The Analysis of Chaotic Behavior in the Chaotic Robot with Hyperchaos Path of Van der Pol(VDP) Obstacle

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ABSTRACT

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Chua's equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In the obstacle, we only assume that all obstacles in the chaos trajectory surface in which robot workspace has an unstable limit cycle with Van der Pol equation

KEYWARD

chaos, Chua's circuit, mobile robot, Lyapunov Exponent

I . Introduction

Chaos theory has been drawing a great deal of attention in the scientific community for almost two decades. Remarkable research efforts have been spent in recent years, trying to export concepts from Physics and Mathematics into the real world engineering applications. Applications of chaos are being actively studied in such areas as chaos control [1-2], chaos synchronization and secure/crypto communication [3-7], Chemistry [8], Biology [9], and robots and their related themes [10].

Recently, Nakamura, Y. et al [10] proposed a *chaotic mobile robot, where a mobile robot is* equipped with a controller that ensures chaotic motion and the dynamics of the mobile robot is represented by Arnold equation. They applied obstacle with chaotic trajectory, but they have not mentioned about the chaotic behavior except Lyapunov exponent.

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Hyperchaos equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In order to avoid obstacles, we assume that all obstacles in the chaos trajectory surface have an unstable limit cycle with Van der Pol equation. When chaotic wandering robots meet obstacles in the hyperchaos trajectory, which is derived using hyperchaos circuit equation synthesized with Chua's equation or SC-CNN, robots are led to the distant region from obstacles because obstacles have unstable limit cycle with Van der Pol equation

II. Chaotic Mobile Robot embedding Chaos Equation

2.1 Mobile Robot

As the mathematical model of mobile robots, we assume a two-wheeled mobile robot as shown in Fig. 1.

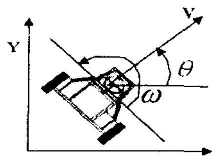


Fig. 1 Two-wheeled mobile robot

Let the linear velocity of the robot v[m/s] and angular velocity ω[rad/s] be the input to the system. The state equation of the four-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where (x,y) is the position of the robot and ω is the angle of the robot.

2.2 Hyperchaos Equation

To create a hyperchaos circuit, we use the n-double scroll using the weak coupling method[15]

1) n-double scroll circuit

In order to synthesize a hyperchaos circuit, we first consider Chua's circuit modified to an n-double scroll attractor. The electrical circuit for obtaining n-double scroll, according to the implementation of Arena et al. [13] is given by

$$\begin{aligned} \dot{x} &= \alpha[y - h(x)] \\ \dot{y} &= x - y + z \\ \dot{z} &= -\beta y \end{aligned} \quad (2)$$

with a piecewise linear characteristic

$$h(x) = m_{2n-1}x + \frac{1}{2}\sum_{i=1}^{2n-1}(m_{i-1} - m_i)(|x + c_i| - |x - c_i|) \quad (3)$$

consisting of 2(2n-1) breakpoint, where n is a natural number. In order to generate n double scrolls one takes α=9 and β=14.286. Some special cases are:

1-double scroll

$m_0 = -1/7,\ m_1 = 2/7,\ c_1 = 1$

2-double scroll

$m_0 = -1/7,\ m_1 = 2/7,\ m_2 = -4/7,$
$m_3 = m_1,\ c_1 = 1,\ c_2 = 2.15,\ c_3 = 3.6$

3-double scroll

$m_0 = -1/7,\ m_1 = 2/7,\ m_2 = -4/7,$
$m_3 = m_1,\ m_4 = m_2,\ m_5 = m_3,\ c_1 = 1,$
$c_2 = 2.15,\ c_3 = 3.6,\ c_4 = 8.2,\ c_3 = 13$

