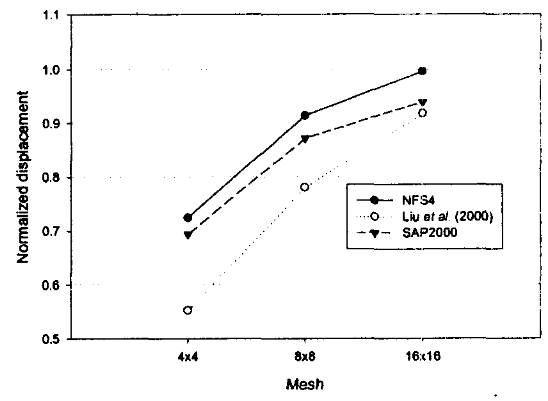


Figure 4. Results of faceted capital

The column capital shaped structure constructed with four panels as depicted in Figure 3 was analyzed by Liu *et al.* (2000). In consideration of symmetry of the structure, only a quarter of the structure is actually analyzed. The 24x24 square column (Figure 3) connected to the capital is replaced by zero-displacement boundary conditions and the rotation of panel edges parallel to the column edges is free. The top of the capital is free and concentrated loads with a magnitude of 100 are applied at the four top corners. A panel is meshed by N x N elements where N varies from 4 to 16. Figure 4 shows the normalized displacements in direction of the load and a reference value is taken as 1.2375 obtained by Liu *et al.* (2000) from 64x64 mesh. The results from commercial software SAP2000 are also listed for comparison. The necessity of drilling degrees of freedom is well demonstrated from this problem.

No mechanisms –Modified Reduced Integration

The modified reduced integration scheme is adopted for the membrane element for removing spurious zero energy mode. Since the hierarchical NC modes have higher order terms, the element NMD4-II formed by reduced integration is rank deficient and shows the spurious zero energy mode. A simple modification of integration scheme in forming NMD4-III, i.e., the use of 5-point integration scheme (Dovey 1974) for the integration of submatrix $\bar{\boldsymbol{B}}^{*T}\boldsymbol{D}\bar{\boldsymbol{B}}^{*}$ (Choi *et al.* 2002), removes the spurious zero energy mode.

The main advantage of the plate-bending elements based on the assumed strain fields is that these elements have proper rank and therefore, no spurious zero energy modes are resulted as in the series of NPB4 elements in this study.

In order to check the presence of spurious zero energy modes, the eigenvalue analyses of a single unconstrained stiffness matrix of the current plate and flat shell element have been also performed. All the presented plate-bending elements and flat shell elements have exactly three and six zero eigenvalues associated with the rigid-body modes, respectively. Thus, all the flat shell elements presented in this study have been proved not to exhibit any spurious mechanism.

Warped Geometry –Rigid link correction

One last problem in a series in this paper is the problem of the inclusion of the warped geometry effects due to the fact that nodes of a 4-node flat shell element in a warped mesh are not coplanar in general. To solve this problem a flat shell element needs to be modified as its formulation is based on the flat geometry and the membrane and plate bending characteristics of the element are uncoupled. In order to take into account the warped geometry, the rigid link correction is simple and easy to use (Taylor 1987). To evaluate the performance and applicability of the presented elements (Table 1) in warped mesh, a twisted cantilever beam of rectangular cross section is tested. This cantilever beam is twisted 90° over its length, and subjected to a concentrated unit load at its free end. The geometry and loading conditions of this example are depicted in Figure 5. The element without using any correction scheme for warped geometry cannot solve this example correctly. The reference solutions in the case of in-plane load and of out-of-plane load are 1.3857 and 0.3427, respectively (Jetteur 1986). The properties and dimensions used are given as Young's modulus $E = 29.0\times10^6$, Poisson's ratio $\nu = 0.22$, thickness $t = 0.05$, side length $L = 12$, and concentrated load $F = 1.0$. All the elements presented in this study show good performance. From these results, the effectiveness of the 'rigid link correction' for the elements in warped geometry is well demonstrated.

