

Analysis of Metal Forming Process Using Meshfree Method

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무요소법에 의한 금속성형공정의 해석

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ABSTRACT

Meshfree approximations exhibit significant potential to solve partial differential equations. Meshfree methods have been successfully applied to various problems which the traditional finite element methods have difficulties to handle, including the quasi-static and dynamic fracture, large deformation problems, contact problems, and strain localization problems. A meshfree method based on the reproducing kernel particle approximation(RKPM) is applied to sheet metal forming analysis in this research. Metal forming examples, such as stretch forming and flanging operation, are analyzed to demonstrate the performance of the proposed meshfree method for largely deformed elasto-plastic material.

Key Words : Meshfree Method, Sheet Metal Forming, RKPM

1. Introduction

Despite its success in the analysis of geometrically and materially nonlinear response, the most widely used finite element methods(FEM) in engineering and science are not suitable for problems such as large deformation, high gradients, strain concentration, grain boundary migration, crack propagation and problems associated with frequently remeshing. These reasons are partially due to the regularity requirement of meshes. Furthermore, mesh generation is a very difficult and lack of robust and efficient 3D mesh generators makes the solution of 3D problems a time-consuming task. To avoid these drawbacks of FEM, so-called meshfree(or meshless) methods have been developed during the past 10 years and in recent years, there has been a growing interest in meshfree methods.

A number of meshfree methods have been developed to deal with these problems successfully by constructing the approximation functions entirely in terms of particles. The essential feature of these

meshfree methods is that the discrete model is completely described by particles. Other noticeable characteristics of the methods are the smooth approximation, ease of adaptability, robustness to large deformations, and robustness to irregularity of particle distributions. Among the meshfree methods, reproducing kernel particle methods(RKPM)¹⁻⁹⁾ appears prominent for its sound mathematical foundation and high accuracy.

In this paper, we employ RK approximation to formulate the discrete nonlinear equilibrium equations, and frictional contact conditions. The basic theory of RKPM is reviewed and we present the application of RKPM to the metal forming problems for largely deformed elasto-plastic material.

2. Theory

2.1 Construction of One-Dimensional RKPM Basic Function

Consider the following kernel estimate of a function $u(x)$:

$$u^k(x) = \int_{-\infty}^{\infty} \Phi_a(x-s)u(s)ds \quad (2.1)$$

where $u^k(x)$ is the kernel estimate of $u(x)$, and $\Phi_a(x-s)$ is the kernel function with the support measure of 'a'. In general, a is defined so that it determines the domain of influence $\Phi_a(x)$ to the neighborhood of $s=x$.

If the kernel function is a Dirac delta function, the $u^k(x)$ exactly generates $u(x)$. In practice, however, the domain is finite in structural problems, and the Dirac delta function is difficult to deal with numerically. Therefore, for a bounded domain, Eq.(2.1) is rewritten by

$$u^k(x) = \int_{\Omega_x} \Phi_a(x-s)u(s)ds \quad (2.2)$$

where $\Phi_a(x-s)$ is a positive function with the following properties:

$$\int_{\Omega_x} \Phi_a(x-s) = 1 \quad (2.3)$$

$$u^x(x) \rightarrow u(x) \text{ as } a \rightarrow 0 \quad (2.4)$$

In fact, a zero-th consistency condition (Eq.(2.3)) can be easily satisfied by the normalization of the kernel function. However, when the domain of interest is finite, Eq.(2.3) does not assure the consistency condition in the discrete form. To study this problem, Liu et al.¹⁾ investigated the reproductivity of kernel estimate using a Taylor series expansion of the function $u(s)$ around x .

Liu et al.¹⁾ introduces a correction function to the kernel estimate:

$$u^R(x) = \int_{\Omega_x} C(x;x-s)\Phi_a(x-s)u(s)ds \quad (2.5)$$

where $u^R(x)$ is the "reproduced" function of $u(x)$, $C(x;x-s)$ is called the correction function that is to be constructed to fulfill reproducing conditions, and Eq.(2.5) is the reproducing kernel approximation, or the reproducing equation.

Eq.(2.5) can be rewritten in the following form,

$$u^R(x) = \int_{\Omega_x} \Phi_a(x;x-s)u(s)ds \quad (2.6)$$

where $\Phi_a(x;x-s) = C(x;x-s)\Phi_a(x-s)$ is called the reproduced kernel. Since Eq.(2.6) exactly reproduces N-th order polynomials, the method fulfills the N-th order consistency conditions.

2.2 Discretization of Reproducing Kernel Approximation

The discretized reproducing equation is obtained by performing numerical integration in Eq.(2.6).

The discrete reproducing conditions are preserved if the numerical integration is consistent with the discretization of the reproducing equation. Since the discretization of the continuous reproducing equation is to obtain the shape functions, the weight of the discretization is set to unity for simplicity.

3. Meshfree Formulation in Elasto-plastic Material with Contact Conditions

Contact conditions are included to handle contact between tools and workpiece. The classical Coulomb law is used to model frictional contact and the penalty method is applied to assure impenetration. The contact traction's t_n and t_t in the normal and tangential directions, respectively, are defined as follows:

$$t_n = -\alpha_n g_n \quad (2.7)$$

$$-\alpha_t g_t \text{ if } |\alpha_t g_t| \leq |\mu_f t_n| \text{ (stick conditions)}$$

$$t_t = -\mu_f t_n \text{sgn}(g_t) \text{ otherwise (slip conditions)} \quad (2.8)$$

where μ is the coefficient of friction, α_n and α_t are the normal and tangential penalty numbers, and g_n and g_t are normal and tangential gaps between contact surfaces.