3) Hyperchaos Circuit Equation

To synthesize a hyperchaos circuit, we consider one-dimension Cellular Neural Network (CNN) with n double scroll cell[ ]. The following equations describe a one-dimensional CNN consisting of identical n-double cell with diffusive coupling as

$$\begin{aligned} \dot{x}^{(j)} &= \alpha[y^{(j)} - h(x^{(j)})] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} \\ \dot{z}^{(j)} &= -\beta y^{(j)}, \quad j = 1,2,\ldots,L \end{aligned} \quad (4)$$

or

$$\begin{aligned} \dot{x}^{(j)} &= \alpha[y^{(j)} - h(x^{(j)})] \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{z}^{(j)} &= -\beta y^{(j)}, \quad j = 1,2,\ldots,L \end{aligned} \quad (5)$$

where L denotes the number of cells. We impose the condition that $x^{(0)} = x^{(L)},\ x^{(L+1)} = x^{(1)}$ for equation (7) and (8). For the coupling constants, $K_0 = 0, K_j = K (j = 1, \ldots, L-1)$ and positive diffusion coefficients D_x, D_y are chosen base on stability theory

2.3. Embedding of Hyperchaos circuit in the Robot

In order to embed the hyperchaos equation into the mobile robot, we define and use the Hyperchaos equation as follows.

$$\begin{aligned} \dot{x}^{(j)} &= \alpha(y^{(j)} - h(x^{(j)} + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ \dot{y}^{(j)} &= x^{(j)} - y^{(j)} + z^{(j)} \\ \dot{z}^{(j)} &= -\beta y^{(j)} \\ \dot{x} &= v\cos x_3 \\ \dot{y} &= v\sin x_3 \end{aligned} \quad (6)$$

Using equation (6), we obtain the embedding hyperchaos *robot trajectories* .

Fig. 2 shows the phase plane of the incremental component of the mobile robot.

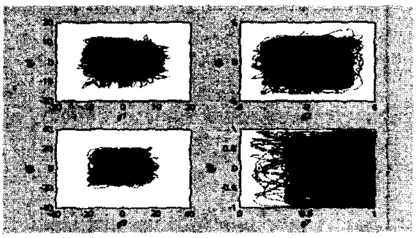


Fig. 2 Phase portrait of variation component (dx, dy, dz) trajectories in Chua's circuit.

2.4 Mirror Mapping

Basically, equation (6) is assumed that the mobile robot moves in a smooth state space without boundary. However, real robot moves in space with boundary like walls or surfaces of obstacles. To avoid boundary or obstacle, we consider mirror mapping when the robot approach walls or obstacles using the Eq. (6) and (7). Whenever the robot approaches a wall or obstacle, we calculated the robot new position

$$A=\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \quad (7)$$

$$A=\frac{1}{1+m}\begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (8)$$

We can use equation (7) when slope is infinitive such as and also use equation (8) when slope is not infinitive.

Fig. 3 Mirror mapping

III. The Chaotic Behavior of embedding Chaos Robot with obstacle

3.1 Fixed obstacle

In this section, we will study the chaotic behavior of a chaos robot with mirror mapping relay on Hyperchaos equation.

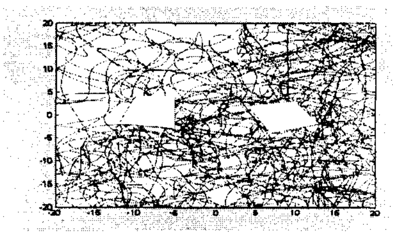


Fig. 4 Trajectories of chaos robot with obstacle embedding Hyperchaos equation

Fig. 4 show the trajectory of chaos robot can avoid obstacles to which mirror- mapping is applied by Eq (6) and (7).

3.2. VDP equation as a obstacle

In order to represent obstacle of the mobile robot, we employ the VDP, which is written as follows:

$$\begin{aligned} \dot{x}&=y \\ \dot{y}&=(1-y^2)y-y \end{aligned} \quad (8)$$

From equation (8), we can get the following limit cycle such as Fig. 5

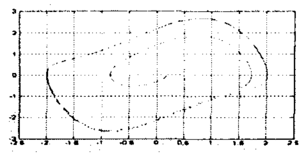


Fig. 5 Limit cycle of VDP

In Fig. 10, computer simulation result show that 1 VDP obstacle at the origin and 1 robot of Hyperchaos equation is working well.