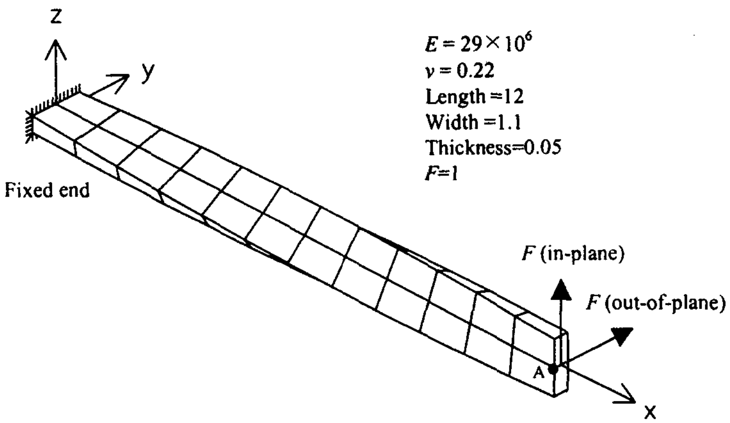


Figure 5. Twisted cantilever beam

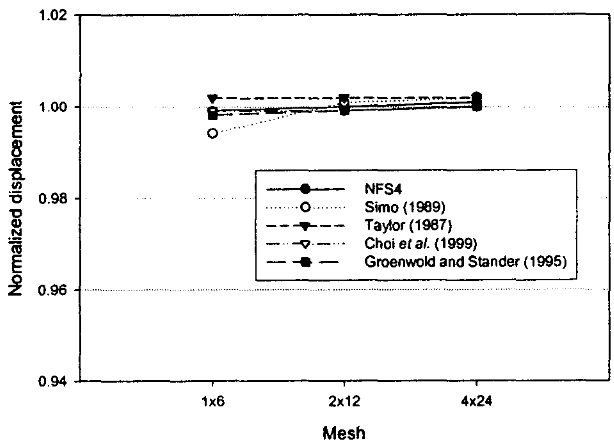


Figure 6. Results of twisted cantilever beam (in-plane)

Practical problems

Scodelis-Lo roof shell

The Scodelis-Lo cylindrical roof subjected to self-weight load is depicted in Figure 7. Membrane contribution to deformation is significant in this problem. An analytic solution for the transverse displacement at the center of the edge (A), as reported by MacNeal and

Harder (1985) is 0.3024. In consideration of symmetry of the structure, only a quarter of the roof is analyzed. Normalized values for the transverse displacement at A are listed in Table 4. The results from other researcher's are also listed for comparison.

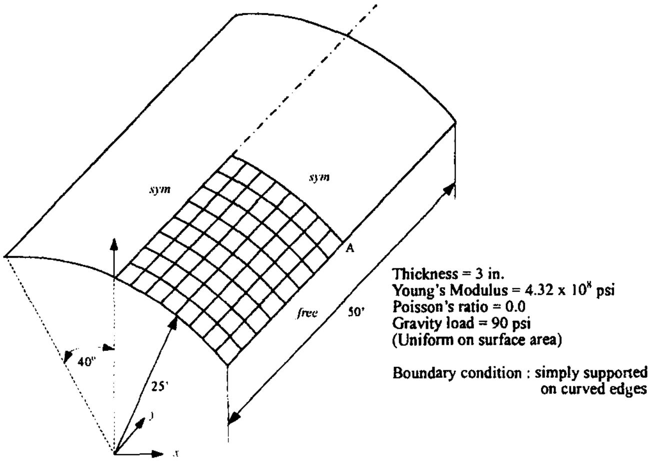


Figure 7. Scodelis-Lo roof shell

Table 4. Results of Scodelis-Lo roof shell

Elements	Displacements at A			Normalized Values		
	4x4	8x8	16x16	4x4	8x8	16x16
NFS4	0.31674	0.30397	0.30159	1.0474	1.0052	0.9973
Belytshko *et al.* (1989)	0.29151	0.29151	0.30210	0.9640	0.9640	0.9990
Bathe and Dvorkin (1985)	0.28547	0.29424	0.29907	0.9440	0.9730	0.9890
Choi and Paik (1994)	0.31571	0.30300	0.30089	1.0440	1.0020	0.9950
Reference	0.3024			1.000		

Box with holes

A box with holes under two opposite concentrated loads at the centers is analyzed (Figure 8). Using the symmetry, a one-eighth segment is actually analyzed. The properties used are as follow: $E = 2{,}100{,}000$, Poisson's ratio $\nu = 0.18$, and concentrated load intensity $P = 2{,}000$. Figure 9 shows the gradually refined meshes obtained under the adaptive process with a prescribed permissible relative error of 7%.