4. Numerical Examples

4.1 Sheet Metal Forming by a Cylindrical Punch

The numerical results from the RKPM are compared with the analytical solutions. A plane-strain sheet metal is stretched by a cylindrical punch as shown in Fig.1 This problem is recommended as a benchmark test of sheet metal forming processes.

In this problem, the sheet metal forming process is considered to be quasi-static, and punch and die are assumed to be perfectly rigid.

The dimension of the problem are $R_p=50.8\text{mm}$, $C_d=59.18\text{mm}$, $R_o=61.30\text{mm}$, $R_d=6.35\text{mm}$, and $h=1.0\text{mm}$. The constitutive law of sheet metal is described using a J_2 plasticity with material constants : Young's modulus $E=69\text{GPa}$, Poisson's ratio $\nu=0.3$, isotropic hardening $\sigma_y(\epsilon^p)=589(10^{-4} + \epsilon^p)^{0.216}$ MPa, and coefficient of friction $\mu=0$. Due to symmetry, only half of the sheet metal is modeled with 4×51 particles and 3×50 integration zones, and Gauss integration order of 4×4 is used. Relatively dense particles are distributed around the die corners in order to capture stress concentrations in those areas. In this analysis, the end of the sheet metal is fixed, and the rigid punch is moved downward with a vertical displacement of 30mm in 50 incremental steps. Reproducing kernel contact formulation and kinematic constraints treatments¹⁰⁾ are employed for the contact analysis.

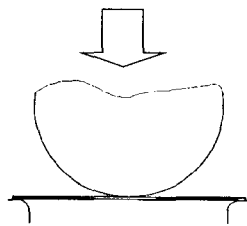


Fig.1 Schematic drawing of plain-strain cylindrical punch problem

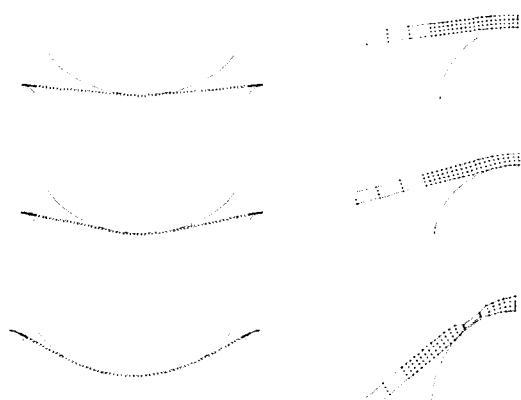


Fig.2 Progressive deformation of a cylindrical punch

The progressive deformation of the sheet metal is shown in Fig.2, and local necking are observed near the die contact areas.

4.2 Springback of a Sheet Metal in Flanging

A straight flanging operation and its springback behavior of a sheet metal is simulated, and the predicted springback angle is compared with experimental data reported in Song et al¹¹⁾. The blank is 150mm in length, 150mm in width and 1mm in thickness. The design parameters of a flanging operation are shown in Fig.3, where the flange length $L=20\text{mm}$, die radius $R=3\text{mm}$, and three different gaps, $G=1.2, 1.6,$ and 2.0mm are considered in this analysis. The material properties are Young's modulus $E=70\text{GPa}$, Poisson's ratio $\nu=0.3$, isotropic hardening $\sigma_y(\epsilon^p) = 146 + 500 \epsilon^p$ MPa.

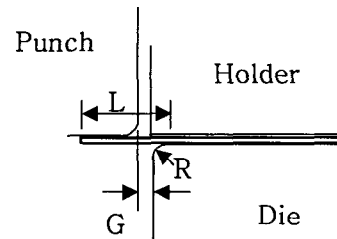
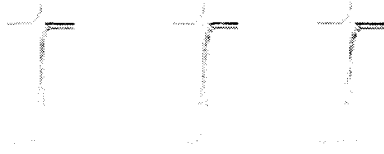


Fig.3 Geometry parameters of flanging problem and description

In meshfree discretization, $3 \times 101, 4 \times 116,$ and 5×131 nodes with three shape function support sizes $r=1.2, 2.0,$ and 3.0 are used and compared. The meshfree results show good agreement with experimental data, where the springback angle increases as the gap dimension increases. The results also demonstrate that the meshfree discretization with larger shape function support size provides a better agreement with the experimental data. Typical flanging progressive deformations using 5×131 nodes and shape function normalized support size of 2.0 are obtained. The comparison of deformation and springback under different gap dimensions are displayed in Fig.4



(a) gap=1.2mm (b)gap=1.6mm (c)gap=2.0mm

Fig.4 Effect of gap dimension on springback

5. Conclusions

A meshfree formulation for loading history-dependent material behavior and frictional contact conditions is developed based on the Reproducing Kernel Producing Method(RKPM) for the metal forming simulation.

The emphasis is on the meshfree treatment of large plastic deformation and complicated contact conditions. The numerical examples show that no mesh distortion difficulties in the finite element analysis are encountered by usage of a smooth kernel function with flexibly adjustable support size.

Due to the use of the Lagrangian reproducing kernel shape functions, the support size of the kernel functions does not require readjustment during the contact computation and the large plastic deformation induced in the metal forming process can be dealt with easily by the proposed method.

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