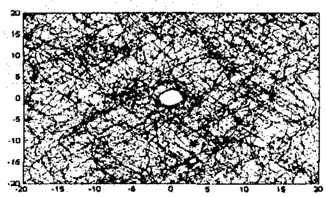


Fig. 6 Computer simulation result of obstacle avoidance with 1 robot and 1 obstacle in Hyperchaos equation

IV. Chaotic Behavior Analysis in the Mobile Robot with Hyperchaotic Path

4.1. Embedding Method

In order to reconstruct phase plane from data of robot's single variable, we applied an embedding method proposed by Takens [12]. The embedding method is referring to the process in which a representation of the attractor can be constructed from a set of scalar time-series. The form of such reconstructed state is given as follows:

$$X_t=[x(t),x(t+\tau),\ldots\ldots,x(t+(m-1)\tau)] \quad (9)$$

Where is a robot trajectory data, is a delay time, and is an embedding dimension. It is significant factor to get reasonable embedding phase plane. In chaos mobile case, we choose is 400 using an auto-correlation time and is chosen 5 because nearest false neighbor disappears in that dimension. Fig. 7 shows the timeseries of chaos robot from equation (5)

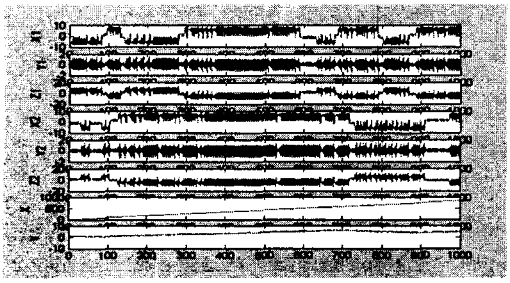
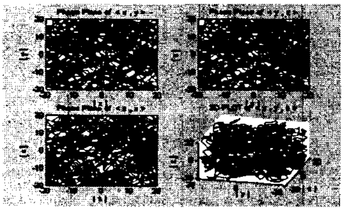


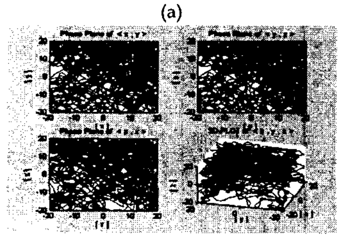
Fig. 7 Time-series of chaotic robot with hyperchaotic path

4.2. Qualitative Analysis

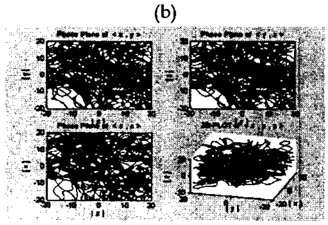
With reconstructed state, the qualitative chaotic degree of chaotic robot path is analyzed in this section using embedding phase plane. Fig. 8 shows phase plane of these embedding state which are originally robot paths when robot has a (a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.



(a)



(b)

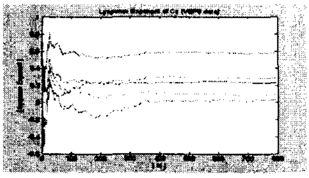


(c)

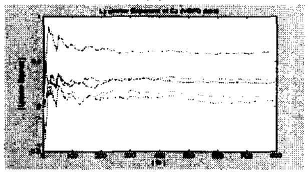
Fig. 8 Reconstructed phase plane(a) no obstacle, (b) fixed obstacle, and (c) VDP obstacle.

4.3. Quantitative Analysis

In this section, we evaluate Lyapunov spectrum [13] in the mobile robot as a quantitative chaos analysis and show the result in Fig 9.



(a)



(b)

Fig. 9 Lyapunov spectrum of mobile robot (a) without obstacle, (b) with obstacle

Ⅴ. Conclusion

In this paper, we propose that the chaotic behavior analysis in the mobile robot of embedding Chua's equation with obstacle. In order to analysis of chaotic behavior in the mobile robot, we apply not only qualitative analysis such as time-series, embedding phase plane, but also quantitative analysis such as Lyapunov exponent in the mobile robot with obstacle. In the obstacle, we only assume that all obstacles in the chaos trajectory surface in which robot workspace has an unstable limit cycle with Van der Pol equation.

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