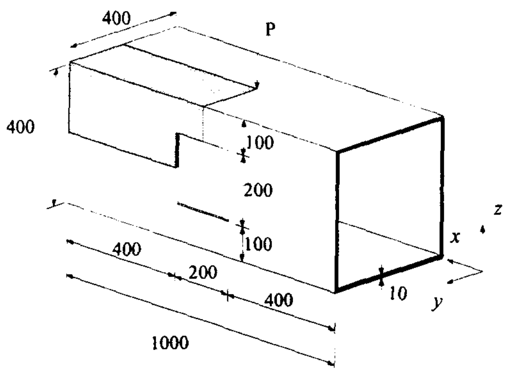


Figure 8 . Configuration of box with holes

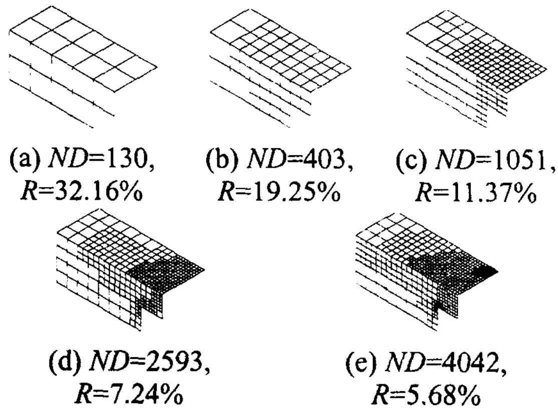


Figure 9. Sequence of refinements

In-ground LNG storage tank

Figure 10 shows the analysis model for LNG storage tank made of various types of elements, i.e. 3D shell elements and 3D solid elements. In the 3D shell model for tank structures, roof shell and side wall are modeled with shell elements while the bottom slab is modeled with 3D solids because of the large thickness of bottom slab. Variable-node shell elements are effectively used in parts A, B and C. The connections of shell structures (parts A and C) can be modeled without any other additional treatment such as the use of rigid area providing refined mesh in the connection area at the same time. Part B is the typical example of the effective use of variable-node elements to fix up the well-known defect of predicted stiffness at the top of roof shell where many elements share the same node. Figure 11 shows the results of analysis of the storage tank under ground water pressure when the large water pressure is applied on the surface of the side wall. The refined mesh using the variable-node elements in corbel pick up the abrupt change of moment distribution effectively.

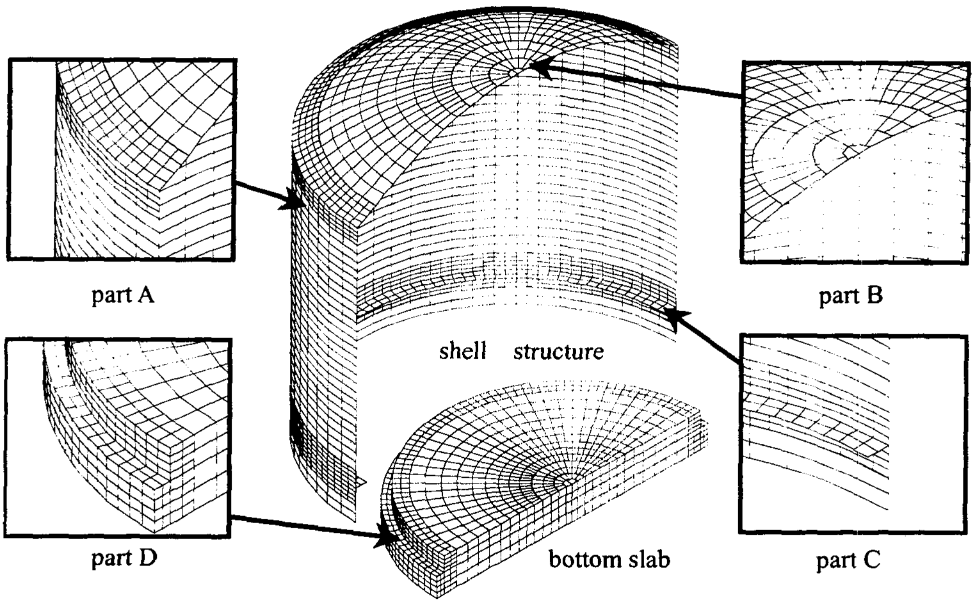


Figure 10. LNG storage tank

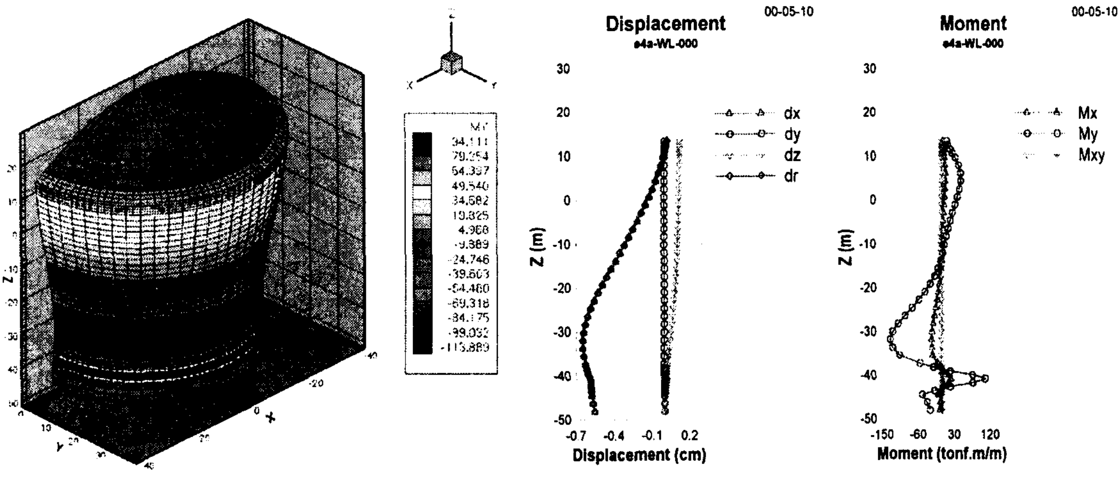


Figure 11. Deformation (scale = 1000) and member forces

Conclusions

In this study, the problems involved in development of the quadrilateral flat shell elements and their possible solutions are discussed. All the existing schemes and a few newly introduced schemes to solve the problems are utilized in an integrated form in the development of a series of high performance flat shell elements. The new schemes introduced in this paper are: (1) the addition of the hierarchical and/or higher order NC modes to the basic NC modes to form the element shape functions and (2) the application of Direct Modification Method (DMM) to the flat shell formulation for the purpose of guaranteeing the element to pass patch tests. These two schemes are successfully incorporated into the existing schemes.

The NC modes are used as a key scheme to achieve the high accuracy and fast convergence of the flat shell element. When the basic NC modes are combined with hierarchical and/or higher order NC modes, the elements become insensitive to mesh distortions and in addition produce the high accuracy and fast convergence of solution. One of the major problems of the NC elements, i.e. the failure of patch test due to the energy variation caused by addition of NC displacement modes, can be effectively solved by using DMM.

The use of Reissner-Mindlin thick plate theory combined with the use of NC modes makes the element more versatile in application to both thin and thick plate/shell problems. When the substitute shear strain fields are used, the shear locking phenomenon disappears. When the membrane component of a flat shell has the hierarchical NC modes in addition to the basic NC modes, the membrane locking can be suppressed. The six degree-of-freedom elements are easily established from membrane element with drilling degrees of freedom based on the functional in which the drilling rotations are introduced as independent variables. The modified integration schemes can be effectively used to remove the spurious zero energy modes of membrane elements discussed herewith. A flat shell element can be used for warped geometry by the use of rigid link correction. The validity of statements in this conclusion is verified by numerical validation. In addition, numerical examples show that a wide range of the folded plate and curved shell problems can be effectively solved by the presented flat shell elements.

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